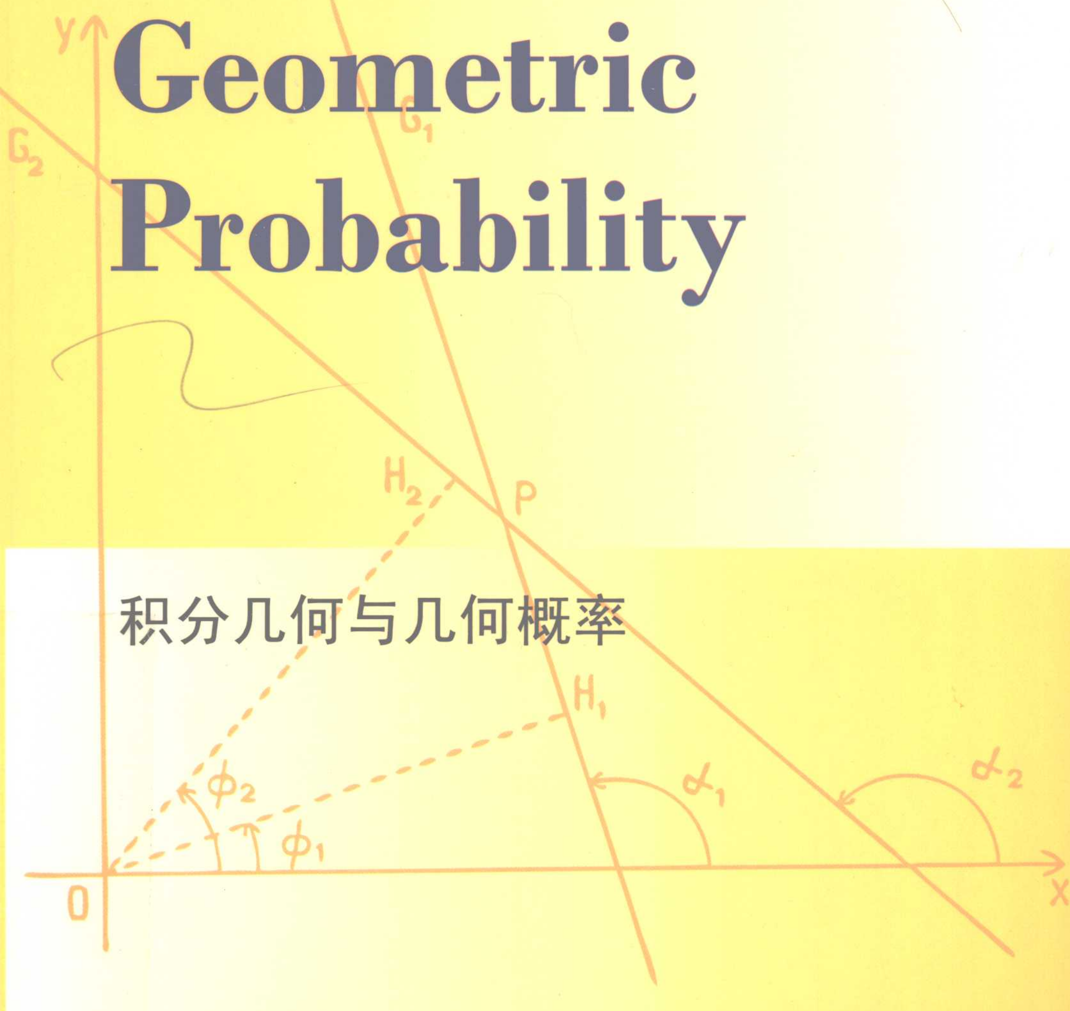
Luis Santaló

Integral Geometry and Geometric Probability

积分几何与几何概率



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Integral Geometry
and
Geometric Probability

Luis A. Santaló
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Second Edition

With a Foreword by
Mark Kac
The Rockefeller University

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Now available in the Cambridge Mathematical Library, the classic work from Luis Santaló. Integral geometry originated with problems on geometrical probability and convex bodies. Its later developments, however, have proved useful in several fields ranging from pure mathematics (measure theory, continuous groups) to technical and applied disciplines (pattern recognition, stereology). The book is a systematic exposition of the theory and a compilation of the main results in the field. The volume can be used to complement courses on differential geometry, Lie groups or probability, or differential geometry. It is ideal both as a reference and for those wishing to enter the field.

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Editor's Statement

A large body of mathematics consists of facts that can be presented and described much like any other natural phenomenon. These facts, at times explicitly brought out as theorems, at other times concealed within a proof, make up most of the applications of mathematics, and are the most likely to survive changes of style and of interest.

This ENCYCLOPEDIA will attempt to present the factual body of all mathematics. Clarity of exposition, accessibility to the non-specialist, and a thorough bibliography are required of each author. Volumes will appear in no particular order, but will be organized into sections, each one comprising a recognizable branch of present-day mathematics. Numbers of volumes and sections will be reconsidered as times and needs change.

It is hoped that this enterprise will make mathematics more widely used where it is needed, and more accessible in fields in which it can be applied but where it has not yet penetrated because of insufficient information.

GIAN-CARLO ROTA

Foreword

This monograph is the first in a projected series on Probability Theory.

Though its title "Integral Geometry" may appear somewhat unusual in this context it is nevertheless quite appropriate, for Integral Geometry is an outgrowth of what in the olden days was referred to as "geometric probabilities."

Originating, as legend has it, with the Buffon needle problem (which after nearly two centuries has lost little of its elegance and appeal), geometric probabilities have run into difficulties culminating in the paradoxes of Bertrand which threatened the fledgling field with banishment from the home of Mathematics. In rescuing it from this fate, Poincaré made the suggestion that the arbitrariness of definition underlying the paradoxes could be removed by tying closer the definition of probability with a geometric group of which it would have to be an invariant.

Thus a union of concepts was born that was to become Integral Geometry.

It is unfortunate that in the past forty or so years during which Probability Theory experienced its most spectacular rise to mathematical prominence, Integral Geometry has stayed on its fringes. Only quite recently has there been a reawakening of interest among practitioners of Probability Theory in this beautiful and fascinating branch of Mathematics, and thus the book by Professor Santaló, for many years the undisputed leader in the field of Integral Geometry, comes at a most appropriate time.

Complete and scholarly, the book also repeatedly belies the popular belief that applicability and elegance are incompatible.

Above all the book should remind all of us that Probability Theory is measure theory with a "soul" which in this case is provided not by Physics or by games of chance or by Economics but by the most ancient and noble of all of mathematical disciplines, namely Geometry.

MARK KAC
General Editor, Section on Probability

Preface

During the years 1935–1939, W. Blaschke and his school in the Mathematics Seminar of the University of Hamburg initiated a series of papers under the generic title "Integral Geometry." Most of the problems treated had their roots in the classical theory of geometric probability and one of the project's main purposes was to investigate whether these probabilistic ideas could be fruitfully applied to obtain results of geometric interest, particularly in the fields of convex bodies and differential geometry in the large. The contents of this early work were included in Blaschke's book *Vorlesungen über Integralgeometrie* [51].

To apply the idea of probability to random elements that are geometric objects (such as points, lines, geodesics, congruent sets, motions, or affinities), it is necessary, first, to define a measure for such sets of elements. Then, the evaluation of this measure for specific sets sometimes leads to remarkable consequences of a purely geometric character, in which the idea of probability turns out to be accidental. The definition of such a measure depends on the geometry with which we are dealing. According to Klein's famous Erlangen Program (1872), the criterion that distinguishes one geometry from another is the group of transformations under which the propositions remain valid. Thus, for the purposes of integral geometry, it seems natural to choose the measure in such a way that it remains invariant under the corresponding group of transformations. This sequence of underlying mathematical concepts —probability, measure, groups, and geometry—forms the basis of integral geometry.

The original work was limited almost entirely to metric (euclidean and noneuclidean) geometry and the probabilistic ideas were those of the classical geometric probability initiated by Crofton [132, 133] and Czuber [134] in the last century. After 1940 the new methods of differential geometry and group theory made it possible to unify and to generalize several questions in integral geometry, which led to new problems and noteworthy progress in this field. Consideration of a differentiable manifold (instead of euclidean space) and of a transitive transformation group operating on it gave rise to integral geometry in homogeneous spaces, and the whole theory was illuminated by the ideas of the theory of locally compact groups and their invariant measures. The inclusion of the methods of integral geometry within the framework of the theory of homogeneous spaces was the work of A. Weil

[710, 711] and S. S. Chern [105]. However, integral geometry generally has been restricted to Lie's transformation groups—more precisely, to matrix Lie groups—for two reasons. First, because they are the most important from the point of view of their geometric applications, and second, because they lead to more computable results. Further, the resulting simplification of the presentation compensates for the loss of generality.

As main references on integral geometry, after the work of Blaschke [51], we have our early introduction [568] and the books of M. I. Stoka [646, 647]. Also closely related are Hadwiger's books [270, 274]. With regard to the theory of geometric probability there is the book of Deltheil [144] and the nice brochure of M. G. Kendall and P. A. P. Moran [335], where a large number of applications to different fields are brought together. The present book intends to provide a synopsis of the main topics of integral geometry, including their origins and their applications, with the aim of showing how the interplay between geometry, group theory, and probability has become fruitful for all of these fields.

In recent times, mainly due to the work of R. E. Miles [410, 411, 414, 418], the field of integral geometry has been enriched by the introduction of the ideas and tools of stochastic processes. In a symposium on integral geometry and geometric probability held at Oberwolfach (Germany) in June 1969, D. G. Kendall, K. Krickeberg, and R. E. Miles suggested the term "stochastic geometry" to indicate precisely those contents of geometry and group theory that are in a sense related to stochastic processes. This constitutes a promising field, to which the present book may be considered an introduction, at least from its geometric point of view (see [294] and G. Matheron's recent book [401a], where the theory of random sets and applications to practical problems are treated in great detail, using deep topological ideas).

The book presupposes only a basic course in advanced calculus, although some elementary knowledge of differential geometry, group theory, and probability is desirable. Publications in which prerequisite material can be found are always indicated where apt.

Part I is concerned with integral geometry on the euclidean plane. It is treated in an elementary way. Most of the problems are handled by specific techniques and the main results are proved directly and independently in each case. We consider this part fundamental in that it exhibits the power of the methods and their usefulness in various fields. Chapters 1 to 4 are classical in the theory of geometric probability. They are devoted to the current notions on the measure of sets of points and lines in the plane, including some fairly recent results, in order to illustrate the breadth of the field of applications. Chapter 5 deals with sets of strips as an immediate generalization of sets of lines. In Chapters 6 and 7 the kinematic measure in the plane is treated in detail in order to emphasize how the measure on groups can be applied to strictly geometric problems. These chapters prepare the ground for the general approach of Chapters 9 and 10. Chapter 8 deals with some discrete subgroups of the motions group and their interpretation from the integral geometric viewpoint.

Part II presents an account of the necessary elements of the theory of Lie groups and homogeneous spaces in order to obtain the invariant measures in these spaces and their properties. The general theory is exemplified by the groups of affine transformations (Chapter 11) and the group of motions in euclidean space (Chapter 12). Several examples are discussed. For instance, it is shown that the affine invariant measure of sets of planes with reference to a fixed convex body permits a geometric interpretation of some inequalities among various characteristics of the convex body – a typical result of integral geometry (Sections 2 and 3 of Chapter 11).

Part III is concerned with integral geometry in euclidean n-dimensional space. Chapter 13 contains a résumé of the main results on convex sets in n-dimensional space. Chapter 14 is devoted to the measure of linear spaces that intersect a convex set or, more generally, a compact manifold embedded in euclidean space. Several integral formulas are obtained and some applications to the theory of geometric probability are mentioned. Chapter 15 is concerned with the so-called kinematic fundamental formula, which includes most formulas in euclidean integral geometry as special or limiting cases. In Chapter 16 the general theory is applied in detail to three-dimensional euclidean space, especially to the question of the size distribution of particles embedded in a convex body when only two-dimensional sections are available, a problem that has several areas of application and has received considerable attention in recent years, giving rise to so-called stereology [166].

Finally, Part IV deals with integral geometry in spaces of constant curvature (noneuclidean integral geometry), in particular integral geometry on the sphere, and some new trends in integral geometry (integral geometry and foliated spaces, integral geometry in complex spaces, symplectic integral geometry, and integral geometry in the sense of Gelfand and Helgason). A survey of these new trends is given, entirely without proofs, but with detailed references to the literature.

Each chapter ends with a section of notes or notes and exercises, including a number of references and theorems without proof and emphasizing applications. These notes increase the amount of material covered and, with the extensive bibliography, establish the book's encyclopedic character.

LUIS A. SANTALÓ

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