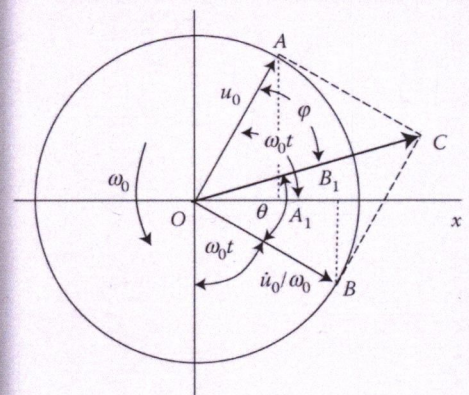
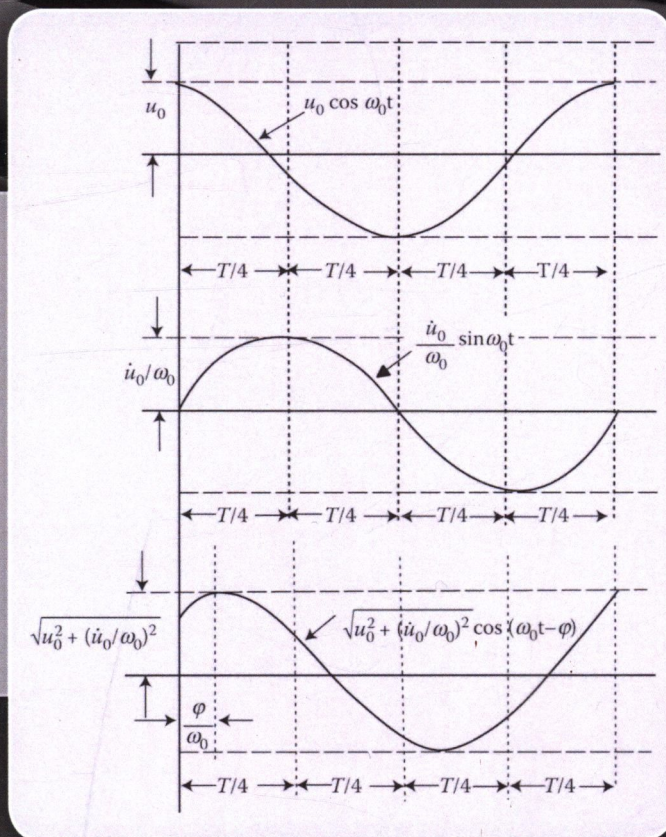


STRUCTURAL DYNAMICS

Concepts and Applications



Projection $OA_1 = u_0 \cos \omega_0 t$
Projection $OB_1 = (\dot{u}_0/\omega_0) \sin \omega_0 t$

Henry R. Busby
George H. Staab

STRUCTURAL DYNAMICS


Concepts and Applications

Structural Dynamics: Concepts and Applications focuses on dynamic problems in mechanical, civil and aerospace engineering through the equations of motion. The text explains structural response from dynamic loads and the modeling and calculation of dynamic responses in structural systems. A range of applications is included, from various engineering disciplines. Coverage progresses consistently from basic to advanced, with emphasis placed on analytical methods and numerical solution techniques. Stress analysis is discussed, and MATLAB® applications are integrated throughout. A solutions manual and figure slides for classroom projection are available for instructors.

Features

- Offers comprehensive coverage of structural dynamic fundamentals
- Includes computational methods and finite element analysis
- Provides in-depth coverage of single and multi-degree-of-freedom systems
- Analyzes frames, beams and shells in structural systems
- Includes MATLAB applications, and MATLAB files online

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**Busby
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Concepts and Applications

Henry R. Busby
George H. Staab



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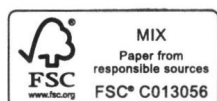
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Structural Dynamics

*Henry R. Busby dedicates this book to his wife Nancy, daughters Corinne, Marlene,
and grandchildren Anna, Thomas, Abigail, Samuel, and Prairie Rain.*

*George H. Staab dedicates this text to his wife Ellen, children Dan
and Ben, daughter-in-law Jen, and granddaughter Claire.*

Preface

The structural dynamics texts currently available present material in a variety of manners and at various technical levels. Some are intended for use by undergraduate students, others for graduate students, and some primarily address specific areas such as continuous systems or thin plates and shells. This text is intended for use by advanced undergraduate and beginning graduate students, and it includes material deemed most pertinent to the anticipated audience by presenting a variety of related topics for single- and multiple-degree-of-freedom systems. No previous knowledge of structural dynamics is required and all necessary background is assumed to come from required undergraduate engineering courses, making the text suitable for self-study. This text is intended to provide the knowledge necessary for establishing the equations of motion and determining the structural responses of systems resulting from dynamic loads. It is applicable to defining and understanding problems relevant to civil, mechanical, and aerospace engineering. Practicing engineers should have no problems using the material in this text as a reference.

In selecting topics for this text, emphasis was placed on the fundamentals of the subject, which include classical analytical methods and modern numerical solution techniques. It is structured so that students can learn the material in a clear forthright manner and develop a firm grasp of mathematical modeling, formulation, and solution of the equations of motion. The text presents an elementary introduction to time-dependent problems using the basic concepts of the single-degree-of-freedom spring-mass systems. Forced and free vibrations as well as damped and undamped systems are considered. Responses to general and arbitrary forcing functions are discussed along with material models (elastic, elastic-plastic, and viscoelastic) and integral transformations (Laplace, Fourier, Dirac, and Heaviside). Random vibrations and their related stochastic processes are presented in an early chapter so that these concepts can be used in subsequent chapters.

Two chapters are dedicated to numerical solution procedures, many of which are addressed using MATLAB® or similar commercially available programs. An early chapter is dedicated to single-degree-of-freedom systems in which finite difference techniques (Euler, Runge-Kutta, etc.), Newmark, Wilson-theta, and HHT-alpha methods are introduced. A later chapter is dedicated to multiple-degree-of-freedom systems, and the finite element method is introduced. Due to the abundance of finite element techniques available for different applications, only selected elements are presented. In addition to finite elements, the numerical techniques presented for one-dimensional problems are expanded upon.

Multiple-degree-of-freedom and continuous systems are presented in Chapters 4 and 5 with discussions of work and energy methods. The subsequent five chapters present topics relevant to specific structural members: strings and bars, beams (Euler-Bernoulli and Timoshenko), frames, plates (including composites), and shells. A brief discussion of continuous fiber-composite laminates is presented in Appendix A, and a relatively comprehensive list of additional references pertaining to structural dynamics is presented in Appendix B.

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