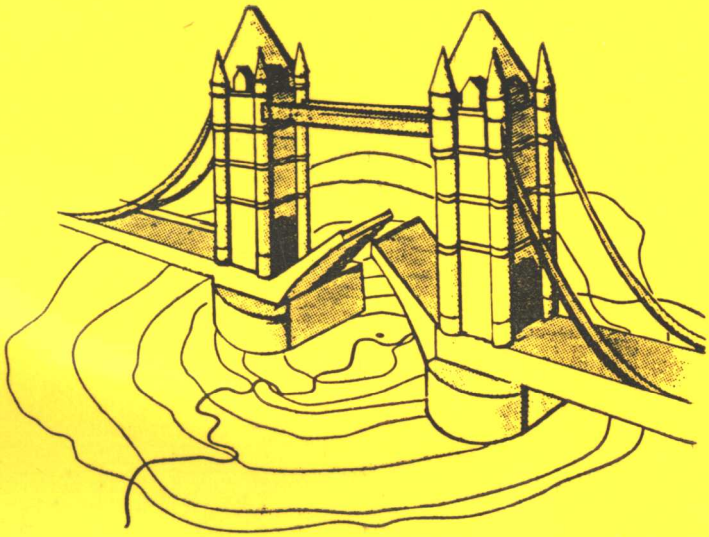
Robert B. Banks

Growth and Diffusion Phenomena

Mathematical Frameworks and Applications

增长和扩散现象的数学模式和应用



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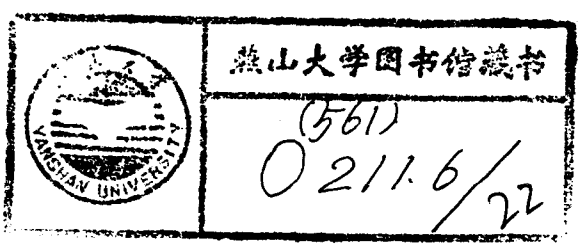
With 216 Figures



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Cover figure. Among the many growth and diffusion models examined by the author is one relating to the temporal-spatial growth of cities. London is utilized as an illustration. The growth of London comensing 1840 is examined in detail in Section 7.5.6

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Texts in Applied Mathematics 14

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To Gunta, Steven and Erik

Series Preface

Mathematics is playing an ever more important role in the physical and biological sciences, provoking a blurring of boundaries between scientific disciplines and a resurgence of interest in the modern as well as the classical techniques of applied mathematics. This renewal of interest, both in research and teaching, has led to the establishment of the series: *Texts in Applied Mathematics (TAM)*.

The development of new courses is a natural consequence of a high level of excitement on the research frontier as newer techniques, such as numerical and symbolic computer systems, dynamical systems, and chaos, mix with and reinforce the traditional methods of applied mathematics. Thus, the purpose of this textbook series is to meet the current and future needs of these advances and encourage the teaching of new courses.

TAM will publish textbooks suitable for use in advanced undergraduate and beginning graduate courses, and will complement the *Applied Mathematical Sciences (AMS)* series, which will focus on advanced textbooks and research level monographs.

Preface

To coincide with the beginning of a book perhaps it is appropriate to mention how the book had its beginning.

Quite a long time ago, when I was a first-year graduate student in environmental engineering, my adviser indicated that I needed more knowledge of the life sciences. So, for starters, I enrolled in an undergraduate course in bacteriology; I was the only engineering student in the class. Furthermore, as the only graduate student in the course, the professor informed me that I would have the additional task of preparing a term paper. So I wrote one entitled "Applications of mathematics in bacteriology". Essentially it consisted of solutions to all the problems in my book on differential equations dealing with "prey-predator" phenomena. The professor evidently liked the paper; I think I got an A.

During the ensuing years my research interests were directed to topics of air and water pollution; aspects of diffusion and dispersion were salient features of these problems. For quite a long period I was involved with graduate students in studies along these lines.

Later on, at universities in Mexico and Thailand, I became interested in innovation diffusion and technology transfer. This activity accelerated during my assignment at ISNAR (International Service for National Agricultural Research) in The Hague. On numerous occasions I received reports dealing with technology transfer, prepared by staff at CIMMYT (International Maize and Wheat Improvement Center) in Mexico, at IRRI (International Rice Research Institute) in the Philippines and at a number of other agricultural R & D institutions.

Over a period of time I observed, as others have, that in many instances virtually identical problems involving growth, transfer and diffusion were being examined by people working in quite different fields of endeavor. It seemed to me that it might be useful to try to help in the enlargement of "information diffusion" among the various fields.

So, with that lengthy preamble, I simply indicate that a major purpose of this book is an attempt to amalgamate some of the many advances made by people working in numerous diverse disciplines on topics relating to growth and diffusion phenomena. Abundantly clear to me is that there are many people much more qualified than I to do this.

The book is arranged as follows. An introductory Chapter 1 is presented which deals with some of the historical aspects of growth and diffusion. Chapters 2 and 3 are devoted to an examination of the numerous frameworks for analysis of growth, transfer and diffusion processes. This is followed, in Chapters 4 and 5, with a number of topics dealing with time dependent growth coefficients and carrying capacities.

The subject of discrete and distributed time delays is presented in Chapter 6 and phenomena involving spatial diffusion are considered in Chapter 7. Some of the many other aspects and problems of spatial–temporal processes are introduced in Chapter 8. A comprehensive list of references is provided at the end of the book.

Throughout the book there are many numerical examples and illustrations. For the latter, an attempt has been made to select specific topics drawn from the numerous fields of endeavor. The following list indicates the various disciplines and respective numbers of illustrations.

Agriculture	6
Economics	7
Biology and physiology	13
Physical sciences and engineering	8
Demography and geography	12
Technology transfer	8
Ecology	6
Other areas	4

It is hoped that the book will be useful to people engaged in planning, analysis and evaluation activities in the various disciplines. It is hoped also that the book will be useful to advanced undergraduate and graduate students in the physical, biological and social sciences as well as to those in some areas of engineering and medicine. Readers should have a course in calculus; introductory differential equations would be helpful.

The entire treatment of the subject features deterministic models with continuous variables as contrasted with stochastic models with discrete variables. For the most part, attention is restricted to "single-species" considerations.

I want to express my gratitude to the numerous people who kindly reviewed portions of the manuscript and offered their corrections, suggestions for improvement and words of encouragement. I thank my former students for their many contributions. I acknowledge, with sincerest appreciation, two people who helped me launch this project: Dr. William K. Gamble, founding Director-General of the International Service for National Agricultural Research in The Hague, and Dr. M. Nawaz Sharif, former Director of the UN/ESCAP Regional Centre for Technology Transfer in Bangalore.

The editorial staff of Springer-Verlag in Heidelberg is warmly thanked. I am grateful also to Dr. Angela Lahee who typeset the book; and Mr. Frank

Ganz, the production editor. I also want to thank Mrs. Pamela Saftler, Head of Cartographics at Texas A & M University, for preparing all the figures. I am much indebted to Dr. M.J.H. Mogridge of University College London for his invaluable help with my illustration concerning the growth of London.

Finally, I want to indicate that my wife, Gunta, struggled with great patience and good humor throughout. During several typings of the manuscript, she occasionally and tactfully brought my attention to the fact that, in her view, the market would be quite limited without a somewhat better plot and a good deal more passion and excitement in the subject matter. In these aspects I know I failed; but in selecting a wonderful lady, I succeeded admirably.

Robert B. Banks

La Jolla, California
Spring 1993

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