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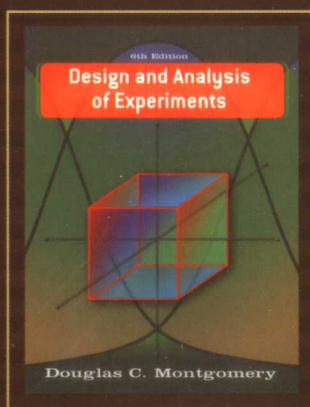
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# Design and Analysis of Experiments

# 试验设计与分析

英文版·第6版

[美] Douglas C. Montgomery 著



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(英文版·第6版)

[美] Douglas C. Montgomery 著

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## 内 容 提 要

本书是关于试验设计与分析的名著, 是作者在亚利桑那州立大学、华盛顿大学和佐治亚理工学院三所大学近30年试验设计教学经验和多年专业顾问经验的基础上编写的. 内容包括简单比较试验、 $2^k$ 因素设计、响应曲面方法和设计、稳健参数设计和过程稳健性研究、随机因素试验、巢和分图设计等.

本书适合作为统计人员、自然科学研究人员、工程技术人员、管理人员和教师进行科学试验设计与分析的参考书, 也可用于农业类、生物类、统计类的高年级本科生、研究生的教学参考用书.

图灵原版数学·统计学系列

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# Preface

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## AUDIENCE

This is an introductory textbook dealing with the design and analysis of experiments. It is based on college-level courses in design of experiments that I have taught nearly 30 years at Arizona State University, the University of Washington, and the Georgia Institute of Technology. It also reflects the methods that I have found useful in my own professional practice as an engineering and statistical consultant in the many areas of science and engineering, including the product realization process.

The book is intended for students who have completed a first course in statistical methods. This background course should include at least some techniques of descriptive statistics, the normal distribution, and an introduction to basic concepts of confidence intervals and hypothesis testing for means and variances. Chapters 10, 11, and 12 require some familiarity with matrix algebra.

Because the prerequisites are relatively modest, this book can be used in a second course on statistics focusing on statistical design of experiments for undergraduate students in engineering, the physical and chemical sciences, mathematics, and other fields of science. For many years I have taught a course from the book at the first-year graduate level in engineering. Students in this course come from all of the fields of engineering, materials science, physics, chemistry, mathematics, operations research, and statistics. I have also used this book as the basis of an industrial short course on design of experiments for practicing technical professionals with a wide variety of backgrounds. There are numerous examples illustrating all of the design and analysis techniques. These examples are based on real-world applications of experimental design and are drawn from many different fields of engineering and the sciences. This adds a strong applications flavor to an academic course for engineers and scientists and makes the book useful as a reference tool for experimenters in a variety of disciplines.

## ABOUT THE BOOK

The sixth edition is a major revision of the book. I have tried to maintain the balance between design and analysis topics of previous editions; however, there are many new topics and examples, and I have reorganized much of the material. There is much more emphasis on the computer in this edition.

### **Minitab and Design-Expert Software**

During the last few years a number of excellent software products to assist experimenters in both the design and analysis phases of this subject have appeared. I have included output from two of these products, Minitab and Design-Expert, at many points in the text. Minitab is a widely available general-purpose statistical software package that has good data analysis capabilities and that handles the analysis of experiments with both fixed and random factors (including the mixed model) quite nicely. Design-Expert is a package focused exclusively on experimental design. It has many capabilities for construction and evaluation of designs and extensive analysis features. Student versions

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of Design-Expert and Minitab are available as a packaging option with this book, and their use is highly recommended. I urge all instructors who use this book to incorporate computer software into your course. (In my course, I bring a laptop computer and a computer projector to every lecture, and every design or analysis topic discussed in class is illustrated with the computer.) To request this book with the student version of Minitab or Design-Expert included, contact your local Wiley representative. You can find your local Wiley representative by going to [www.wiley.com/college](http://www.wiley.com/college) and clicking on the tab for "Who's My Rep?"

### **Empirical Model**

I have continued to focus on the connection between the experiment and the model that the experimenter can develop from the results of the experiment. Engineers (and physical and chemical scientists to a large extent) learn about physical mechanisms and their underlying mechanistic models early in their academic training, and throughout much of their professional careers they are involved with manipulation of these models. Statistically designed experiments offer the engineer a valid basis for developing an *empirical* model of the system being investigated. This empirical model can then be manipulated (perhaps through a response surface or contour plot, or perhaps mathematically) just as any other engineering model. I have discovered through many years of teaching that this viewpoint is very effective in creating enthusiasm in the engineering community for statistically designed experiments. Therefore, the notion of an underlying empirical model for the experiment and response surfaces appears early in the book and receives much more emphasis.

### **Factorial Designs**

I have also made an effort to get the reader to the critical topics involving factorial designs much faster. To facilitate this, the introductory material on completely randomized single-factor experiments and the analysis of variance has been condensed into a single chapter (Chapter 3). I have expanded the material on factorial and fractional factorial designs (Chapters 5–9) in an effort to make the material flow more effectively from both the reader's and the instructor's viewpoint and to place more emphasis on the empirical model. There is new material on a number of important topics, including follow-up experimentation following a fractional factorial, and small, efficient resolution IV and V design.

### **Additional Important Topics**

The chapter on response surfaces (Chapter 11) immediately follows the material on factorial and fractional factorial designs and regression modeling. I have added a new chapter (12) on robust parameter design and process robustness experiments. Chapters 13 and 14 discuss experiments involving random effects and some applications of these concepts to nested and split-plot designs. Because there is expanding industrial interest in these designs, Chapters 13 and 14 have several new topics. Chapter 15 is an overview of important design and analysis topics: nonnormality of the response, the Box–Cox method for selecting the form of a transformation, and other alternatives; unbalanced factorial experiments; the analysis of covariance, including covariates in a factorial design, and repeated measures.

### **Experimental Design**

Throughout the book I have stressed the importance of experimental design as a tool for engineers and scientists to use for product design and development as well as process de-

velopment and improvement. The use of experimental design in developing products that are robust to environmental factors and other sources of variability is illustrated. I believe that the use of experimental design early in the product cycle can substantially reduce development lead time and cost, leading to processes and products that perform better in the field and have higher reliability than those developed using other approaches.

The book contains more material than can be covered comfortably in one course, and I hope that instructors will be able to either vary the content of each course offering or discuss some topics in greater depth, depending on class interest. There are problem sets at the end of each chapter (except Chapter 1). These problems vary in scope from computational exercises, designed to reinforce the fundamentals, to extensions or elaboration of basic principles.

## COURSE SUGGESTIONS

My own course focuses extensively on factorial and fractional factorial designs. Consequently, I usually cover Chapter 1, Chapter 2 (very quickly), most of Chapter 3, Chapter 4 (excluding the material on incomplete blocks and only mentioning Latin squares briefly), and I discuss Chapters 5 through 8 on factorials and two-level factorial and fractional factorial designs in detail. To conclude the course, I introduce response surface methodology (Chapter 11) and give an overview of random effects models (Chapter 13) and nested and split-plot designs (Chapter 14). I always require the students to complete a term project that involves designing, conducting, and presenting the results of a statistically designed experiment. I require them to do this in teams because this is the way that much industrial experimentation is conducted. They must present the results of this project, both orally and in written form.

## THE SUPPLEMENTAL TEXT MATERIAL

For the sixth edition I have prepared supplemental text material for each chapter of the book. Often, this supplemental material elaborates on topics that could not be discussed in greater detail in the book. I have also presented some subjects that do not appear directly in the book, but an introduction to them could prove useful to some students and professional practitioners. Some of this material is at a higher mathematical level than the text. I realize that instructors use this book with a wide array of audiences, and some more advanced design courses could possibly benefit from including several of the supplemental text material topics. This material is in electronic form on the World Wide Website for this book, located at [www.wiley.com/college/montgomery](http://www.wiley.com/college/montgomery).

## WEBSITE

Current supporting material for instructors and students is available at the website [www.wiley.com/college/montgomery](http://www.wiley.com/college/montgomery). This site will be used to communicate information about innovations and recommendations for effectively using this text. The supplemental text material described above is available at the site, along with electronic versions of data sets used for examples and homework problems, a course syllabus, and some representative student term projects from the course at Arizona State University.

### Student Companion Site<sup>1</sup>

The student's section of the textbook website contains the following:

1. The supplemental text material described above
2. Data sets from the book examples and homework problems, in electronic form
3. Sample Student Projects

### Instructor Companion Site

The instructor's section of the textbook website contains the following:

4. Solutions to the text problems
5. The supplemental text material described above
6. PowerPoint lecture slides
7. Figures from the text in electronic format, for easy inclusion in lecture slides
8. Data sets from the book examples and homework problems, in electronic form
9. Sample Syllabus
10. Sample Student Projects

The instructor's section is for instructor use only, and is password-protected. Visit the Instructor Companion Site portion of the website, located at [www.wiley.com/college/montgomery](http://www.wiley.com/college/montgomery), to register for a password.



### Student Solutions Manual

The purpose of the Student Solutions Manual is to provide the student with an in-depth understanding of how to apply the concepts presented in the Design and Analysis of Experiments, 6th Edition, by Douglas C. Montgomery. Along with detailed instructions on how to solve the selected chapter exercises, insights from practical applications are also shared.

Solutions have been provided for problems selected by the author of the text, Douglas C. Montgomery. Occasionally a group of "continued exercises" is presented and provides the student with a full solution for a specific data set. Problems that are included in the Student Solutions Manual are indicated by an icon appearing in the text margin next to the problem statement.

This is an excellent study aid that many text users will find extremely helpful. The Student Solutions Manual may be ordered in a set with the text, or purchased separately. Contact your local Wiley representative to request the set for your bookstore, or purchase the Student Solutions Manual from the Wiley website.

## ACKNOWLEDGMENTS

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and Dr. William DuMouche were particularly valuable. My Department Chair, Dr. Gary Hogg, has provided an intellectually stimulating environment in which to work.

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**Douglas C. Montgomery**  
*Tempe, Arizona*

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# 1

## Introduction

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### CHAPTER OUTLINE

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1-2 SOME TYPICAL APPLICATIONS	TECHNIQUES IN EXPERIMENTATION
OF EXPERIMENTAL DESIGN	SUPPLEMENTAL MATERIAL FOR CHAPTER 1
1-3 BASIC PRINCIPLES	S1-1 More about Planning Experiments
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The supplemental material is on the textbook website [www.wiley.com/college/montgomery](http://www.wiley.com/college/montgomery).

### 1-1 STRATEGY OF EXPERIMENTATION

Investigators perform experiments in virtually all fields of inquiry, usually to discover something about a particular process or system. Literally, an experiment is a **test**. More formally, we can define an **experiment** as a test or series of tests in which purposeful changes are made to the input variables of a process or system so that we may observe and identify the reasons for changes that may be observed in the output response.

This book is about planning and conducting experiments and about analyzing the resulting data so that valid and objective conclusions are obtained. Our focus is on experiments in engineering and science. Experimentation plays an important role in **product realization** activities, which consist of new product design and formulation, manufacturing process development, and process improvement. The objective in many cases may be to develop a **robust** process, that is, a process affected minimally by external sources of variability.

As an example of an experiment, suppose that a metallurgical engineer is interested in studying the effect of two different hardening processes, oil quenching and saltwater quenching, on an aluminum alloy. Here the objective of the experimenter is to determine which quenching solution produces the maximum hardness for this particular alloy. The engineer decides to subject a number of alloy specimens or test coupons to each quenching medium and measure the hardness of the specimens after quenching. The average hardness of the specimens treated in each quenching solution will be used to determine which solution is best.

As we consider this simple experiment, a number of important questions come to mind:

1. Are these two solutions the only quenching media of potential interest?
2. Are there any other factors that might affect hardness that should be investigated or controlled in this experiment (such as, for example, the temperature of the quenching media)?