国外数学名著系列

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Jacob Korevaar

Tauberian Theory

A Century of Developments

陶伯理论

百年进展

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《国外数学名著系列》(影印版)序

要使我国的数学事业更好地发展起来，需要数学家淡泊名利并付出更艰苦地努力。另一方面，我们也要从客观上为数学家创造更有利的发展数学事业的外部环境，这主要是加强对数学事业的支持与投资力度，使数学家有较好的工作与生活条件，其中也包括改善与加强数学的出版工作。

从出版方面来讲，除了较好较快地出版我们自己的成果外，引进国外的先进出版物无疑也是十分重要与必不可少的。从数学来说，施普林格（Springer）出版社至今仍然是世界上最具权威的出版社。科学出版社影印一批他们出版的好的新书，使我国广大数学家能以较低的价格购买，特别是在边远地区工作的数学家能普遍见到这些书，无疑是对推动我国数学的科研与教学十分有益的事。

这次科学出版社购买了版权，一次影印了 23 本施普林格出版社出版的数学书，就是一件好事，也是值得继续做下去的事情。大体上分一下，这 23 本书中，包括基础数学书 5 本，应用数学书 6 本与计算数学书 12 本，其中有些书也具有交叉性质。这些书都是很新的，2000 年以后出版的占绝大部分，共计 16 本，其余的也是 1990 年以后出版的。这些书可以使读者较快地了解数学某方面的前沿，例如基础数学中的数论、代数与拓扑三本，都是由该领域大数学家编著的“数学百科全书”的分册。对从事这方面研究的数学家了解该领域的前沿与全貌很有帮助。按照学科的特点，基础数学类的书以“经典”为主，应用和计算数学类的书以“前沿”为主。这些书的作者多数是国际知名的大数学家，例如《拓扑学》一书的作者诺维科夫是俄罗斯科学院的院士，曾获“菲尔兹奖”和“沃尔夫数学奖”。这些大数学家的著作无疑将会对我国的科研人员起到非常好的指导作用。

当然，23 本书只能涵盖数学的一部分，所以，这项工作还应该继续做下去。更进一步，有些读者面较广的好书还应该翻译成中文出版，使之有更大的读者群。

总之，我对科学出版社影印施普林格出版社的部分数学著作这一举措表示热烈的支持，并盼望这一工作取得更大的成绩。

王　元

2005 年 12 月 3 日

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To Pia

Preface

Summability methods have been used at least since the days of Euler to assign a reasonable sum to an infinite series, whether it is convergent or not. In its simplest form, Tauberian theory deals with the problem of finding conditions under which a summable series is actually convergent. A first condition of this kind, which applies to Abel summability (the power series method), was given by Alfred Tauber in 1897. However, Tauberian theory began in earnest only around 1910 with the work of Hardy and Littlewood. Over a period of thirty years they obtained a large number of refined 'Tauberian theorems', and they gave the subject its euphonious name.

A summability method for a series typically involves an averaging process of the partial sums. The step from summability to convergence requires a reversal of the averaging. For this one generally needs an additional condition on the series, known as a Tauberian condition. There is an endless variety of summability methods, and a corresponding variety of possible Tauberian theorems. However, the subject acquires a certain unity by similarities among optimal Tauberian conditions. One may also note the frequent appearance of the Riemann–Lebesgue lemma.

The aim of the book is to treat the principal Tauberian theorems in various categories and to provide attractive proofs. We sometimes use more than one approach and occasionally generalize the results in the literature.

The arrangement of the material roughly follows the historical development. The first three chapters deal with the basic theory. Chapter I describes the major Tauberian results of Hardy and Littlewood. They involve power series and related transforms. Over the years, many of the difficult original proofs have been simplified considerably. Karamata's surprising approach by polynomial approximation receives ample attention. The famous 'high-indices theorem', which involves lacunary series with 'Hadamard gaps', is treated by a variation on Ingham's peak function method. The proof of some other difficult theorems is postponed till later chapters.

An important impulse for Tauberian theory came from number theory, in particular, the search for relatively simple proofs of the prime number theorem. In this area the Tauberian work of Hardy and Littlewood had not been definitive. The unsatisfactory situation was one of the factors that led Wiener to his comprehensive Tauberian theory of 1932. Here the central theme is the comparison of different limitation methods. In Chapter II, Wiener's theory is developed on the basis of a convenient testing equation, which is treated both classically and by the author's distributional method. With Wiener's theory in hand we describe several paths to the prime number theorem. Additional proofs of the PNT may be found in other chapters; cf. the Index.

For some of the proofs the natural setting is 'complex Tauberian theory', which is treated in Chapter III. Here conditions in the complex domain play an essential role. The complex theory had two roots in the early 1900's. One was Fatou's theorem, which involves power series, the other was Landau's treatment of the prime number theorem, based on Dirichlet series. A common framework is provided by Laplace transforms. The beautiful extension of Landau's theorem by Wiener and Ikehara is treated both in a classical manner and in an elegant contemporary way. Another attractive approach to the prime number theorem uses the newer complex analysis method due to Newman. Adapted to the Fatou area, this method has recently been applied in operator theory. The effect of conditions in the complex domain is illustrated also by results in Tauberian remainder theory. An earlier version of Chapter III has appeared as a survey under the title 'A century of complex Tauberian theory', Bulletin of the American Mathematical Society (N.S.), vol. 39 (2002), pp 475–531.

After Hardy, Littlewood and Wiener, the principal actor in Tauberian theory was Karamata. Chapter IV deals with his heritage involving 'regular variation', which has become indispensable in asymptotics of all kinds, including probability theory. Besides the standard theory, the chapter contains a variety of Tauberian theorems involving large Laplace transforms. Regular Variation is now a subject in its own right, witness the 1987 book with that title by Bingham, Goldie and Teugels.

Chapter V treats other extensions of the classical theory. The first part deals with the Banach algebra approach to Wiener theory. Going back to Beurling and Gelfand, it has led to important generalizations. Other parts of the chapter serve to reduce 'general' Tauberian theorems to the easier case involving the limitation of bounded functions or sequences. After an important boundedness theorem of Pitt, we discuss the functional-analytic approach initiated by the Polish school, and greatly developed by Zeller and Meyer-König. The chapter concludes with some interesting special theorems.

One of the more difficult Tauberian areas concerns the Borel method of summation, which is best known as a tool for analytic continuation. Although the Tauberian theory for this method was started by Hardy and Littlewood, several basic results are of relatively recent origin. In Chapter VI we present a new unified Tauberian theory for Borel summability and the related 'circle methods', of which Euler summability is the best known representative. The treatment includes a common theory for lacunary series with 'square-root gaps'.

Chapter VII is devoted to (real) Tauberian remainder theory. The basic question is to obtain remainder estimates for convergent series, given the order of approximation provided by a summability process. The chapter starts with the case of power series and Laplace transforms, for which Freud and the author refined the method of polynomial approximation. It continues with the broader approach by Ganelius and others which is based on refinement of Wiener theory. The final part of the chapter treats some difficult nonlinear problems of the type, first considered by Erdős in connection with the elementary proof of the prime number theorem. The material based on Siegel's unpublished 'fundamental relation' was taken from the author's article 'Tauberian theorem of Erdős revisited', which appeared in Combinatorica, vol. 21 (2001), pp 239–250.

The idea of a 'Tauberian book' came up in the sixties when I lectured at Stanford and Oregon, but the project became dormant soon after I moved back from the U.S. to Amsterdam in the seventies. This notwithstanding a solemn promise, made to Springer series editor Sz.-Nagy (on Tolstoy's grave, of all places) to complete a Tauberian book. Naturally, the present volume has evolved a great deal from the early notes. In the meantime, a few of the topics have been treated very nicely in small books by A.G. Postnikov [1979] and Jan van de Lune [1986]. Prior to these, the only 'general' Tauberian book had been Pitt's monograph [1958].

Although the present book deals with a large variety of results, it does not aspire to completeness. The Tauberian literature is just too extensive – there are so many summability methods! This is clear already from Hardy's book *Divergent Series* of 1949 and the 'Ergebnisse' book by Zeller and Beekmann of 1970. The present Bibliography lists a substantial number of contributions (both old and new), but by no means all. Our emphasis is on Tauberian theorems for the principal summability methods and on results that belong to the area of classical analysis. We do not consider multidimensional theory or absolute summability; the book does not deal with Tauberian theorems for topological groups or generalized functions. There is no systematic treatment of the many applications, but a number of them are scattered through the book; see the Index. For all that is missing, the interested reader is referred to the well-known reference journals and databases.

The various chapters of the book are largely independent of each other. For the newcomer to Tauberian theory, the first ten sections of Chapter I may serve as orientation. Beyond that, every chapter has its own introduction. The Index refers to a few challenging open problems.

ACKNOWLEDGEMENTS. Under the German occupation (1940-45), students in the Netherlands had a difficult time. When university attendance was impossible, one tried to study from notes taken by older students. I am very grateful to my former high-school teacher C. Visser in Dordrecht and my later Ph.D. advisor H.D. Kloosterman at Leiden for help during this period. Visser encouraged me to explore Tauberian theorems that were stated without proof in notes of Kloosterman's introductory analysis course. Kloosterman continued to receive students at his home for examinations and encouraged my early independent work. For many years, my heroes were Hardy and Littlewood, next to Pólya, Szegő and Landau; a list soon extended to Karamata and Wiener.

Shortly after the war, I was fortunate to meet Paul Erdős in Amsterdam, where he lectured on the elementary proof of the prime number theorem. He challenged me with related Tauberian questions, also after I moved to the U.S. Some of my early papers owe a great deal to his suggestions.

Jumping to the last few years, it is a pleasure to mention some of the many friends and colleagues who have helped with the book. After Fred Gehring rekindled my interest, Nick Bingham provided constant support and encouragement. Nick, Harold Diamond, Kenneth Ross, anonymous referees and other experts commented on early drafts of several chapters, and Ronald Kortram read through the whole book. I thank all of them for their useful suggestions. However, even after several rounds of editing, some of my mistakes and omissions are likely to remain, for which my apologies.

On the technical side, notably LaTeX questions, my former student and junior colleague Jan Wiegerinck was always ready to assist with day-to-day problems. He and Jan van de Craats skillfully executed the drawings. At Springer Verlag, Dr. Byrne and her staff kindly met my wishes on the styling of the book.

Special thanks are due to the Mathematical Institute at the University of Amsterdam, which provided a desk and access to its facilities after my retirement. Our library is well supplied with older material, and librarian Sjoerd Lashley was always willing to go after newer items. The library of the CWI (Centrum voor Wiskunde en Informatica) in Amsterdam was very helpful too, as were friends at the University of Wisconsin and elsewhere; Armen Sergeev in Moscow provided me with a (photo)copy of Subhankulov's hard-to-get book on remainder theory.

The interest of the mathematical community in Tauberian theory has been relatively constant over the years. That the book may provide a new impulse!

Amsterdam, January 2004 *Jaap Korevaar*

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