Soviet Scientific Reviews, Section A

HYSICS REVIEWS

Volume 2 (1980)

Edited by

I. M. KHALATNIKOV

Soviet Scientific Reviews, Section A

PHYSICS REVIEWS

Volume 2 (1980)

Edited by

I. M. KHALATNIKOV

Director, L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR, Moscow

SOVIET SCIENTIFIC REVIEWS

Soviet Scientific Reviews are published under license and distributed by:

Harwood Academic Publishers GmbH
Poststrasse 22
CH-7000 Chur, Switzerland

Editorial Office for the United Kingdom:
61 Grays Inn Road
London WC1X 8TL

Editorial Office for the United States of America:
Post Office Box 786
Cooper Station
New York, New York 10276

ISBN 3-7186 0017-X, ISSN 0143-0394.

Printed in the United States of America

Preface to the Series

In the last few years many important developments have taken place in Soviet science which may have not received as much attention as deserved among the international community of scientists because of language problems and circulation problems.

In launching this new series of *Soviet Scientific Reviews* we are motivated by the desire to make accounts of recent scientific advances in the USSR more readily and rapidly accessible to scientists who do not read Russian. The articles in these volumes are meant to be in the nature of reviews of recent developments and are written by Soviet experts in the fields covered. Most of the manuscripts are translated from Russian. In the interest of speedy publication neither the authors nor the volume editors have an opportunity to see the translations or to read proofs. They are therefore absolved of any responsibility for inaccuracies in the English texts.

Soviet Scientific Reviews will appear annually, with the average of specific subject areas in each of the sciences varying from year to year. In 1979 we published volumes in Chemistry and Physics. In 1980 we are expanding the series with the addition of annual volumes in Biology, Mathematical Physics, and Astrophysics and Space Physics.

We are much indebted to the volume editors and individual authors for their splendid cooperation in getting these first volumes put together and sent to press under considerable time pressure.

The future success of this series depends, of course, on how well it meets the readers' needs and desires. We therefore earnestly solicit readers' comments and particularly suggestions for topics and authors for future volumes.

By taking this initiative we hope to contribute to the development of scientific cooperation and the better understanding among scientists.

FOREWORD

I. M. Khalatnikov,
Director, L.D. Landau Institute of Theoretical Physics, Academy of Sciences of the U.S.S.R.

The second volume of "PHYSICS REVIEWS" in the series of Soviet Scientific Reviews contains a collection of both experimental and theoretical works done by Soviet authors in 1978 and written in 1979. To our regret, it appears with a considerable delay due to purely technical difficulties of the translation. Unfortunately, we have to accept the fact that a review itself and its translation into English take about a year. This it is not surprising considering the long established practice of translating articles in such physical journals like JETP where the English version appears a year later than the Russian original.

The second volume includes three reviews of experimental works on solid state physics. In his article V. Edelman investigates properties of electrons localized on the liquid helium surface. The work by I. Krylov is dedicated to macroscopic electrodynamics of the transient state of superconductors. V. Tsoi in his paper develops an original method of transverse focusing of conduction electrons to study their interaction with a conductor's surface. The collection of theoretical reviews consists of four works. The article by V. Mineev systematically surveys homotopic topology methods applicable for investigating singularities, domain walls, solitons in superfluid phases of ^{3}He and other systems with spontaneously broken symmetry. The author was among the first to apply these methods to physics. In his paper A. Zamolodchikov studies properties of S-matrices for two-dimensional models, where he succeeds in deriving exact expressions for S-matrices. The large contribution by E. Bogomol'nyi et al. is devoted to new methods of calculating high orders of the perturbation theory in the quantum field theory, which prove extremely effective in obtaining exact results in the asymptotical region. And finally P. Wiegmann contributes a review of the results of his own research on the character of phase transitions in two-dimensional systems with the Abelian symmetry group—i.e., of problems associated with the physics of films, lattice field theories, and the theory of one-dimensional fermion systems.

The editor would also like to announce that the third volume of "PHYSICS REVIEWS" will contain articles dealing with experimental nuclear and elementary-particle physics.

CONTENTS

Factorized S Matrices and Lattice Statistical Systems

A. B. Zamolodchikov

L. D. Landau Institute of Theoretical Physics
Academy of Sciences of the U.S.S.R.

Contents

The general characteristics of relativistic factorized S matrices in two-dimensional space-time are examined. It is shown that for Euclidean values of external momenta, multiparticle elements in such S matrices formally define special model systems of lattice-statistics model systems (S models) that have a number of characteristics in common with the Baxter eight-vertex model.

1. Introduction

Models exist of the relativistic scattering theory permitting precise computation of all elements of the total S matrix in (1 + 1)-dimensional space-time. Explicit calculation of the total S matrix is bascially possible because of its special property known as *factorization:* An arbitrary L-particle element of the S matrix is expressed in standard form in terms of the product of L(L − 1)/2 two-particle S matrices. Factorized S matrices contain only purely elastic scatterings. Nevertheless, they are far from trivial and are not, in general, diagonal. Nondiagonal processes are permissible in theories with spectrum degeneracy (i.e., containing several types of particle of identical mass); they are responsible for the redistribution of momenta among particles of different types. Essentially, in the non-diagonal case, the factorization condition of the total S matrix imposes definite limitations on the two-particle S matrix (in addition to the usual requirements of analyticity and unitarity), namely, the *factorization equations*. (See review [1] and references therein.)

Factorized S matrices first arose in connection with non-relativistic problems [2-4]. The first relativisitic example appears in [5] (see also [6]). The factorization equations were first used in [7]. At the present time, a number of factorized S matrices have been constructed; these include S matrices with intrinsic symmetries $O(2)$ [7-10], $O(N)$ with $N \geq 3$ [11,12], U(N) with $N \geq 2$ [13,14], and Z_4 [15]; other examples occur in [16,17].

The structure of the multiparticle elements of the factorized S matrix corresponds to the idea of the scattering process as a succession of pair collisions, with the particles moving freely between collisions, while the pair collisions are described by the two-particle S matrices. Under the condition of the particle spectrum degeneracy, the construction of the multiparticle S matrix includes summation over the types of particles in intermediate states.

According to the method used to construct them from two-particle S matrices, the L-particle scattering amplitudes (analytically continued to Euclidean values of all the external momenta, when they become real) can for large L be interpreted as model systems of lattice statistics connected with a definite (generally irregular) lattice. The number of lattice vertices is L(L − 1)/2, while the structure of the lattice depends on the values of the external momenta (regular lattices correspond to some particular choices of the external momenta). These statistical systems are similar in their formulation to the "ice" model [18-20] and to the Baxter "eight-vertex" model [21-24]: fluctuating variables are assigned to the edges of the lattice and the partition function is computed as the sum over all possible states of all the edges, the specified statistical weights of all possible "vertex configurations" (vertex weights) being taken into account. The elements of the two-particle S matrix act as the vertex weights.

The eight-vertex model (including the ice model, the Ising model, and other physicially interesting systems as particular cases) is exactly solvable. In [21] Baxter found an exact expression for the partition function in the case of a rectangular lattice. The possibility of obtaining the exact solution is basically due to the existence of a parametric family of transfer matrices that commute at different values of the parameter.

Baxter [22] also investigated the exactly solvable eight-vertex model on an irregular lattice. He discovered a remarkable symmetry of this model, which he called Z-invariance: The partition function of the model (as well as some correlation functions) are left unchanged upon special deformations of the lattice that essentially change the coordination structure. The existence of a parametric family of commuting transfer matrices turns out to be the simple consequence of Z-invariance. In particular, the Z-invariance permits direct extension of the exact solution of the rectangular lattice model to the case of an infinite irregular lattice [22].

In fact, all the statistical systems connected (in the manner indicated above) with factorized S matrices possess Z-invariance; this is the effect of the factorization equations for the corresponding two-particle S matrix. All such systems, apparently, are exactly solvable.

Some of the known factorized S matrices correspond to known exactly solvable lattice models. For example, the S matrix with $O(2)$ symmetry [7-10] is connected with the ice model [18-20], the Z_4 symmetrical S matrix [15] is connected with the eight-vertex Baxter model. Other known examples of factorized S matrices seem to correspond to new lattice systems.

This paper provides a short review of the general characteristics of factorized S matrices and explains their formal connection with statistical lattice systems.

II. Factorized S Matrices

1. Conservation laws and factorization

Because of the specific characteristics of particle kinematics in (1 + 1) dimensional space-time, the existence of scattering theories with the following simplifying properties is possible.

(*a*). The scattering is restricted by an infinite series of selection rules ensuring the conservation of a set of asymptotic momenta of all particles:

$$\{P_a;\ a \in \text{in}\} = \{P_b;\ b \in \text{out}\} \tag{2.1}$$

Here, $P_a(P_b)$ are the spatial components of the momenta of particles in the *in* (*out*) state. In particular, the total number of particles is conserved.

(*b*). The L-particle S matrix is factorized into L(L − 1)/2 two-particle S matrices as if the process of L-particle scattering were reduced to a sequence of pair collisions.

Because of the characteristic (*b*), such a scattering theories are called factorized.

Factorized S matrices are encountered in the study of particle scattering in (1 + 1)-dimensional of quantum field theory models connected with completely integrable nonlinear equations such as the nonlinear Schrödinger and sine-Gordon equations [5,6,25]. Such dynamic systems are characterized by the existence of an infinite series of independent integrals of motion that reduce at $t \to \pm\infty$ to the sums of integer powers of asymptotic particle momenta:

$$Q_n = \sum_{a \in \text{in}} P_a^n = \sum_{b \in \text{out}} P_b^n;\ n = 1,2,\ldots \tag{2.2}$$

The property (*a*) of the corresponding scattering theory is dictated by equalities (2.2).

Actually, characteristic (*b*) is, in essence, the kinematic consequence of the selection rules (2.1). The proof and explanation of this fact can be found in papers [1,26]; we shall not discuss it in detail here. We note only that the origin of this property of the S matrix can be understood in the following way. The conservation laws (2.2) can be thought to be the consequence of an infinite series of dynamic symmetries of the theory. In view of the asymptotic form (2.2) of the Q_n operators, it is natural to assume that there exist particular symmetry transformations that reduce at $t \rightarrow \pm \infty$ to independent translations of the asymptotic coordinates of individuals particles.* By performing such *translations,* one can get an arbitrarily large space-time separation of all the regions in which the pair collisions occur. This permits one to express the amplitude of any multiparticle process in terms of two-particle amplitudes. Obviously, for every multiparticle amplitude there are several (generally different) formal expressions in terms of two-particle amplitudes, since the pair collisions making up the whole pro cess can be differently ordered in time. The sequence of the pair collisions is determined by the values of the asymptotic coordinates. The symmetry mentioned above therefore means that the various formal expressions for the multiparticle amplitude must be equal. This requirement, which reduces to definite functional equations for the two-particle S matrix (the factorization equations) plays the important part in the factorized scattering theory.

Any factorized S matrix satisfying the conditions of unitarity and analyticity represents a self-consistent 1 + 1 dimensional scattering theory. It can be considered formally by positing the properties of (*a*) and (*b*) above, without raising the question of their dynamic origin and completely disregarding the specific dynamics of interaction. In what follows, we shall adhere to this point of view.

2. *General structure of the relativistic factorized S matrix*

In (1 + 1)-dimensional space-time, the asymptotic states of the particles are characterized, besides the types of the particles, by the values of their two-momenta P_a^μ:

$$(P_a^\mu)^2 = (P_a^0)^2 - (P_a^1)^2 = m_a^2 \tag{2.3}$$

*It is implied that the scattering states (say, the *in*-states) are slightly smeared over the asymptotic momenta in a way that an assembly of converging wave packets, well separated in space, exist at $t \rightarrow -\infty$. The asymptotic coordinates specify in the usual manner the relative arrangement of the packets in the infinite past.

where the subscript a is the number of the individual particles in the given state, and m_a is the mass of the a-th particle. It is convenient to use instead of the momenta the rapidities Θ_a, defined by the following formulas:

$$P_a^0 = m_a \cosh \Theta_a;\ P_a^1 = m_a \sinh \Theta_a \qquad (2.4)$$

The scattering amplitudes of the particles, which depend only on scalar products of the form $P_a^\mu P_b^\mu$ because of relativistic invariance, will in terms of the Θ variables be functions of the rapidity differences $\Theta_a - \Theta_b$. We shall use the notation $\Theta_{ab} = \Theta_a - \Theta_b$.

To consider the general case of the factorized scattering theory, we assume n different particle types, which we shall designate $A_i = (A_1, A_2, \ldots, A_n)$; their masses are m_i. For the sake of simplicity, we assume all the A_i particles to be real: $CA_i = A_i$, where C is the charge conjugation. We write the L-particle asymptotic *in* (*out*) state containing particles $A_{i_1}, A_{i_2}, \ldots, A_{i_L}$ with rapidities $\Theta_1, \Theta_2, \ldots, \Theta_L$, respectively, in the form

$$|A_{i_1}(\Theta_1), A_{i_2}(\Theta_2), \ldots, A_{i_L}(\Theta_L)\rangle_{\substack{\text{in}\\ \text{(out)}}} \qquad (2.5)$$

We assume also that the of the scattering theory of the A_i particles has the properties (*a*) and (*b*) mentioned in article 1 of this section. In particular, the selection rule (1) means that for any L, an *in* state of the form (2.5) can be expanded into a finite superposition of the *out* states:

$$|A_{i_1}(\Theta_1), A_{i_2}(\Theta_2), \ldots, A_{i_L}(\Theta_L)\rangle_{\text{in}} =$$
$$S_{i_1 i_2 \ldots i_L}^{j_1 j_2 \ldots j_L}(\Theta_1, \Theta_2, \ldots \Theta_L)\,|A_{j_1}(\Theta_1)O, A_{j_2}(\Theta_2), \ldots, A_{j_L}(\Theta_L)\rangle_{\text{out}} \qquad (2.6)$$

Here and in what follows, summation from 1 ton over the repeated indices numbering the types of particles is implied. The coefficients $S_{i_1 \ldots i_L}^{j_1 \ldots j_L}(\Theta_1, \Theta_2, \ldots \Theta_L)$ of the expansion (2.6) are the elements of the L-particle S matrix.

Actually, in the right-hand side of the expansion (2.6) we have only those terms for which

$$m_{j_1} = m_{i_1};\ m_{j_2} = m_{i_2};\ \ldots;\ m_{j_L} = m_{i_L} \qquad (2.7)$$

[This also follows from properties (*a*) and (*b*).] In other words, the redistribution of rapidities is possible only between particles of the same mass. If all the masses m_i are different, expansion (2.6) contains only one term with $j_1 = i_1, j_2 = i_2, \ldots, j_L = i_L$, which means that S is a diagonal matrix. The case of the diagonal S matrix turns out to be less interesting. We shall therefore assume that there is a mass degeneracy among the A_i particles; i.e., the mass spectrum consists of a number of degenerate multiplets. Moreover, we shall assume for the sake of simplicity that all the A_i particles have the same mass m. Since in the theory of the factorized S matrix the limitation (2.7) is the unique kinematic manifestation of the particle masses, the case of identical masses is, essentially, the most general.

For L = 2, formula (2.6) takes the form

$$| A_i(\Theta_1)A_j(\Theta_2) \rangle_{in} = S_{ij}^{kl}(\Theta_1 - \Theta_2) \, | A_k(\Theta_1)A_l(\Theta_2) \rangle_{out} \qquad (2.8)$$

and defines the two-particle S matrix $S_{ij}^{kl}(\Theta)$. The two-particle S matrix is conveniently expressed by the diagram of Fig. 1. Each of the two intersecting lines in the drawing represents one of the values of the rapidities Θ_1 and Θ_2. The two upper legs of the diagram (it might be

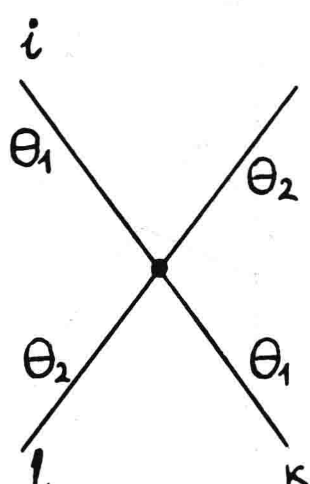

Figure 1. Diagram Representing the Two-Particle S Matrix $S_{ij}^{kl}(\Theta_1 - \Theta_2)$.

better to call them arms) represent the initial particles while the two lower legs represent the final ones. The time is assumed to be running from top to bottom. As shown, the indices i, j, k, and l are assigned to the legs.

The arbitrary element of the L-particle factorized S matrix $S_{i_1 \ldots i_L}^{j_1 \ldots j_L}(\Theta_1, \ldots, \Theta_L)$ defined by the relation (2.5) can be expressed in standard form in terms of the two-particle S matrices $S_{ij}^{kl}(\Theta)$ according to the following rule. Consider a diagram made up of L straight lines of various slopes (see Fig. 2 which corresponds to the case L = 4). To each line we assign one rapidity value from $\{\Theta_1, \Theta_2, \ldots, \Theta_L\}$ such that the order of increasing rapidities corresponds to the order of increasing slopes (counted counterclockwise from the vertical). To be specific, let us consider the order of rapidities as $\Theta_1 > \Theta_2 > \Theta_3 > \ldots > \Theta_L$. We also attach the two-particle S matrix $S_{ij}^{kl}(\Theta_{ab})$ to each intersection point of the straight lines, Θ_a and Θ_b ($\Theta_a > \Theta_b$) being the

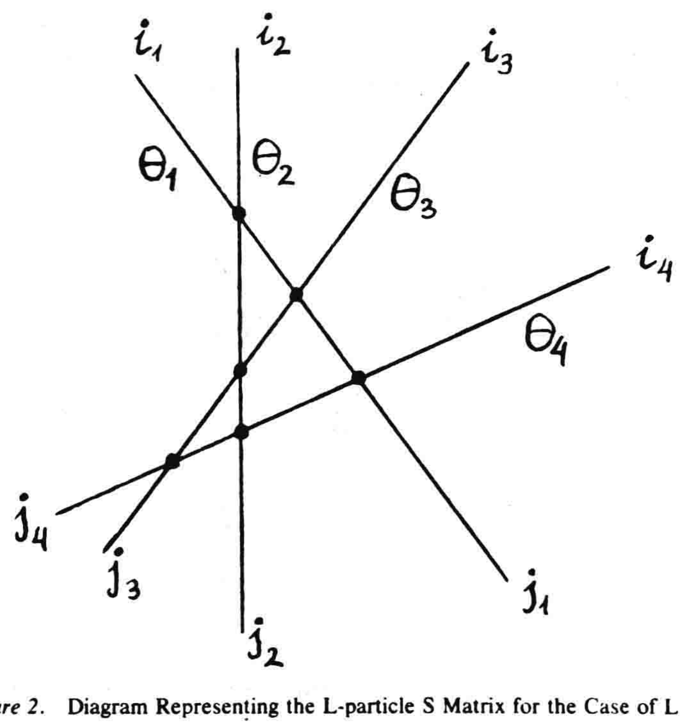

Figure 2. Diagram Representing the L-particle S Matrix for the Case of L = 4.

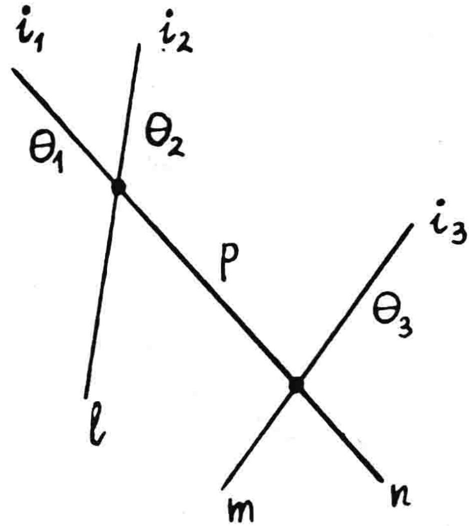

Figure 3. Fragment of a Diagram. It Corresponds to the Analytic Expression (2.9).

rapidities of the intersected lines. The legs of some of these matrices turn out to be linked. For such a situtation, we imply the summation from 1 to n over the common index corresponding to the linked legs. For example, the expression

$$S^{pl}_{i_1 i_2}(\Theta_{12})\, S^{nm}_{pi_3}(\Theta_{13}) \tag{2.9}$$

corresponds to a fragment of the diagram shown in Fig. 3. The L-particle diagram also has 2L external (unlinked) legs: L upper, and L lower. The indices numbering the types of initial particles $\{i\} = \{i_1, i_2, \ldots, i_L\}$ corresponds to the upper external legs while the indices of the final particles $\{j\} = \{j_1, j_2, \ldots, j_L\}$ corresponds to the lower ones. The expression corresponding to the entire diagram will then be the L-particle S matrix $S^{j_1 \cdots j_L}_{i_1 \cdots i_L}(\Theta_1, \ldots, \Theta_L)$.

The described rule is not unambiguous for a general matrix $S^{kl}_{ij}(\Theta)$. Actually, there are several different diagrams corresponding to one and the same L-particle S matrix element. These diagrams differ in that one or several lines are shifted. For example, the element of the S matrix shown in Fig. 2 can alternatively be represented by the dia-

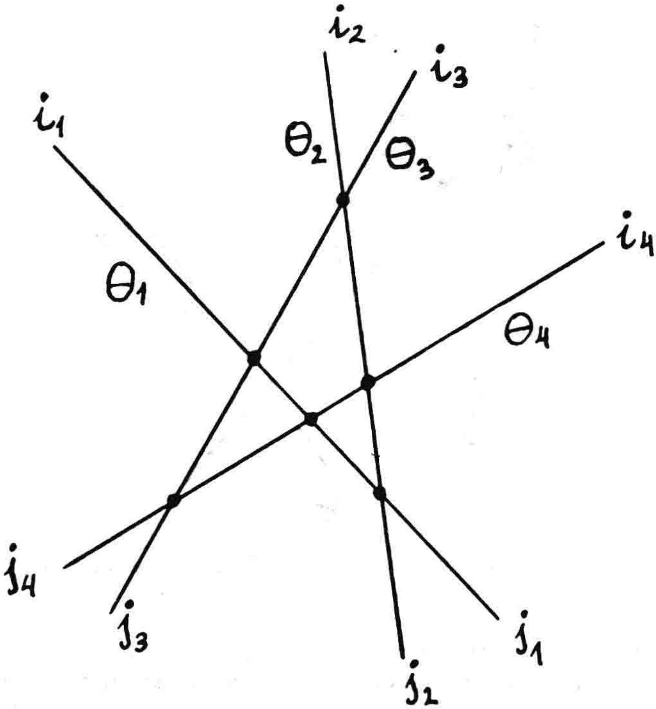

Figure 4. Alternative Representation of the Same Element of the Four-Particle S Matrix as Shown in Fig. 2.

gram of Fig. 4 (as well as some other diagrams). Various expressions of the S matrix in terms of the two-particle amplitudes $S_{ij}^{kl}(\Theta)$ obviously correspond to various diagrams.

These diagrams can be considered as representing the space-time map of particle scattering. In such an interpretation, parallel shifts of the straight lines correspond to shifts of the asymptotic coordinates of the particles. On the basis of the concepts of symmetry given in article 1 of this section,* diagrams differing by a parallel shift of the straight lines must actually be equal.

For this requirement to be satisfied, the equality (for any values of Θ_1, Θ_2, and Θ_3) of the three-particle diagrams, shown in Fig. 5 is necessary and sufficient. The last can be written in the form

*A more rigorous but also more cumbersome reasoning is given in reference [1].