COMPUTATIONAL MODELLING OF REINFORCED CONCRETE STRUCTURES

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Ernest Hinton and Roger Owen

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Ernest Hinton and Roger Owen
Department of Civil Engineering,
University College of Swansea, U.K.

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PREFACE

This book evolved from the conference 'Computer-aided analysis and design of reinforced concrete structures' which was held in Split, Yugoslavia in September 1984. The conference attracted 260 participants and it became apparent during discussions in Split that there was a real need for a more considered view of recent developments in this general field of research. bearing this in mind, we invited a number of contributions for a state-of-the-art text on the theme of 'Computational modelling of reinforced concrete structures'. The resulting text contains chapters from authors present at the conference and others who are also actively involved in research aimed at the development of efficinet, accurate and reliable computational models for reinforced concrete structures.

The book has two main themes: the first six chapters deal with constitutive modelling of concrete whereas the last six concentrate on the development of numerical models and their application in the analysis and design of reinforced concrete structures. Practitioneers and research workers have long recognised the importance of all aspects of computational modelling. To obtain reliable and realistic solutions from analytical and computational procedures, the constitutive and numerical models and also nonlinear solution techniques must be carefully checked for accuracy and consistency. The contributions in this text reflect the considerable activity in this research area in recent years.

We wish to express our sincere thanks to all of the authors for their cooperation in producing a set of stimulating contributions.

Ernest Hinton and Roger Owen
Swansea, June 1986

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PART I

CONSTITUTIVE MODELLING

CHAPTER I

NON-LINEAR ANALYSIS OF REINFORCED CONCRETE SLABS

R. J. Cope
Department of Civil Engineering
Plymouth Polytechnic

SUMMARY

Constitutive models for use in non-linear analytical procedures to predict the post-cracking response of reinforced concrete slabs are reviewed. Many of the proposed models of concrete behaviour are based on the results of tests of plain concrete specimens. In view of the inherent scatter in such results, the variation in material properties throughout a slab, and the influences of reinforcing bars, it is argued that the pursuit of great accuracy is not warranted. The main influences of reinforced concrete behaviour on slab responses are discussed and the current trends and difficulties in the formulation of constitutive models for the cracked composite material are described.

1. INTRODUCTION

To date, procedures for the analysis and design of reinforced concrete slabs have been based mainly on empirical rules, or on simplified treatments of material properties, such as those assumed for linear and rigid-plastic methods [1]. Non-linear methods of analysis could be used to predict, more realistically, the behaviour of non-standard slabs with known patterns of loading. They could also be used to assess the performance of existing slabs under particular overloads and when damaged. Because of the inherent uncertainties concerning concrete properties, analytical predictions are unlikely to be very reliable when the structural response is dependent on those properties. However, for most slabs, structural response is determined mainly by structural cracking and by yielding of reinforcement at high load levels. In such circumstances, non-linear methods of analysis have much to offer.

In this chapter, information on concrete properties in slabs and material models for use in non-linear analytical procedures are reviewed. The text attempts to explain the current state of the art and the current lines of development.

Concrete in a slab is subjected to a variety of stress states. In the immediate vicinity of heavy concentrated loading and near supports, it may be necessary to consider triaxial conditions. However, reliable test data on triaxial reinforced concrete properties are scarce and triaxial stress-strain relationships are not considered in detail here. Away from concentrated loading, it is generally assumed that consideration of biaxial effects is sufficient. However, most biaxial test data are from plain concrete specimens, the behaviour of which may not accurately represent that of a concrete element in a slab which is restrained by reinforcement and by surrounding, less highly stressed concrete. Also, it is well established that the tensile strength of a plain concrete beam or slab is strongly influenced by the strain gradient over its depth. Close to cracks and slab edges, and in zones of slabs subjected to predominantly one way bending, uni-axial conditions may prevail.

2. UNI-AXIAL, COMPRESSIVE STRESS-STRAIN RESPONSE

Consideration of uni-axial stress-strain equations for plain concrete under short term loading is a useful starting point for a discussion of more complex conditions, and a considerable quantity of test data is available. Popovics [2] has reviewed the work of many researchers, and he noted that the average strain is the result of many, small, varying, discrete deformations, that occur in the various constituents of a concrete. Deduced material properties based solely on the cylinder (or cube) strength are, therefore, likely to be in error. This finding has been confirmed by Mirza et al [3], who presented data, obtained from hundreds of laboratory tests, on the scatter of measured tensile strengths and initial elastic moduli about predicted values. When target or characteristic compressive strengths of standard specimens are all that is available to a designer, precision and great sophistication are not warranted in a model of material behaviour. Popovics also noted that the inelastic part of the average strain is greater for the first application of load to a certain level, than for a subsequent application to the same level. This observation implies that constitutive equations based on test data for progressively increasing loading are, strictly, only applicable to the first loading of the concrete in a structure.

In a subsequent paper, Popovics [4] noted that stress-strain curves had different characteristics according to whether the load was applied in stress or strain increments, see Fig. 1a. Using triaxial test data, Gerstle et al [5] have argued that the falling part of the stress-strain curve is mainly dependent on the nature of the testing machine, and, in particular, on the stiffness of its interface with the specimen. If there were no restraint, the concrete would fail completely at its maximum stress. It follows that test data can only be valid for strains below the strain level corresponding to the maximum stress. In the general case, this probably corresponds to all strain states up to a strain state corresponding to the minimum volume of a specimen.

In a concrete structure, a failing element is restrained by surrounding concrete, which may be less highly stressed, and by steel reinforcement. To date, there is no test data known to the author that relates the falling branch of the stress-strain curve to the stiffness of the surrounding material. There is ample test data, however, to show that concrete in the compression zone of a beam, or slab, does not fail completely when the strain corresponding to the maximum uni-axial stress is reached, and that considerably greater strains can be accommodated.

From the test data he studied, Popovics proposed the following uni-axial stress-strain formula

$$\sigma = \sigma_0 \left(\frac{\varepsilon}{\varepsilon_0}\right) \frac{n}{(n - 1 + (\varepsilon/\varepsilon_0)^n)} \qquad (1)$$

where $n = 0.58\ \sigma_0 + 1.0$ for normal weight concretes; σ_0 is the maximum stress (N/mm^2) and ε_0 is the corresponding strain. When ε_0 is not known, which is the usual case facing a designer, Popovics suggested that, for normal weight concretes, it could be evaluated using:

$$\varepsilon_0 = 9.368 \times 10^{-4}\ \sqrt[4]{\sigma_0} \qquad (2)$$

Although Eq. (2) was fitted to data exhibiting considerable scatter, Popovics demonstrated that when combined with Eq. (1), it led to good predictions of uni-axial stress-strain curves for normal weight concretes of varying strength, including their descending branch (sic).

The tangent modulus is given by differentiating Eq. (1). Thus,

$$E_t = \frac{d\sigma}{d\varepsilon} = \frac{\sigma_0}{\varepsilon_0} \frac{(1 - (\varepsilon/\varepsilon_0)^n\ n(n - 1)}{(n - 1 + (\varepsilon/\varepsilon_0)^n)^2} \qquad (3)$$

Tests on plain concrete specimens under eccentric loading [6] have indicated that the stress-strain curves for concrete in a flexural member are similar to those derived from axially

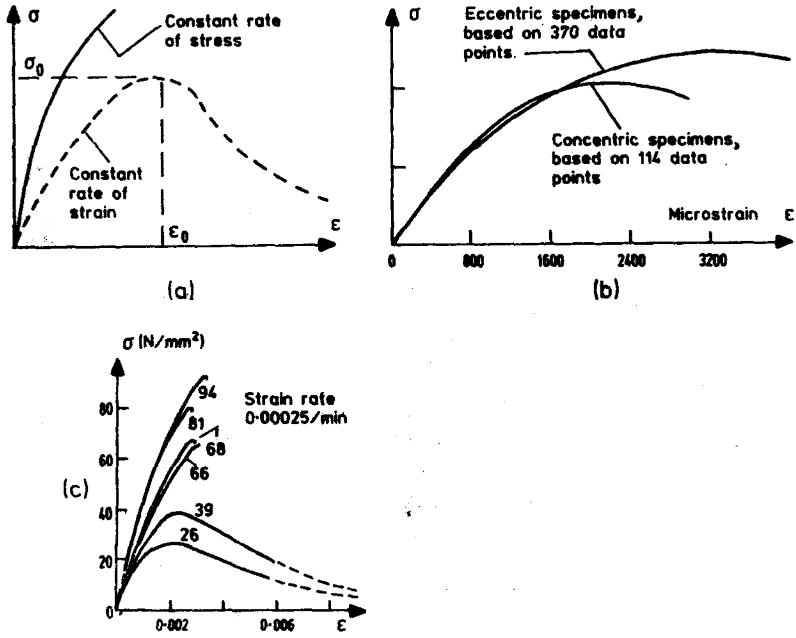

Fig. 1 Uni-axial stress-strain for plain concrete
(a) based on [4]
(b) based on [7]
(c) based on [9,26]

loaded cylinders. The results also showed that, up to a maximum compressive strain of about 0.003, there was no visible cracking or spalling. Other tests, see, for example, [7] have indicated even greater ductility and strength in the presence of strain gradient see Fig. 1b. These improvements in behaviour are probably due to the confining actions of the less highly stressed concrete, which delays the appearance of transverse micro-cracking. Blume et al [8] recommend the use of a maximum strain of 0.004 for ultimate curvature calculations involving concrete not confined by reinforcement.

Caution should be exercised when extrapolating stress-strain equations to concretes outside the range for which they have been validated. High strength concretes have recently been introduced for bridges in the USA and Japan and, when unconstrained, these may be relatively brittle [9]. Figure 1c compares typical stress-strain curves, from 100 x 200 mm cylinder tests, for different strength concretes made with the same aggregates. Although the strength range is large, it can be seen that there are relatively small changes in the initial modulus and in the strain corresponding to the peak failure stress. Although the higher strength concretes appear to be relatively brittle, the lack of a descending branch may be more a function of the testing machine than of the material [26].

The author has obtained reasonable predictions of slab behaviour using Eq. (1) for concrete, and by assuming either complete loss of strength for $\varepsilon \geq \varepsilon_0$ [10], or by assuming a constant compressive stress for $\varepsilon_0 < \varepsilon \leq 0.0035$, followed by complete loss of strength for $\varepsilon > 0.0035$ [11]. However, it should be noted that the examples studied were not very sensitive to the compressive strength or stiffness of concrete.

3. UNI-AXIAL, TENSILE STRESS-STRAIN RESPONSE

The uni-axial tensile stress-strain response of plain concrete is approximately linear. When tested under strain control, however, some softening becomes apparent, and specimens can resist a falling tensile force with strains that are several times the strain at maximum stress.

The flexural-tensile strength determined from a modulus of rupture test tends to be higher than the tensile strength of a split cylinder, which in turn tends to be higher than the tensile strength obtained from a direct tension test. The range of values obtained from specimens cast from the same concrete can be large, as can the scatter about values predicted from the compressive strength [3].

There is a number of contributory reasons for these variations. In a direct tension test, concrete on the weakest plane fails and there is a large number of potential failure planes. In a split cylinder test, the stress is not uniform and the test is effectively performed on a single plane. Concrete in a modulus of rupture test is subjected to a linearly distributed strain over the depth. The theory assumes a corresponding linear stress distribution, which is not strictly true. Also, it is known that a strain gradient enhances the strength of a specimen. There are many potential failure planes, but concrete fibres subjected to the maximum strain are limited to the soffit zone of a test beam.

The tensile behaviour of concrete in a structure may be different to that of a standard specimen. When tensile concrete in a structure is restrained by adjacent portions of concrete which are less highly stressed (or are in compression, as may be the case with composite construction) the ultimate tensile strain may be increased significantly. However, when concrete is reinforced, the bars may act as stress raises and adversely effect compaction locally. Also, restraint to early thermal contraction, while the concrete is weak, may lead to the presence of additional internal tensile stresses and micro-cracking. Indirect evidence that the direction of weakening of the effective tensile strength is biased by the reinforcement directions in a slab has been reported [12].

The predicted, non-linear response of a reinforced member to the first application of loading is quite sensitive to the specified tensile strength of the concrete [12]. In the author's experience of beam and slab analyses, reasonable predictions are usually obtained when the split cylinder strength of a standard specimen is used for the tensile strength of the concrete. However, a lower value may be needed when there is transverse reinforcement positioned close to the soffit (as in a model of a slab), and running in a direction close to that of the expected cracking.

From the above discussion, it can be appreciated that the effective tensile strength of the concrete in a structure may be difficult to determine. Even its average value and coefficient of variation cannot be determined with any degree of precision.

3.1 Tensile Toughness

Concrete is not a perfectly brittle material and possesses a property that Hillerborg has called, 'tensile toughness' [13]. This toughness plays a substantial role in controlling the shear capacity of a member and the spacing and width of bending and shrinkage cracks.

For a detailed analysis, a single stress-strain curve cannot be used for tension, as the difference in behaviour between concrete in a fracture zone and that outside the zone is too great. Hillerborg has proposed the use of a stress-strain curve for concrete outside the fracture zone and a stress "crack width" relationship for the cracking zone. The concept is illustrated for a tensile member loaded under displacement control in Fig. 2a-b. The deformation on a gauge length ℓ outside the fracture zone is $\ell\varepsilon$, whereas that on the same gauge length including a fracture zone is $\ell\varepsilon + w$. After the peak tensile stress, concrete between fracture zones is predicted to experience reducing strains, while the cracks form.

Hillerborg has argued that it is reasonable to assume the $\sigma - w$ curve to be a material property for concrete. A typical stress-deformation curve over a gauge length of 100 mm is shown in Fig. 2c. The area below the curve gives the energy absorbed per unit area over a 100 mm length and is a measure of the tensile toughness. From the shape of the curve, it can be appreciated that the toughness is mainly dependent on the $\sigma - w$ curve.

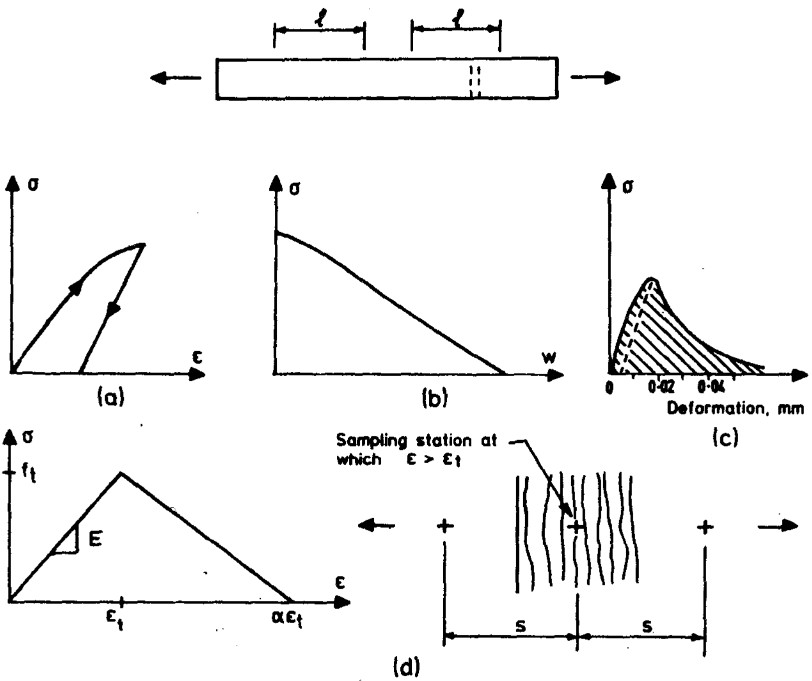

Fig. 2 (a) Stress-strain response outside fracture zone; (b) Stress-deformation response of fracture zone; (c) Typical stress-deformation curve for concrete on a gauge length of 100 mm (based on [13]) (d) Equivalent stress-strain curve.

Experiments indicate that the average length of a fracture zone in the stress direction is of the same order as the maximum aggregate size [14]. For analytical purposes, it can therefore be considered as a fictitious crack that precedes the formation of a discrete crack in a tensile member and as a fictitious extension of a flexural crack in a bending member. In the smeared crack analytical approach, the σ - w curve has to be converted to an equivalent stress-strain curve.

The material parameters used to do this are the modulus of elasticity, E, the tensile strength, f_t and the fracture energy G_f. The latter quantity is given by the area under the σ - w curve and can be obtained from test data. However, unfortunately, these exhibit considerable scatter. For normal weight concretes, typical values for G_f lie in the range 200 f_t^2/E to 400 f_t^2/E N/mm.

Nallathambi et al [15] have studied the effects of member and crack sizes, water-cement ratio and coarse aggregate texture on the fracture toughness of concrete. Based on their experimental data from beams of width b, depth d, length L and with a notch of depth g, they suggest that G_f could be obtained from

$$G_f = 0.125 \left(\frac{fc^2 b}{E}\right)\left(\frac{g}{b}\right)^{0.173}\left(\frac{d}{L}\right)^{0.631+0.4(a/d)} \tag{4a}$$

where fc and E are the compressive strength and elastic modulus of the concrete obtained from standard cylinder tests and g is the maximum size of the coarse aggregate. All parameter values are in N and m units. Presumably, for a slab, b would be set to the dimension of the cracking area normal to the tensile stress and a to the depth of an existing crack. However, application of the formula to a slab with a varying moment field could be accomplished, at best, only approximately.

Bazant and Oh [16] have suggested use of the formula

$$G_f = (2.72 + 0.0214 f_t) f_t^2 \, g/E \tag{4b}$$

where f_t is the tensile strength of the concrete and all of the parameters are in lb and inch units.

When an equivalent stress-strain curve is used in a Finite Element analysis, the strains are obtained at a discrete number of sampling stations. Constitutive equations in the form of stress-strain curves are imposed at these stations and the spacings of the stations give the gauge lengths over which average deformations are considered. To obtain mesh independent solutions, it is necessary to take the sampling station spacing into account when specifying the equivalent post peak, tensile stress-strain curve [16].

If the spacing of the sampling stations is s, and a bi-linear equivalent stress-strain curve is adopted, see Fig. 2d, the area under the stress-deformation curve is given by $A = \int \sigma(s d\varepsilon) = \alpha s f_t \varepsilon_t / 2$. Making the assumption that this area is a constant for the material, G_f, gives

$$\alpha = 2G_f/(s f_t \varepsilon_t) \tag{4c}$$

Bazant [17] has suggested that when α makes the descending branch vertical, a reduced value of f_t should be used to retain the essential shape of the bi-linear curve.

4. BI-AXIAL, STRESS-STRAIN RESPONSE

In general, for concrete in compression, the presence of an orthogonal compressive stress leads to a stiffer response and to greater strength. This is due to the effect of Poisson's ratio and, at high stress levels, to the delay of transverse micro-cracking. In tension, the presence of an orthogonal compressive stress reduces the tensile stiffness and strength, as cracking is encouraged. Bi-axial tensile strengths are similar to the uni-axial strength.

The constitutive equations for concrete under plane-stress conditions have the general form:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau \end{bmatrix} \qquad (5)$$

where σ_1 and σ_2 are the orthogonal in-plane stresses; ε_1 and ε_2 are the corresponding direct strains; σ_3 and ε_3 are the normal stress and strain, respectively; and τ and γ are the in-plane shear stress and strain, respectively.

There are too many coefficients to handle economically and simplifying assumptions are usually made. For plane stress conditions, $\sigma_3 = 0$. Therefore, the coefficients in the third column of the matrix are not of interest. If it is assumed that energy is conserved (no frictional losses, for example), the matrix of coefficients must be symmetric about the leading diagonal. If it is further assumed that the concrete is orthotropic, with the directions of material orthotropy coinciding with the principal stress directions, and that no shear strains are induced by the application of increments of the principal stresses, then

$$b_{41} = b_{42} = b_{43} = b_{34} = 0$$

With these assumptions, the constitutive equations reduce to:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} \qquad (5a)$$

$$\varepsilon_3 = b_{31}\,\sigma_1 + b_{32}\,\sigma_2 \qquad \text{and} \qquad \gamma = b_{44}\,\tau \qquad (5b, 5c)$$

The coefficients of these equations can be expressed in terms of material properties by considering uni-axial loadings. (i) Apply σ_1, with $\sigma_2 = \tau = 0$, then $b_{11} = \varepsilon_1/\sigma_1 = 1/E_1$, $b_{21} = \varepsilon_2/\sigma_1 = -\nu_{21}/E_1$, and $b_{31} = \varepsilon_3/\sigma_1 = -\nu_{31}/E_1$, where E_1 is the