

SUPERVISED STUDY  
IN  
MATHEMATICS AND SCIENCE

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**To the Memory of**

**MY FATHER**

**WHO BELIEVED AN EDUCATION WAS THE  
RICHEST HERITAGE A PARENT COULD  
BEQUEATH TO HIS CHILDREN**

## PREFACE

IN attempting a book on supervised study which will cover even approximately the subjects of mathematics and science, it is impossible to do more than give suggestive lessons. This, therefore, has been my plan — to give only one or two typical outlines of a topic or subject, but to leave an intimation of its application whenever or wherever the teacher may elect. Thus, only one Red Letter Day lesson is presented in Algebra, but the teacher will undoubtedly desire to use many such plans during the year. The material in the lessons mentioned may be suggestive for the planning of others.

I have not tried to add another learned book in pedagogy to the many already on the market. It has rather been my aim to write a book that may be of explicit and direct value to the teacher or principal who is daily striving to teach his children *how to study* and *how to learn*. I have tried to write it in simple language, so that the reader may get the meat, if there be any, without too much stuffing.

It is needless to say that I am a firm believer in supervised study. It has done much for our children; I am sure it will do more as we progress in the proficiency of its administration. It is not a panacea for all pedagogical ills, but it is valuable for what it claims to be, and it holds great promise for the future.

I am greatly indebted to Professor Alfred L. Hall-Quest of the University of Cincinnati, who, as editor of this series, has not only made it possible for this volume to be, but who,

through the reading and criticism of the manuscript and through innumerable other suggestions, has been of inestimable help to me.

Deep appreciation is also here expressed to Professor Charles M. Rebert of St. Lawrence University, for valuable suggestions and advice; to Mr. A. E. Breece of the Hughes High School, Cincinnati, Ohio, who made a very careful and valuable critical review of the manuscript as relating to mathematics; to my teachers at Canton, N. Y., who made it possible to actually try out many of the lessons; and to my wife, for her constant counsel and encouragement.

In addition, I wish to acknowledge my thanks for the courtesy of The Macmillan Company, the American Book Company and the Charles E. Merrill Company for permission to quote more or less extensively from their publications.

S. CLAYTON SUMNER.

PALMYRA, N. Y.

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## INTRODUCTION BY THE EDITOR

### SUPERVISED STUDY A MORAL IMPERATIVE

MILLIONS of words have been written about education. Theories have abounded and still are fertile. The visitor at educational conventions, especially in the department of school superintendents, is impressed, however, with the rapid multiplication of devices for visualizing educational practice. A rich variety of moving picture machines and already voluminous catalogues of educational films witness to the dawn of a new era in the technic of teaching. Later we shall no doubt find boards of censors passing upon these films — boards composed of theorists, critic teachers, educational scientists, *et al.*, — but at present the field is open for all. Doubtless many teachers will find in this form of visual education an opportunity for enlargement of income as well as for the demonstration of teaching skill.

*Increasing Emphasis on Demonstration.* Demonstration and description are rapidly coming to the front in discussions of methods of teaching. One carefully prepared and successfully performed demonstration is of more value than many verbal descriptions, however clear these may be. A series of vivid verbal descriptions makes definite and concrete a volume of abstractions and theorizings on educational practice. Theory is important; it must not be discounted; but here, as elsewhere, an illustration turns on light and makes

objective and easily understood the necessarily vaguer discussions of abstract theory.

This series of volumes on Supervised Study attempts to visualize one form of study supervision. Each book is written by a teacher who has had considerable experience in this type of work. The emphasis in each discussion is to make concrete in as detailed description as possible, what the author has actually done in his own classroom. Very briefly each author states the theory underlying his practice, but beyond this brief statement he refrains from a discussion of principles. Teachers desire to see how theory is applied. One cannot be too clear and too definite in describing the mode of procedure in supervision.

*At Present No Generally Accepted Meaning of Supervised Study.* In answer to those who believe that Supervised Study as described in this and other volumes of the series is different from the general understanding of the term, it should be emphasized that at present there is no generally accepted form of Supervised Study. It is the conviction of the editor of this series that a standardized form is undesirable. The main objective is teaching children — all children — how to study and guiding them while they apply the principles of correct studying. It is of comparatively little importance how this is done, providing it is done effectively. If the teacher makes this type of teaching superlatively significant, it follows that the management of the class and the method of presenting subject matter will change accordingly. But each teacher must be the final judge of how to adapt this new point of view to local needs.

*The Imperative Need of Preventing Failures in School.* It should be said, however, that any plan which seeks to prevent

failures and which aims to train *all pupils* to study as effectively as native ability permits is superior to plans that simply correct improper methods of work and that are concerned only with the retarded pupils. If school work is limited to the assigning and hearing of lessons, only a few — the highly endowed — will permanently profit by such experience. There are well-meaning people who sincerely believe that the school is the place for eliminating society's mentally unfit, and that the surest way of such elimination is to assign lessons, long and hard. Those who can will; those who cannot will not. Those who will and can are the fit!

Some there are who learn to swim by the "sink or swim" method; they are destined, forsooth, to be swimmers if they do not sink. But how many of you who read these pages learned to swim by this fatalistic method? There are children who early judge themselves incapable of school work. Nobody cares! They either can or cannot study. By means of the hard, soulless machinery of assigning and hearing lessons they are cast out. We call this a safe test and out they go labeled mentally weak, unfit to partake in a world of thrilling knowledge, unfit to climb to altitudes of self-revelation and social worth. If, however, they could have been taught how to use their minds, how to partake in the feast of knowledge, who knows but that many of them would have found a new meaning of their destiny!

*Supervised Study Is Not Only an Intellectual Necessity; It Is a Moral Imperative.* As teachers it is our plain duty to teach children how to study. The whole class period must be conducted in this spirit. The specific aim of every class period must be to so direct the pupils that their grasp of the new work is adequate for independent application. The teacher

is preëminently a director of study and not primarily a dispenser of subject matter.

*The Point of View in This Volume.* The author of this volume is convinced of the effectiveness of Supervised Study. He and his teachers have tried it long enough to know its advantages. The subjects of mathematics and science are especially favorable to this method. A comprehensive view of the courses in the high school is given in a series of typical lessons describing in great detail how children may be directed in beginning, continuing, and reviewing their study of particular units of subject matter. The author is well aware of the movement for reorganization of courses especially in ninth grade mathematics, but inasmuch as such revision is not likely to be possible in all schools for some time to come the usual division of courses is considered in this volume. It is believed also that even where general mathematics is taught, not a few pupils will elect additional special courses in the field of mathematics. Inasmuch as general science is at present little more than a combination of various special sciences the separate treatment used in this volume seems preferable. It is hoped that general science will evolve increasingly along the lines of natural correlations through which the pupil will be able to understand the intimate relationships that exist among the phenomena of nature.



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**PART ONE**  
**MATHEMATICS**



# SUPERVISED STUDY IN MATHEMATICS AND SCIENCE

## CHAPTER ONE

### THE MANAGEMENT OF THE SUPERVISED STUDY PERIOD IN MATHEMATICS

**Causes of Failures in Mathematics.** — There are a number of contributory causes which, together or separately, might account for the high mortality in mathematics classes. That it is high is so commonly accepted among the profession, that a large percentage of failures has almost come to be an established expectation. In eleven high schools near Chicago, the percentage of failures in algebra and geometry was found to be greater than in any other subject.<sup>1</sup> In the report of the New York State Education Department on statistics for Regents Academic Examinations, the failures in mathematics for the past five years have been between thirty-three per cent and forty per cent.<sup>2</sup> The nearest competitors for scholastic dishonors are the commercial subjects which are largely mathematical in content.

The causes of these failures are psychological, pedagogical, and physical. Psychologically, mathematics has been by

<sup>1</sup> *School Review*, June, 1913, p. 415.

<sup>2</sup> *Annual Report of the State Department of Education* (10th to 14th inclusive), New York State.

almost common opinion accorded the position of being the hardest subject in the school curriculum. This estimate of the subject, persisted in by pupils, teachers, and the laity, has inevitably resulted in a state of mind that predetermines a large percentage of failures. Until we teachers succeed in dispelling this opinion, pupils in many instances will expect to fail, and they will fail. There is no sane reason why mathematics should be so considered, and with the new vision of teaching the subject and with the readjustment of the course of study, combined with its scientific treatment (which will emphasize the functional and practical side instead of the formal aspect), this view of the severity of mathematics doubtless will gradually disappear.

**Mathematics Taught with Deliberate Unattractiveness.** — It is repeating a platitude to refer to the fact that mathematics has been very poorly taught in the public school. There has been no serious lack of scholarship and of emphasis on the acquirement of knowledge of subject matter, but this very emphasis has tended toward the serious neglect of training pupils to apply mathematical rules and formulas to practical reasoning. Too much emphasis has been laid on the formal examination, the "spectacular" effects according to Schultze.<sup>1</sup> Too much is attempted in the time allotted, with insufficient assimilation of the matter studied. Pupils are not taught *how to study* mathematics. They are only drilled on abstract formulas. The result is an *overdeveloped* memory and *undeveloped* powers of reasoning.

Because of the above noted unsound pedagogical methods, with the resulting formal examinations, and because the

<sup>1</sup> Arthur Schultze, "The Teaching of Mathematics in Secondary Schools"; The Macmillan Company, 1912.



pupils are graded chiefly on mechanical ability, their progress in mathematics can be determined to a highly refined nicety. They have failed to "do" a certain number of problems. Ergo, they are just that much deficient in ability and improvement. There is no leeway for difference of opinion, for the exercising of the reasoning faculty, for the training of individual characteristics and differences. Being largely a fact subject, as now taught, it resolves itself mainly into a question of "yes" or "no," and this accentuates the probability of failure. Individuals differ vastly in their ability to memorize, and therefore the poor memorizer is placed at a disadvantage. The pupil who can reason out a new demonstration in geometry knows infinitely more geometry than he who can transcribe on paper every one of the prescribed demonstrations in a book on this subject.

**The Value and Place of Supervised Study.** — This leads us logically to a discussion of the value of supervised study in mathematics. Unsupervised study is inefficient study because much time and energy are lost in misdirected effort. Pupils do not know how to attack a lesson any more than they know how to perform the mechanical processes, until they are carefully taught. Class exercises avail little for the majority of the pupils because no two minds react in the same way. To clinch the class exercise individual guidance is required. The unsupervised recitation as a rule does not provide for this. Problems in algebra and originals in geometry are entirely dependent upon the characteristics of the individual mind, which can be developed and trained only through the individual himself. To quote from an article by the author,<sup>1</sup> "the school must teach its pupils not to be perfect

<sup>1</sup> *Journal of the New York State Teachers' Association*, November, 1918.