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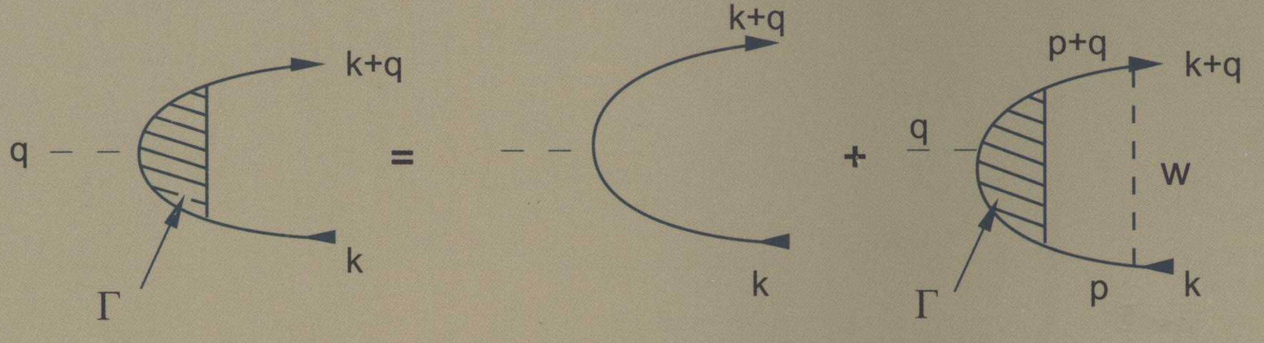

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Preface

The first, second, and third editions of this book seem to occur at ten year intervals. The intent is to keep the book up-to-date. Many-body theory is a field which continually evolves in time. Journals only publish new results, conferences only invite speakers to report new phenomena, and agencies only fund scientists to do new physics. Today's physics is old hat by tomorrow. Students want to learn new material, and textbooks must be modified to keep up with the times.

The early chapters in this book teach the techniques of many-body theory. They are largely unchanged in format. The later chapters apply the techniques to specific problems. The third edition increases the number of applications. New sections have been added, while old sections have been modified to include recent applications.

The previous editions were set in type using pre-computer technology. No computer file existed of the prior editions. The publisher scanned the second edition and gave me a disk with the contents. This scan recorded the words accurately and scrambled the equations into unintelligible form. So I retyped the equations using LaTeX. Although tedious, it allowed me to correct the infinite numbers of typographical errors in the previous edition. The earlier typesetting methods did not permit such corrections. The entire book was edited sentence-by-sentence. Most old sections of the book were shortened by editing sentences and paragraphs.

I also contemplated removing entirely some old sections. Each time I did this, and told somebody, they always remarked that the deleted section was their favorite, and I simply could not remove it. While it is gratifying to have so many sections be everyone's favorite, it does make shortening the book somewhat hard! In the end I gave up, and no sections were removed. Many were rewritten to shorten them. Since many new sections were added, the book gets longer with each edition. The reference list was updated.

New sections include: Bethe lattice, different mean-free-paths, Hubbard model, Coulomb blockade, Landauer transport, and the Quantum Hall effect. The big problem is what to say about high-temperature superconductivity. Although much experimental information is available regarding this important topic, the theoretical picture is quite uncertain. There is no agreed understanding of the pairing mechanism which causes the high transition temperature. It is hard to write a text book on a topic for which there is little agreement regarding fundamental theory. In the end, I mentioned only some important experiments and their results, and added little new information on the theory mechanisms. The section on the gap equation was rewritten to use the modern method of solving it in complex frequency space, rather than the older method of real frequency space.

I thank Steve Girvin for his proofreading twice the various versions of the section on the Quantum Hall effect, and Koung-An Chao for teaching me about quantum dots. I also very much thank my wife Sally for letting me spend every evening and weekend for one year preparing this new edition.

Contents

Chapter 1

Introductory Material

1.1. HARMONIC OSCILLATORS AND PHONONS

First quantization in physics refers to the property of particles that certain operators do not commute:

$$[x, p_x] = i\hbar \tag{1.1}$$

$$E \rightarrow i\hbar \frac{\partial}{\partial t} \tag{1.2}$$

Later it was realized that forces between particles were caused by other particles: photons caused electromagnetic forces, pions caused some nuclear forces, etc. These particles are also quantized, which leads to second quantization. The basic idea is that forces are caused by the exchange of particles, and the number of particles is quantized: one, two, three, etc. The quantization imparts a quantum nature to the classical force fields.

In solids the vibrational modes of the atoms are quantized because of first quantization (1.1.1). These quantized vibrational modes are called phonons. An electron can interact with a phonon, and this phonon can travel to another electron, interact, and thereby cause an indirect interaction between electrons. Indeed, the phonon need not move but can vibrate until the next electron comes by. The induced interaction between electrons is an example of second quantization. The phonons play a role in solids similar to the classical fields of particle physics. They cause quantized interactions between electrons.

Phonons in solids can usually be described as harmonic oscillators. A fuller description of the effects of anharmonicity is introduced later. But, for the moment, this idea should be sufficient motivation to study the harmonic oscillator. The one-dimensional harmonic oscillator has the Hamiltonian

$$H = \frac{p^2}{2m} + \frac{K}{2}x^2 \tag{1.3}$$

To solve this Hamiltonian, introduce a dimensionless coordinate ξ:

$$\omega^2 = \frac{K}{m} \tag{1.4}$$

$$\xi = x\left(\frac{m\omega}{\hbar}\right)^{1/2} \tag{1.5}$$

$$-i\frac{\partial}{\partial\xi} = \frac{p}{\sqrt{\hbar m\omega}} \tag{1.6}$$

and

$$H = \frac{\hbar\omega}{2}\left(-\frac{\partial^2}{\partial\xi^2} + \xi^2\right) \tag{1.7}$$

The harmonic oscillator Hamiltonian has a solution in terms of Hermite polynomials. The states are quantized such that

$$H\psi_n = \hbar\omega\left(n + \frac{1}{2}\right)\psi_n \tag{1.8}$$

where n is an integer. Use Dirac notation for the eigenstates $|n\rangle = \psi_n$. One can also learn by direct calculation that the following matrix elements exist for the operators x and p:

$$\begin{aligned}\langle n'|x|n\rangle &= \left(\frac{\hbar}{2m\omega}\right)^{1/2}[(n')^{1/2}\delta_{n'=n+1} + (n)^{1/2}\delta_{n'=n-1}] \\ \langle n'|p|n\rangle &= i\left(\frac{m\hbar\omega}{2}\right)^{1/2}[(n')^{1/2}\delta_{n'=n+1} - (n)^{1/2}\delta_{n'=n-1}]\end{aligned} \tag{1.9}$$

It is customary to define two dimensionless operators as follows:

$$\begin{aligned}a &= \frac{1}{\sqrt{2}}\left(\xi + \frac{\partial}{\partial\xi}\right) = \left(\frac{m\omega}{2\hbar}\right)^{1/2}\left(x + \frac{ip}{m\omega}\right) \\ a^\dagger &= \frac{1}{\sqrt{2}}\left(\xi - \frac{\partial}{\partial\xi}\right) = \left(\frac{m\omega}{2\hbar}\right)^{1/2}\left(x - \frac{ip}{m\omega}\right)\end{aligned} \tag{1.10}$$

They are Hermitian conjugates of each other. They are sometimes called *raising* and *lowering operators*, but here they are called *creation* ($a^\dagger$) and *destruction operators* (a). The Hamiltonian (1.7) may be written with them as

$$H = \frac{\hbar\omega}{2}[aa^\dagger + a^\dagger a] \tag{1.11}$$

$$= \frac{\hbar\omega}{2}\left[\frac{1}{2}\left(\xi + \frac{\partial}{\partial\xi}\right)\left(\xi - \frac{\partial}{\partial\xi}\right) + \frac{1}{2}\left(\xi - \frac{\partial}{\partial\xi}\right)\left(\xi + \frac{\partial}{\partial\xi}\right)\right] \tag{1.12}$$

$$H = \frac{\hbar\omega}{2}\left(-\frac{\partial^2}{\partial\xi^2} + \xi^2\right) \tag{1.13}$$

A very important property of these operators is called *commutation relations*. These are derived by considering how they act, sequentially, on any function $f(\xi)$. The two operations a and $a^\dagger$ in turn give

$$aa^\dagger f(\xi) = \frac{1}{2}\left(\xi + \frac{\partial}{\partial\xi}\right)\left(\xi - \frac{\partial}{\partial\xi}\right)f(\xi) = \frac{1}{2}(\xi^2 f + f - f'') \tag{1.14}$$

while the reverse order gives

$$a^\dagger a f(\xi) = \frac{1}{2}\left(\xi - \frac{\partial}{\partial\xi}\right)\left(\xi + \frac{\partial}{\partial\xi}\right)f(\xi) = \frac{1}{2}(\xi^2 f - f - f'') \tag{1.15}$$

These two results are subtracted,

$$[aa^\dagger - a^\dagger a]f(\xi) = f(\xi) \tag{1.16}$$

and yield the original function. The operator in brackets is replaced by a bracket with a comma,

$$[aa^\dagger - a^\dagger a] \equiv [a, a^\dagger] \tag{1.17}$$

which means the same thing. The relationship (1.16) is usually expressed by omitting the function $f(\xi)$:

$$[a, a^\dagger] = 1 \tag{1.18}$$

In a similar way, one can prove that

$$[a, a] = 0 \tag{1.19}$$

$$[a^\dagger, a^\dagger] = 0 \tag{1.20}$$

These three commutators, plus the Hamiltonian

$$H = \frac{\hbar\omega}{2}[aa^\dagger + a^\dagger a] = \frac{\hbar\omega}{2}[aa^\dagger - a^\dagger a + 2a^\dagger a] = \hbar\omega[a^\dagger a + \tfrac{1}{2}] \tag{1.21}$$

completely specify the harmonic oscillator problem in terms of operators. With these four relationships, one can show that the eigenvalue spectrum is indeed (1.8), where n is an integer. The eigenstates are

$$|n\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}}|0\rangle \tag{1.22}$$

where $|0\rangle$ is the state of no phonons which obeys

$$a|0\rangle = 0 \tag{1.23}$$

and where the $n!$ is for normalization. Operating on this state by a creation operator gives

$$a^\dagger|n\rangle = \frac{(a^\dagger)^{n+1}}{\sqrt{n!}}|0\rangle = \frac{(n+1)^{1/2}}{\sqrt{(n+1)!}}(a^\dagger)^{n+1}|0\rangle \tag{1.24}$$

$$= (n+1)^{1/2}|n+1\rangle \tag{1.25}$$

the state with the next highest integer. The only matrix element between states are

$$\langle n'|a^\dagger|n\rangle = (n+1)^{1/2}\delta_{n'=n+1} \tag{1.26}$$

$$\langle n'|a|n\rangle = (n)^{1/2}\delta_{n'=n-1}. \tag{1.27}$$

The second expression is derived from the first by taking the Hermitian conjugate of the first, and then exchanging dummy variables n and n'. Alternately,

$$a|n\rangle = (n)^{1/2}|n-1\rangle \tag{1.28}$$

So the destruction operator a lowers the quantum number. Then operating by the sequence

$$a^\dagger a|n\rangle = a^\dagger(n)^{1/2}|n-1\rangle = (n)^{1/2}a^\dagger|n-1\rangle = n|n\rangle \tag{1.29}$$

gives an eigenvalue n, which verifies the eigenvalue (1.8). Furthermore, using the original definitions (1.10) permits us to express x and p in terms of these operators,

$$x = \left(\frac{\hbar}{2m\omega}\right)^{1/2}(a + a^\dagger) \tag{1.30}$$

$$p = i\left(\frac{m\hbar\omega}{2}\right)^{1/2}(a^\dagger - a) \tag{1.31}$$

and the matrix elements (1.9) follow immediately:

$$\begin{aligned}
\langle n'|x|n\rangle &= \left(\frac{\hbar}{2m\omega}\right)^{1/2}[\langle n'|a|n\rangle + \langle n'|a^\dagger|n\rangle] \\
&= \left(\frac{\hbar}{2m\omega}\right)^{1/2}[(n)^{1/2}\delta_{n'=n-1} + (n+1)^{1/2}\delta_{n'=n+1}] \\
\langle n'|p|n\rangle &= i\left(\frac{m\hbar\omega}{2}\right)^{1/2}[\langle n'|a^\dagger|n\rangle - \langle n'|a|n\rangle] \\
&= i\left(\frac{m\hbar\omega}{2}\right)^{1/2}[(n+1)^{1/2}\delta_{n'=n+1} - (n)^{1/2}\delta_{n'=n-1}]
\end{aligned}$$

The description of the harmonic oscillator in terms of operators is equivalent to the conventional method of using wave functions $\psi_n(\xi)$ of position.

The time dependence of these operators is often important. In the Heisenberg representation of quantum mechanics, the time development of operators is given by ($\hbar = 1$)

$$O(t) = e^{iHt}Oe^{-iHt} \tag{1.32}$$

so that the operator obeys the equation

$$i\frac{\partial O(t)}{\partial t} = i[H, O(t)] \tag{1.33}$$

For the destruction operator, this equation becomes

$$\frac{\partial}{\partial t}a = i[H, a] = i\omega[a^\dagger aa - aa^\dagger a] = i\omega[a^\dagger, a]a = -i\omega a \tag{1.34}$$

which has the simple solution

$$a(t) = e^{-i\omega t}a \tag{1.35}$$

The reference point of time may be selected arbitrarily, so that the operators have an arbitrary phase factor associated with them. This phase is unimportant, since it cancels out of all final results. The Hermitian conjugate of this expression is

$$a^\dagger(t) = e^{i\omega t}a^\dagger \tag{1.36}$$

The time development of the position operator can be represented as

$$x(t) = \left(\frac{\hbar}{2m\omega}\right)^{1/2}(ae^{-i\omega t} + a^\dagger e^{i\omega t}) \tag{1.37}$$

This result for $x(t)$ will be used often in discussing phonon problems.

Another familiar problem which can be solved with operators is a charged harmonic oscillator in a constant electric field F:

$$H = \frac{p^2}{2m} + \frac{K}{2}x^2 + eFx = \omega(a^\dagger a + \tfrac{1}{2}) + \lambda(a + a^\dagger) \tag{1.38}$$

$$\lambda = eF\left(\frac{\hbar}{2m\omega}\right)^{1/2} \tag{1.39}$$

This Hamiltonian may be solved exactly. First consider the equation of motion for the time development of the destruction operator:

$$\frac{\partial a}{\partial t} = i[H, a] = -i(\omega a + \lambda) \tag{1.40}$$

The right-hand side is no longer just proportional to a, since there is the constant term. However, let us define a new set of operators by the relationships

$$A = a + \frac{\lambda}{\omega} \tag{1.41}$$

$$A^\dagger = a^\dagger + \frac{\lambda}{\omega} \tag{1.42}$$

They obey the equation

$$\frac{\partial A}{\partial t} = -i\omega A \tag{1.43}$$

so they have the simple time development

$$A(t) = e^{-i\omega t}A \tag{1.44}$$

$$A^\dagger(t) = e^{i\omega t}A^\dagger \tag{1.45}$$