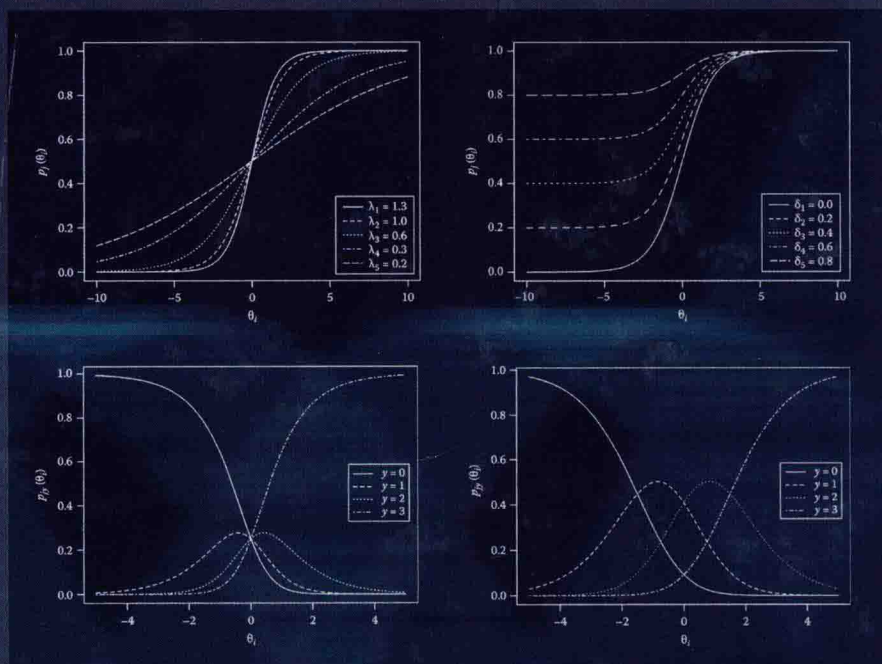


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Interdisciplinary Statistics Series

Statistical Analysis of Questionnaires

A Unified Approach Based
on R and Stata



Francesco Bartolucci
Silvia Bacci
Michela Gnaldi



CRC Press
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Preface

Questionnaires are fundamental tools of data collection in many fields and, in particular, in education, psychology, and sociology. They are becoming important tools of data collection even in economics, management, and medicine, especially when an individual characteristic that is not directly observable, such as the level of satisfaction of a customer or the quality of life of a patient, is of main interest.

A questionnaire is typically composed of a series of items that are scored on a binary or ordinal scale and are obviously correlated within each respondent. Therefore, data collected by a questionnaire require special statistical methods to be analyzed. This area has been the object of an increasing interest in the past decades, soliciting a growing number of academic research studies focused on theoretical and practical advances. In particular, in this book, we deal with the foundations of classical test theory (CTT), item response theory (IRT) fundamentals, IRT for dichotomous and polytomous items, up to the most recent IRT extensions, such as multidimensional IRT models, and latent class IRT models. Estimation methods and diagnostics—including graphical diagnostic tools, parametric and nonparametric tests, and differential item functioning—are also described.

The book is intended to have an applicative cut to mirror the growing interest and use of statistical methods and software in several fields of knowledge. Other than covering CTT and IRT basics, the book comprehensively accounts for the most recent theoretical IRT developments and provides *Stata* and *R* software codes for each method described within the chapters. For *Stata* we use version 12 and for *R* we use version 3.1.2. To enhance comprehension, real datasets are employed in the examples and the resulting software outputs are illustrated in detail.

The book is specifically designed for graduate students in applied statistics and psychometrics, and practitioners involved in measurement tasks within the education, health, and marketing fields, among others. For these figures, the book can be a reference work that will provide both the theoretical framework behind testing and measuring and a practical guide to carry out relevant analyses also based on the most advanced extensions and developments.

The prerequisites to properly understand the content of this book are at the level of a basic course of statistics which one can learn from the main parts of textbooks such as Mood et al. (1974) or Casella and Berger (2006). Moreover, a modest knowledge of *Stata* is required, which one can learn from the manuals for this software or from books such as Accock (2008). We use the main *Stata* module for the examples and suggest using *gllamm* (Rabe-Hesketh et al. 2004) for analyzing data from a questionnaire. Similarly, a basic knowledge of *R* is required which one may acquire from textbooks

such as Dalgaard (2008). In this regard, the main packages we use are `ltm` (Rizopoulos, 2006) and `MultiLCIRT` (Bartolucci et al., 2014), even though we acknowledge that many R packages exist which may be used alternatively.* Finally, the datasets used for the illustrative examples in this book as well as the main updates of Stata and R codes are available from the authors' webpage.†

The book is organized into six chapters. Chapter 1 addresses preliminary aspects related to measurement in the psychological and educational fields and to the logic, development, and use of questionnaires. The chapter also provides a description of the notation and datasets used throughout the book.

Chapter 2 examines the theory of reliability from the perspective of CTT, the procedures used to determine reliability with test data and the estimation of true scores, together with some indices commonly used for item analysis. After a brief description of the conceptual basis of test validity, the remainder of the chapter deals with test bias and with extensions of CTT (i.e., generalizability theory).

Chapter 3 is aimed at illustrating the main assumptions characterizing IRT models for dichotomously scored items and the concept of item characteristic curve. The chapter also describes the three main IRT models—that is, the well-known Rasch model, the two-parameter logistic model, and the three-parameter logistic model—entangling them in the framework of random-effects models under both the normality assumption for the latent trait and the discreteness assumption.

Chapter 4 is devoted to a generalization of IRT models for binary data to account for the categorical nature of item responses. It is very common for psychological and educational tests to be characterized by polytomous items to supply more precise information about the response process. The most relevant case we typically refer to concerns items with ordered categories. Specifically, Chapter 4 illustrates the assumptions of IRT models for polytomous items and the main criteria used to classify these models, with special attention on their statistical properties.

Chapter 5 deals with two fundamental steps of IRT analyses. The chapter's first part is devoted to estimation methods such as the joint maximum likelihood method, the conditional maximum likelihood method, and the marginal maximum likelihood method. The second part is devoted to diagnostic instruments; we distinguish: graphical tools, methods focused on the

*See <http://cran.r-project.org/web/views/Psychometrics.html> for a list of R packages, many of which contain functions for the methods that are used to analyze data collected by questionnaires.

†See <https://sites.google.com/site/questionnairesbook/datasets> for the datasets and <https://sites.google.com/site/questionnairesbook/software-updates> for the software updates.

goodness-of-fit measurement and based on parametric and nonparametric hypothesis tests, and methods focused on differential item functioning.

In Chapter 6, we illustrate several extensions of the models discussed previously. Traditional IRT models assume that the distribution of individual abilities, either continuous or discrete, is the same for all subjects. The first way to extend classical IRT models is to enclose individual covariates in the model so that the distribution of the ability becomes individual specific. Another way to extend classical IRT models is to allow the individual abilities to be dependent as a result of sharing common factors in the case of multi-level or longitudinal data. In addition, according to traditional IRT models, all dependencies among individual responses are accounted for by only one latent trait. However, questionnaires are often made up of several subsets of items measuring different constructs (or dimensions). Therefore, the chapter also deals with multidimensional models. It concludes with an overview of structural equation models in which the relationships between these models and IRT models are outlined.

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We specially thank Roberto Ricci and Patrizia Falzetti of the Italian Institute for the Evaluation of the Education System (INVALSI) for allowing us to use some of the data they produce in our examples.

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* See <https://sites.google.com/site/latentfirb/>.

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