

国外数学名著系列

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Natali Hritonenko Yuri Yatsenko

Mathematical Modeling in Economics,
Ecology and the Environment

经济、生态与环境科学
中的数学模型



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《国外数学名著系列》(影印版)序

要使我国的数学事业更好地发展起来,需要数学家淡泊名利并付出更艰苦地努力。另一方面,我们也要从客观上为数学家创造更有利的发展数学事业的外部环境,这主要是加强对数学事业的支持与投资力度,使数学家有较好的工作与生活条件,其中也包括改善与加强数学的出版工作。

从出版方面来讲,除了较好较快地出版我们自己的成果外,引进国外的先进出版物无疑也是十分重要与必不可少的。从数学来说,施普林格(Springer)出版社至今仍然是世界上最具权威的出版社。科学出版社影印一批他们出版的好的新书,使我国广大数学家能以较低的价格购买,特别是在边远地区工作的数学家能普遍见到这些书,无疑是对推动我国数学的科研与教学十分有益的事。

这次科学出版社购买了版权,一次影印了23本施普林格出版社出版的数学书,就是一件好事,也是值得继续做下去的事情。大体上分一下,这23本书中,包括基础数学书5本,应用数学书6本与计算数学书12本,其中有些书也具有交叉性质。这些书都是很新的,2000年以后出版的占绝大部分,共计16本,其余的也是1990年以后出版的。这些书可以使读者较快地了解数学某方面的前沿,例如基础数学中的数论、代数与拓扑三本,都是由该领域大数学家编著的“数学百科全书”的分册。对从事这方面研究的数学家了解该领域的前沿与全貌很有帮助。按照学科的特点,基础数学类的书以“经典”为主,应用和计算数学类的书以“前沿”为主。这些书的作者多数是国际知名的大数学家,例如《拓扑学》一书的作者诺维科夫是俄罗斯科学院的院士,曾获“菲尔兹奖”和“沃尔夫数学奖”。这些大数学家的著作无疑将会对我国的科研人员起到非常好的指导作用。

当然,23本书只能涵盖数学的一部分,所以,这项工作还应该继续做下去。更进一步,有些读者面较广的好书还应该翻译成中文出版,使之有更大的读者群。

总之,我对科学出版社影印施普林格出版社的部分数学著作这一举措表示热烈的支持,并盼望这一工作取得更大的成绩。

王 元

2005年12月3日

Preface

The problems of interrelation between human economics and natural environment include scientific, technical, economic, demographic, social, political and other aspects that are studied by scientists of many specialities.

One of the important aspects in scientific study of environmental and ecological problems is the development of mathematical and computer tools for rational management of economics and environment.

This book introduces a wide range of mathematical models in economics, ecology and environmental sciences to a general mathematical audience with no in-depth experience in this specific area. Areas covered are: controlled economic growth and technological development, world dynamics, environmental impact, resource extraction, air and water pollution propagation, ecological population dynamics and exploitation.

A variety of known models are considered, from classical ones (Cobb-Douglass production function, Leontief input-output analysis, Solow models of economic dynamics, Verhulst-Pearl and Lotka-Volterra models of population dynamics, and others) to the models of world dynamics and the models of water contamination propagation used after Chernobyl nuclear catastrophe. Special attention is given to modelling of hierarchical regional economic-ecological interaction and technological change in the context of environmental impact.

The authors use a unique block-by-block approach to model analysis that explains how all these models are constructed from common simple components (blocks) describing elementary physical processes. Much attention is given to the choice of control influences, step-by-step model construction, analysis of arising mathematical problems with allowance for their mutual influence, qualitative behavior of model trajectories and their meaningful interpretation. Role and stages of mathematical modelling are also analyzed.

The book is reasonably concise and contains a number of new results that previously appeared in research journals only. Some sections (in Chapters 2, 4, 7) contain generally known results needed for further setting forth and are written briefly. Despite limited volume, many mathematical and related issues are discussed: system analysis, model aggregation, discrete and continuous models, differential and integral equations, optimization and bifurcation analysis, and so on.

The state-of-art of economic-environmental modeling is characterized by an increasing number of theoretical and applied publications on various mathematical models and methods, new information techniques and software, modelling of separate environmental subsystems (atmosphere, water, soil), etc. A broader application of mathematical modelling in ecological and environmental problems reflects tendencies of modern science.

We tried to consider all relevant tendencies in economic, ecological and environmental modeling, but it has appeared to be impossible in such a wide application field. So, we have had to restrict ourselves with several important directions of the modeling close to our scientific interests. The book mainly reflects the experience collected in European mathematical modeling of economic and environmental problems during the last three decades.

Main goals of the monograph are:

- to expose current practice of applied mathematical modeling in economics, ecology and environmental sciences;
- to find out interconnections between various economic, ecological and environmental models, to explain how complex mathematical models are constructed from simpler elements;

- to teach the reader to construct, verify and analyze mathematical models for real-life problems with no assistance;
- to provide theoretical insights to guide the development of practical models.

The presentation level requires mathematical knowledge (calculus, elements of differential and integral equations) of one - two years university science faculties. The book is suitable for introductory and advanced courses in mathematical modeling.

The book is a research monograph rather than a textbook. However, It provides excellent material for a course in mathematical modeling. The authors used it as a textbook for graduated courses and hope that the book would be useful for such a purpose. There are such courses in mathematical modeling for 3-5 year university students. To start working on a specific problem, such students need a global integrated vision of mathematical modeling. We do our best to reach this goal in our book.

The book presents a self-contained introduction for those coming to the subject for the first time. University basic mathematics courses are sufficient for understanding most of the considered models. Authors try to avoid using more advanced terminology and concepts whose definition is missing in the book.

The book is intended for graduate and postgraduate students and specialists in modelling, mathematical economics, mathematical ecology, environmental sciences and a wide mathematical audience.

Review of Book Content

During the past decades the concept of economic-ecological system (EES) arose and various models were developed for prediction and optimization of ESS evolution. The models describe economic processes, taking into account environmental contamination, and allow us to control such processes. The notion of EES is thoroughly explained in Chapter I and Part III. In particular, this notion includes two-way interactions between economic and environmental systems (corresponding models are illustrated in Chapters 10-12).

The first chapter differs from the rest of the book and covers two main topics : the role of mathematical models in EES control and classification of the mathematical models. The remaining chapters are separated into three parts.

Part I "MODELS OF CONTROLLED ECONOMIC SYSTEMS" of the monograph consists of Chapters 2-6 and is devoted to mathematical modeling of economic systems. This area of modeling is pretty stable and has its own terminology, classification and investigation methods.

The models considered here and in the next part are used later in Part III as blocks for construction of more complicated models of integrated systems. This fact determined the choice of the models and topics in Part I (it is mostly economic growth theory) and Part II.

Chapters 2 and 3 analyze aggregate nonlinear economic-mathematical models based on production functions. A brief description of classic linear multisector economic models (the Leontief model and the Neumann-Gale model) is given in Chapter 4. In Chapters 5 and 6 we concentrate on the models of technological renovation described by integral or partial differential equations.

Optimization analysis is important for many considered models and the turnpike properties are emphasized for basic models of Part I. The turnpike properties describe some "efficient" trajectories (turnpikes) which are close to optimal model trajectories but have a simpler structure. They indicate basic tendencies and laws of economic development.

Part II "MODELS IN ECOLOGY AND ENVIRONMENT" are a mixture of various mathematical models used in ecological and environmental problems. Three large topics are covered : models of biological communities (Chapter 7), models of air (Chapter 8) and water (Chapter 9) contamination propagation.

Many basic models (more precisely, model blocks) of Chapters 2-10 are exposed briefly because they are given in other books and textbooks in the field. However, more complex models constructed from these blocks are explained in detail.

Part III "MODELS OF ECONOMIC-ECOLOGICAL SYSTEMS" is devoted to construction of integrated models for joint description of economic and environmental dynamics.

Each chapter of Part III is dealing with a set of similar models that focus on a detailed consideration of a specific economic-environmental process (rather than on the model scale or a mathematical similarity):

Chapter 10 studies simple aggregate economic-environmental models (based on the models of Chapters 2 and 3);

Chapter 11 explores models of world dynamics which take into account several new key features of economic-environmental interaction (as compared with Chapter 10);

Chapter 12 studies economic-environmental models with a detailed description of pollution propagation (based on the environmental models of Chapters 8 and 9);

Chapter 13 investigates controlled technological renovation in the integral models of economic-environmental interaction (based on the models of Chapters 5 and 6);

Chapter 14 exposes economic control of rational exploitation of ecological populations (biological communities), based on the models of Chapter 7.

Of course, the list of considered economic-environmental problems is not complete but represents enough to show how new problems and processes under consideration affect selection of mathematical tools. The models of Chapters 10 and 11 are based on ordinary differential equations, models of Chapter 12 - on partial differential equations, Chapter 13 - on integral equations, etc.

The models of resource extraction introduced in Chapter 10 are used as blocks in the world dynamics models of Chapter 11. The simplest models with environmental control of Chapters 10 and the world models of Chapter 11 consider environmental pollution in a very aggregated form. The next chapter concentrates on more detailed description of this process in EES.

It should be stressed that no new model is introduced in the book. The authors analyzed only well known models. As a rule, the models were

previously discussed in several journal publications, monographs or in textbooks. Chapters 5,6, and 13 are connected with the authors' results and the models exposed here were previously published in international journals as well as in the monograph (N.Hritonenko & Yu.Yatsenko, 1996).

Analysis of selected mathematical questions (solvability, qualitative dynamics, etc.) arising in exposed models is provided only if it is necessary for explaining a gist of the models. In different cases, the investigated theoretical questions are phase portraits and bifurcation analysis (ecological systems), analysis of stationary solutions (waste propagation models), optimization analysis and turnpike properties (economic growth and technological development), etc.

This typology used in the monograph is a question under discussion and possibly is not complete. It reflects the authors' experience and is helpful for this book goal.

Some Basic Notions

The terminology of the book combine basic notions of mathematics (differential and integral equations, linear algebra, optimization theory, and some others), theory of economic growth, population ecology and environmental sciences. All mathematical definitions are given as needed.

As we already mentioned, the terminology of mathematical economics is stable and we provide necessary definitions (of production function, total and final outputs, technological change, etc.) directly in corresponding chapters.

Unfortunately, it is not so in the case of ecological and environmental sciences. These sciences are still developing and use different, sometimes confusing terminology. In addition, various terms might be used for the same environmental notion by different researchers. Let us introduce some general ecological and environmental notions related to the book.

Environment is a set (complex) of forces and phenomena of the nature and human activity external with respect to an object considered, which are directly in contact with the object.

Natural environment (or "*habitat*") is a narrower notion that corresponds to the set of natural factors only. However, this is the most common understanding of the *environment* notion. In this book, *environment* usually means the *natural environment*.

Areal [lat. *area* - square, space] is an earth surface area of spreading some phenomenon, species, plants, etc.

Biological community (*biocoenosis*) is a set of individuals (animals, plants) which inhabit the same territory (areal) and participate in a common self-reproduction process.

Population is a biological community of one separate species.

Biogeocoenosis can be defined as a part of the earth surface on which biocoenosis is still homogeneous along with its corresponding parts of atmosphere, lithosphere, and hydrosphere. In total they form integral and internally consistent complex.

Ecological system (*ecosystem*) is a functional system that includes a biological community and its habitat. This concept is similar to the biogeocoenosis but is more general and can apply to artificial natural-human systems (agrosystems, urban environment, etc.). A.Tensley introduced the term in 1935.

As opposed to the *environment*, the *ecosystem* and *biogeocoenosis* notions have a clearly defined territorial (regional) aspect.

Economic-ecological system (*EES*) is a regional system that includes the the economic system and the ecosystem of the region with their two-way interaction.

The set of all biogeocoenosis (ecosystems) of our planet forms the global ecosystem called *biosphere*. This term was introduced by the famous Russian scientist V.Vernadsky

According to the Vernadsky's doctrine, the biosphere is transforming into the *noosphere* - a new evolution state of the biosphere connected with a development of human society. The *noosphere* is the sphere of nature and society interactions in which a human activity is the main factor.

From a practical viewpoint, the noosphere may be considered as a philosophic reflection of the EES notion on global level.

Close to the noosphere are the concepts of *sociosphere*, *antroposphere*, *technosphere*, and *geobiosociosphere*. However, these concepts do not reflect a new theory that would justify their introduction.

Ecology (in a narrow sense) [greek. oicos - habitation + logos] is a science about interrelation of organisms and their communities among themselves and with natural environment. E.Heckel introduces the term in his famous two-volume work "General morphology of organisms" in 1866.

In connection with enlarging the area of ecological research, more general definition of ecology has been recently formed.

Ecology (in a broad sense) is a science about the structure and functions of living stratum of the Earth as a theoretical basis of human behaviour in natural environment. Such science is still in the beginning stages of its development and has other names (*science of biosphere*, *noology*, etc.). It contains different parts (global ecology, social ecology, human ecology, and others) with sometimes confusing terminology.

Ecologization of human science means consideration of environmental consequences of human nature-changing activity on both global and local levels. It has resulted in appearance of new applied sciences close to ecology: ecological geology, radioecology and geochemical ecology, cosmic ecology and agricultural ecology, evolutionary ecology, ecological physiology of man, medical ecology, geo-hygiene, medical geography, and others.

The authors are grateful to anonymous referees for useful remarks and high evaluation of our manuscript.

We retain responsibility for all possible errors. We would love to hear from the readers of our book.

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