

# PROGRESS IN METAL PHYSICS

4

*Editor*

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## FOREWORD TO VOLUME 1

THE study of the physical properties of metals has developed through a number of stages. The first was that in which the mechanical properties were correlated empirically with the heat treatment to which the metal had been subjected and, sometimes, to the chemical composition. At this stage the successful treatment of metals was an art, in the sense that experience rather than understanding led to the most satisfactory results. The next stage, in which the internal structure of the metal was examined, was based originally on the use of the microscope and it was found that many experimental facts could be explained in terms of effects that were of the right size to be seen under magnifications of less than about two thousand. The development of the x-ray diffraction techniques allowed phenomena of a smaller order of magnitude to be examined, and much of the existing information was found to be comprehensible in terms of the geometry of the crystal structure of the various phases that were visible under the microscope.

More recent development can perhaps best be discussed by a division of the field into what may be termed 'statics' and 'dynamics'. Under the former heading is the study of the conditions which govern the structure of a metal or alloy when it is in thermodynamic and mechanical equilibrium. The theories of the phases that are present in equilibrium and of the elastic constants have made remarkable progress in terms of rapidly developing theory of the part played by electrons in the metal.

Under the heading 'dynamic' effects we may include both the conditions governing the approach to equilibrium in respect of the phases that are present, in which diffusion plays an important part, and the response of a metal to forces which are sufficient to cause non-recoverable or plastic mechanical deformation.

These and associated subjects have advanced so rapidly that it has become difficult for research workers in one part of the field to remain up to date in other branches. It is the purpose of this volume, which is the first of an annual series, to present authoritative reviews of the present state of knowledge in specialized aspects of the field that includes both physical metallurgy and metal physics. It is not intended that any one volume should form a complete textbook on these subjects. It is hoped rather that a few subjects of current interest should be discussed rather fully so as to cover, in the course of several years, all the more important aspects in which progress is being made. In order to make the series reasonably self contained it is proposed that the necessary 'historical' background should be included the first time a particular subject is discussed. Subsequent articles on such subjects will generally only cover the more recent progress.

B. CHALMERS

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## INTERNAL FRICTION IN METALS

*A. S. Nowick*

THE capacity of a vibrating solid to convert its mechanical energy of vibration into heat, even when so well isolated that energy losses to its surroundings are negligible, is called internal friction. The most common manifestation of internal friction is the damping, or loss of vibration amplitude, of a freely vibrating body. Interest in internal friction in metals undoubtedly extends back to prehistoric times, to the earliest cymbals and bells. Modern investigation has followed two different channels. On the one hand there has been widespread interest in damping for engineering applications; depending on the particular application, one may wish to obtain either a very high or a very low rate of damping. In work of this kind, the effect of heat treatments and alloying elements on the damping characteristics of engineering materials is investigated and commonly reported in terms of a quantity called "damping capacity." Vibration amplitudes employed are commensurate with those that occur in practice. An adequate review of this phase of the subject would in itself require a sizeable article. A second approach to internal friction, with which the present review is concerned, is the use of damping as a tool to study internal structure and atomic movements in solids. This is a relatively new field of investigation and has been developing rapidly. Interest is focused on the origin and mechanism of internal friction rather than its desirability, and the vibration amplitudes employed are very small.

A perfectly elastic material will not produce damping since, under oscillatory conditions, stress and strain are always in phase, and consequently no mechanical hysteresis or loss of vibrational energy can take place. Damping is, therefore, a result of the non-elastic behaviour of materials. The fact that it can be observed at stress levels far below those at which plastic flow occurs, shows that there is really no such thing as an "elastic range." In recent years there has been a great increase in understanding of those physical processes that result in non-elastic behaviour, and, therefore, in damping, of metals at low stress levels. Review articles on this subject, written in the period 1939-1943,<sup>1-3</sup> emphasize three main sources of damping: thermo-elasticity, ferromagnetic effects, and the internal stresses associated with cold work, or dislocations. Of these, the last was least understood, in the sense that the mechanism by which these internal distortions act to produce damping was not established. During the war and early

post-war period intensive investigations of internal friction, particularly by ZENER and co-workers, brought to light other sources of internal friction, including stress relaxation across grain boundaries and stress-induced ordering. These last two mechanisms together with thermo-elastic internal friction fitted into a scheme of non-elastic behaviour which ZENER<sup>4</sup> called *anelasticity*, the development of which culminated in his book<sup>5</sup> in 1948. In the treatment of anelasticity it is shown that not only internal friction but several related static effects (recoverable creep, stress relaxation, and elastic after-effect at low stress levels) result from this type of non-elastic behaviour. Only internal friction that does not depend on the amplitude of vibration is considered in this treatment. The impact of this important work has been so great as to lead to the widespread misconception that all internal friction at low amplitudes can be explained in terms of anelasticity. The internal friction originating in cold work is an appropriate example; theories attempting to explain observed effects in terms of the scheme of anelastic phenomena,<sup>6-8</sup> have not met with notable success. In fact, after READ<sup>9, 10</sup> showed that cold work internal friction in single crystals is strongly amplitude dependent, the writer demonstrated<sup>11, 12</sup> that this behaviour originates in a mechanism of static hysteresis, which is quite different from anelasticity.

In the present article, an attempt is first made to provide a phenomenological description of the general features of non-elastic deformation which is sufficiently general to include, as special cases, anelasticity as well as static hysteresis and amplitude dependent internal friction. The first three main sections deal with this phenomenological approach. It is shown that internal friction cannot be divorced from an equally important effect: the change in the effective elastic modulus resulting from the non-elastic behaviour, which is called "the  $\Delta M$ -effect". Following the phenomenological treatment, the various experimental methods used to measure internal friction are reviewed. The remainder of the article is devoted to the physical origins of internal friction. All sources of damping involve an internal re-arrangement that takes place under stress. There are three types of re-arrangements that have been studied: *thermal* (the thermo-elastic effect), *magnetic*, and *atomic* re-arrangements. All of these are considered in the present article, although emphasis is placed upon the latter category, since most recent developments are in this field and it is the least adequately reviewed. Included in the category of atomic re-arrangements are diffusion effects, relaxation across interfaces, and the motion of dislocations.

#### PHENOMENOLOGICAL THEORY: HOMOGENEOUS STRAIN

In most vibration experiments the specimen under investigation is subjected to a non-homogeneous strain. A discussion of the elastic and

non-elastic behaviour of the specimen is, therefore, complicated by the dependence of stress and strain on position in the specimen. In this section, the principles of non-elastic behaviour will be introduced under the assumption that we are dealing either with a small element of a specimen within which the strain is uniform, or with a complete specimen throughout which the strain is the same at any instant. The latter condition can be realized, for a specimen of appropriate shape, by the introduction of an auxiliary inertia member relative to which the inertia of the specimen itself is negligible. For example, it is possible to set up a homogeneous shear strain by torsional vibration of a hollow tube to which is attached a heavy inertia member.

For simplicity of notation, it is assumed that the stress system within the specimen can be defined by a single component  $\sigma$ . Furthermore, only the principal strain  $\varepsilon$ , corresponding to the stress  $\sigma$ , need be considered, since other strain components do not enter into expressions for the work done by the stress. The symbol  $M$  is used to represent the appropriate elastic modulus that relates stress and strain.

As already mentioned, in order for internal friction to occur, the oscillatory stress and strain must not be in phase with each other and, in fact, in order that energy be dissipated, the strain must lag behind the stress. Let us represent this angle of lag by  $\phi$ . Then  $\phi$  is a convenient measure of internal friction, since the energy dissipated in a cycle of vibration goes to zero when  $\phi$  approaches zero. It is convenient to use complex notation to represent the various phase relationships. Thus, the variation of stress with time is expressed as

$$\sigma = \sigma_1 e^{i\omega t} \quad . . . . \quad (1)$$

where  $\omega$  is the angular frequency of vibration ( $\omega = 2\pi f$ , where  $f$  is the number of vibrations per second) and  $\sigma_1$  is the stress amplitude. Correspondingly, the strain is

$$\varepsilon = (\varepsilon_1 - i\varepsilon_2) e^{i\omega t} \quad . . . . \quad (2)$$

where the quantities  $\varepsilon_1$  and  $\varepsilon_2$  are the amplitudes of the components of strain in phase with and  $90^\circ$  behind the stress, respectively. Clearly,

$$\tan \phi = \varepsilon_2 / \varepsilon_1 \quad . . . . \quad (3)$$

Throughout this article it will be assumed that the internal friction is small, i.e.  $\varepsilon_2 \ll \varepsilon_1$  and  $\tan \phi \cong \phi$ . This approximation greatly simplifies the relations between various measures of internal friction, and there have been very few experiments to which it does not apply. An appropriate definition for the *dynamic modulus*,  $M$ , is

$$M = \sigma_1 / \varepsilon_1 \quad . . . . \quad (4)$$

It is also convenient to define the *complex modulus*,  $\tilde{M}$ , as the ratio of



stress to strain, using the complex notation of equations (1) and (2). Thus,

$$\tilde{M} = \frac{\sigma_1}{\varepsilon_1 - i\varepsilon_2} = \frac{\sigma_1}{\varepsilon_1(1 - i \tan \phi)} \cong M(1 + i\phi) \quad \dots \quad (5)$$

where the approximation of small  $\phi$  has been introduced. The real part of the complex modulus is  $M$  and the ratio of the imaginary part to the real part is the phase angle  $\phi$ , which is our present measure of internal friction. The complex modulus, therefore, contains the essential parameters pertaining to the vibration.

The occurrence of non-elastic behaviour means more than the fact that  $\phi \neq 0$ . In order to discuss the concept of non-elastic behaviour we must consider the law that describes the interrelation of stress and strain, not only under conditions of oscillation, but also under static conditions, e.g. the application of a constant stress. A *perfectly elastic substance* is one for which the strain is uniquely determined by the stress at any instant, i.e. a monotonic functional relationship exists between the two quantities. This function only deviates from Hooke's law of proportionality when the strain is large. Since we are concerned entirely with small deformations, a simple law of proportionality may always be used to represent the stress-strain equation for a perfectly elastic substance. A material that shows *non-elastic behaviour* is one for which the stress-strain equation is not a simple proportionality but assumes a generalized form, in which, for example, there may be terms involving time derivatives of stress and strain or terms that represent multiple-valued functional relationships. For self consistency with the requirement of small damping, we may limit ourselves to stress-strain equations where all these non-elastic terms are very much smaller than the elastic terms. If the stress-strain equation is linear in stress, strain, and time derivatives of these quantities, it can be solved by the substitution of equations (1) and (2) to obtain the complex modulus directly. If, as a result of the non-elastic behaviour, terms which are nonlinear appear in the stress-strain equation, an exact solution can no longer be obtained by the substitution of a simple harmonic stress and strain, but requires complete Fourier series for these quantities, i.e. stress and strain may be *periodic* but not sinusoidal. An approximate solution can be obtained<sup>13</sup> if we ignore everything but the fundamental Fourier components of stress and strain and determine the complex modulus from these quantities. The parameters  $M$  and  $\phi$  obtained from the real and imaginary parts of this complex modulus will depend on the amplitude of vibration, as a consequence of the nonlinear terms. We then speak of "amplitude dependent internal friction" and, in general, also of an "amplitude dependent dynamic modulus." This method, which converts a nonlinear problem into an approximately equivalent linear

problem with parameters dependent on the amplitude of vibration, is known as the *method of equivalent linearization*.<sup>13</sup> The approximation is a good one when the nonlinear terms are small, a condition which is certainly valid in the present case, since all nonlinear terms are among the small non-elastic terms.

We will now consider two well-known theories of non-elastic behaviour which lead to very simple stress-strain equations.

### *The Voigt Solid*

In order to account for non-elastic behaviour of materials, VOIGT<sup>14</sup> made the assumption (previously introduced by STOKES<sup>15</sup> and others) that, in addition to the elastic stress  $M\varepsilon$  there is a frictional resistance proportional to the rate of strain  $\dot{\varepsilon}$ . We, therefore, obtain the stress-strain equation

$$\sigma = M\varepsilon + \eta\dot{\varepsilon} \quad . . . . \quad (6)$$

This is a linear equation and, therefore, leads to an amplitude independent internal friction. A material that can be described in terms of equation (6) is called a Voigt solid. The complex modulus is  $\tilde{M} = M + i\omega\eta$ , so that the internal friction is given by

$$\phi = (\eta/M)\omega \quad . . . . \quad (7)$$

The internal friction of a Voigt solid is, therefore, proportional to the frequency of vibration and its dynamic modulus is a constant (independent of frequency).

The Voigt assumptions are most often used in textbooks on theoretical physics,<sup>16</sup> to discuss the damped oscillations of a mass point attached to a weightless spring. The purpose of such treatments is usually to show how internal friction manifests itself in free and forced vibrations. No attention is usually paid to the question of the frequency dependence of internal friction.

### *The Maxwell Solid*

An alternative description of non-elastic behaviour is due to MAXWELL.<sup>17</sup> Rather than use the nebulous concept of an internal frictional force, this approach first assumes that the strain is made up of two parts:

$$\varepsilon = \varepsilon' + \varepsilon'' \quad . . . . \quad (8)$$

where  $\varepsilon'$  is the perfectly elastic strain which obeys Hooke's law

$$\varepsilon' = \sigma/M' \quad . . . . \quad (9)$$

and  $\varepsilon''$  is the *non-elastic strain* which results from internal re-arrangement and is not proportional to the stress. Equation (9) is to be regarded as the definition of  $M'$ . MAXWELL further assumes that the non-elastic

term  $\epsilon''$  obeys the Newtonian viscosity equation, i.e. the flow  $\epsilon''$  is proportional to the stress, or

$$\epsilon'' = \xi \sigma \quad . . . . (10)$$

The stress-strain equation is obtained by combining equations (8), (9) and (10):

$$\dot{\epsilon} = \frac{\dot{\sigma}}{M'} + \xi \sigma \quad . . . . (11)$$

From this equation and the assumption of small internal friction, we obtain

$$M = M'; \quad \phi = \xi M' / \omega \quad . . . . (12)$$

Again the dynamic modulus is a constant, but now  $\phi$  is inversely proportional to the frequency. The amplitude independence of  $\phi$  is related to the linearity of equation (11).

It has been known for many years that the experimentally observed frequency dependence of internal friction, for most materials, is in agreement with neither the VOIGT nor the MAXWELL descriptions of non-elastic behaviour. Another shortcoming of these descriptions is the prediction, in both cases, that the dynamic modulus of the system is unaffected by the non-elastic behaviour, except for terms of second order in the internal friction which are omitted here. Experimental observations show that the occurrence of internal friction is often accompanied by a fractional decrease in the dynamic modulus, of the same order of magnitude as the internal friction.

Various methods have been suggested for the generalization of the Voigt and Maxwell solids to obtain a more widely applicable theory. These methods regard the properties of an actual solid as a superposition of those of a large collection (finite or infinite) of Voigt or of Maxwell solids each with different parameters. In this way it is possible to introduce into the description of the non-elastic behaviour of a material as many arbitrary parameters as are necessary to describe its observed behaviour. Such superposition methods have been particularly popular in the study of mechanical properties of high polymers.<sup>18, 19</sup> They are limited, however, to the treatment of materials whose internal friction is independent of the amplitude of vibration, and also, cannot be used for the study of static hysteresis (described in a later section). It will, therefore, be more suitable for us to introduce a different approach, which is sufficiently general to include all phenomena that lead to internal friction in metals.

#### *The Generalized Maxwell Approach*

The first assumption of the Maxwell description is that a material under stress may undergo internal re-arrangement leading to a non-

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elastic strain  $\varepsilon''$  in addition to the elastic strain  $\varepsilon'$ . This assumption, expressed by equations (8) and (9), is very generally applicable to non-elastic phenomena. On the other hand, we will drop the highly restrictive second assumption expressed by equation (10) and, for the present, leave unspecified the equation that describes the non-elastic strain  $\varepsilon''$ . Without such an equation, we cannot obtain the stress-strain relation required to complete the description of the behaviour of a material. Possible equations for  $\varepsilon''$  will be discussed in a later section; at present, phase relationships will be discussed in general terms.

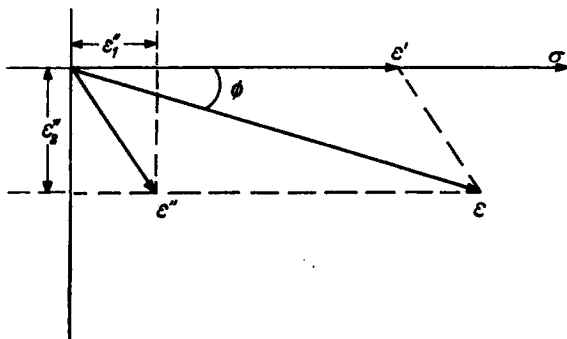


Fig. 1. Phase relationships between the stress and the elastic and non-elastic strain

If the stress (or its fundamental Fourier component) is represented by equation 1, the elastic strain is,

$$\varepsilon' = \varepsilon_1' e^{i\omega t} \quad . . . . (13)$$

The non-elastic strain  $\varepsilon''$  will, in general, be resolvable into a component in phase with the stress and one lagging by  $90^\circ$ , or

$$\varepsilon'' = (\varepsilon_1'' - i\varepsilon_2'') e^{i\omega t} \quad . . . . (14)$$

where the quantities  $\varepsilon_1'$ ,  $\varepsilon_1''$  and  $\varepsilon_2''$  are appropriate amplitudes and are real and positive quantities. Fig. 1 shows the phase relationships between stress and components of strain. This diagram is not drawn to scale, since the assumption of small internal friction and non-elastic effects means that  $|\varepsilon''| \ll |\varepsilon'|$ . With this approximation  $\varepsilon_1 \cong \varepsilon_1'$  (to first order), and equation (3) becomes

$$\phi \cong \varepsilon_2'' / \varepsilon_1' \quad . . . . (15)$$

The internal friction is, to the first approximation, dependent only on the component of  $\varepsilon''$  out of phase with stress. The result is not surprising since only the component of strain out of phase with the stress will

contribute to energy dissipation. The dynamic modulus  $M$  is given by (see equation (4))

$$M = \frac{\sigma_1}{\varepsilon_1' + \varepsilon_1''} \cong M'(1 - \varepsilon_1''/\varepsilon_1') \quad \dots \quad (16)$$

where  $M'$  is defined by equation (9). Thus, since the presence of non-elastic behaviour may increase the in-phase component of strain, the dynamic modulus is less than (or equal to)  $M'$ , and the difference,  $\Delta M = M' - M$  is given by

$$\Delta M/M = \varepsilon_1''/\varepsilon_1' \quad \dots \quad (17)$$

This lowering of the elastic modulus because of the existence of a non-elastic strain will be called the  $\Delta M$  effect. This discussion has, therefore, shown that internal friction and the  $\Delta M$  effect are closely related consequences of the occurrence of non-elastic strain, the first resulting from the component out of phase with stress, and the second from the component in phase with the stress. It is also clear that if the equation, thus far unspecified, that describes  $\varepsilon''$  is nonlinear, the ratios  $\varepsilon_2''/\varepsilon_1'$  and  $\varepsilon_1''/\varepsilon_1'$  will be functions of strain amplitude. Specific forms for this equation will be considered in the section called "Viscoelasticity and Static Hysteresis."

#### *Measures of Internal Friction*

The relationship between various measures of internal friction, as well as of the  $\Delta M$  effect, will now be considered. As long as the discussion of non-elastic behaviour is restricted to the use of quantities such as the complex modulus and phase angle  $\phi$ , it is possible to treat in the same way, either a small element of material or a complete specimen under homogeneous strain. If quantities such as the logarithmic decrement and the width of a resonance peak are to be considered, it is necessary to deal with an entire specimen in a particular mode of vibration. For simplicity, we will discuss here only specimens vibrating under homogeneous strain, and in the next section will show how the present discussion is generalized when the strain is not homogeneous. It is required, then, that a relatively large inertia member be attached to the specimen under consideration. If  $\theta$  represents the displacement of the inertia member,  $I$  its "inertia,"  $F_s$  and  $F_a$  the "force" exerted upon this member by the specimen and the externally applied "force," respectively, then the equation of motion of this member is

$$I\ddot{\theta} = F_s + F_a \quad \dots \quad (18)$$

(In the example of the torsion of a hollow tube mentioned earlier,  $\theta$  is simply the angle of twist,  $I$  the moment of inertia, and the quantities

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$F$  are torques. On the other hand, if the vibration is under a longitudinal load,  $\theta$  becomes the longitudinal displacement,  $I$  the attached mass, and  $F$  a force.) The quantity  $\theta$  is essentially the strain,  $\varepsilon$ , throughout the specimen, except for a geometric constant of proportionality, while  $F_e$  is proportional to the stress  $\sigma$ , but opposite in sign. Inasmuch as  $\sigma = \tilde{M}\varepsilon$  (by the definition of  $\tilde{M}$ , equation (5)), with  $\tilde{M} = M(1 + i\phi)$ , we may also write

$$F_e = -\tilde{k}\theta; \quad \tilde{k} = k(1 + i\phi) \quad . . . . (19)$$

where  $\tilde{k}$  is a complex "force constant." Equation (18) may then be solved in the case of *forced vibrations* with an impressed angular frequency  $\omega$ , i.e. when the applied "force" is taken as sinusoidal of the form  $F_e = F_0 e^{i\omega t}$ . Under steady state conditions, the equation will have a solution of the form  $\theta = \theta_0 e^{i\omega t}$ , where  $\theta_0$  is, in general, complex. (If the stress-strain equation is nonlinear,  $\theta$  will actually be representable by a complete Fourier series, but as mentioned earlier, we will ignore all but the fundamental Fourier components.) When this solution and equation (19) are substituted into equation (18), an equation for  $\theta_0$  is obtained:

$$\theta_0 = \frac{F_0/I}{(\omega_r^2 - \omega^2) + i\omega_r^2\phi} \quad . . . . (20)$$

where

$$\omega_r^2 = k/I \quad . . . . (21)$$

The real quantity  $|\theta_0|^2$  is given by

$$|\theta_0|^2 = \frac{(F_0/I)^2}{(\omega_r^2 - \omega^2)^2 + \omega_r^4\phi^2} \quad . . . . (22)$$

This is the well-known equation for a resonance curve (a plot of  $|\theta_0|^2$  versus  $\omega$ ) with a resonant angular frequency (at which the amplitude of vibration is a maximum) equal to  $\omega_r$ . If the frequencies at which  $|\theta_0|^2$  falls to half the maximum value are denoted by  $\omega_1$  and  $\omega_2$  it follows that

$$\phi = \frac{\omega_2 - \omega_1}{\omega_r} \equiv \frac{1}{Q} \quad . . . . (23)$$

The internal friction  $\phi$  may, therefore, be obtained in forced vibration as the ratio of the width of the resonance curve (square of displacement amplitude versus frequency) at half maximum divided by the resonant frequency. Inasmuch as the reciprocal of this ratio is called the " $Q$ " of an electrical circuit, the symbol  $Q^{-1}$  has been adopted by some authors<sup>1, 20</sup> as a measure of internal friction; from equation (23),  $Q^{-1}$  is equal to the phase angle  $\phi$  between stress and strain. The study of internal friction by means of a resonance curve is complicated when the

internal friction is amplitude dependent. When  $\phi$  is itself a function of  $|\theta_0|$ , and in fact (as commonly observed experimentally) an increasing function of  $|\theta_0|$ , the height of the resonance peak will be determined by the value of  $\phi$  at maximum amplitude, but the peak is broader than for the case of constant internal friction. Although the resonance peak is broadened by the amplitude dependence of  $\phi$ , it may still be symmetrical about  $\omega = \omega_r$  (to terms of first order in the internal friction) so long as there is no  $\Delta M$  effect. In general, however, amplitude dependent internal friction is accompanied by an amplitude dependent dynamic modulus  $M$ . This means that  $k$  is a function of amplitude and, therefore, that  $\omega_r$  as well as  $\phi$ , in equation (22), is dependent on  $|\theta_0|$ . In the usual case, where the  $\Delta M$  effect increases (and, therefore,  $\omega_r$  decreases) with increasing amplitude, it is readily shown from equation (22) as well as experimentally<sup>11</sup> that the resonance curve falls off faster on the low frequency side than on the high frequency side. If the amplitude dependence of  $\omega_r$  is sufficiently large, this asymmetry may become so great that the resonance curve bends over on itself and becomes double valued on the low frequency side. Thus, discontinuities may be observed in tracing this curve experimentally.<sup>21</sup>

When amplitude dependent effects are studied by forced vibration methods, it is usually preferable to work at exactly the resonant frequency where, from equation (22), the amplitude at resonance  $|\theta_0^m|$  is

$$|\theta_0^m| = F_0 / (I\omega_r^2\phi) \quad . . . . \quad (24)$$

It is necessary to determine the maximum amplitude  $|\theta_0^m|$  and the force  $F_0$  in order to obtain  $\phi$ ; both  $\phi$  and  $\omega_r$  are then known at the specific amplitude  $|\theta_0^m|$ . By variation of  $F_0$  it is then possible to obtain these values at any other displacement amplitude. Whereas, in principle this method is a very simple one for determination of the internal friction and  $\Delta M$  effect as a function of amplitude, the quantities  $F_0$  and  $|\theta_0^m|$  are often difficult to obtain experimentally. In the piezoelectric method (to be discussed in a subsequent section) these quantities are readily calculated, so that this method has been particularly suitable to the study of amplitude dependent internal friction. In other methods of forced vibration, it is more convenient to keep  $F_0$  constant and determine  $Q^{-1}$  (equation (23)), which requires only a knowledge of *relative* amplitudes and does not require that  $F_0$  be known. The latter methods are, therefore, only useful when internal friction is independent of amplitude.

In addition to the possibility for studying internal friction by the determination of the resonance curve in forced vibrations, the oldest and most popular methods are related to determination of the *damping of free vibrations*, where vibrations, once excited, are allowed to continue,

## INTERNAL FRICTION IN METALS

with no external forces applied to the system; the rate of decrease of amplitude is then determined. In order to solve the equation of motion 18, for  $F_a = 0$  (no external forces), we substitute a solution of the form

$$\theta = \theta_0 e^{i\omega t}; \quad \tilde{\omega} = \omega(1 + i\delta/2\pi)$$

which represents exponentially damped oscillations, if  $\delta$  is a constant. ( $\theta_0$  is now real.) When this solution is substituted into equation (18), and cognizance taken of equation (19), we obtain:  $I\tilde{\omega}^2 = \tilde{k}$ ; if real and imaginary parts are now separately equated, and second order terms in internal friction neglected, it is found that

$$\omega^2 = k/I; \quad \delta = \pi\phi \quad . . . . (25)$$

The quantity  $\delta$  represents the natural logarithm of the ratio of amplitudes in two successive vibrations and is called the *logarithmic decrement* (or simply, the "decrement"). When this quantity is small (as always assumed in this discussion) it is also equal to the fractional decrease in vibrational amplitude per cycle. In order to obtain  $\delta$  from experiment, it is simply necessary to plot the logarithm of the amplitude of vibration against the number of cycles of vibration. If internal friction is independent of amplitude a straight line is obtained whose slope gives the decrement directly. However, if internal friction is amplitude dependent, this curve will not be a straight line, but its slope at any amplitude will give the decrement at that particular amplitude. Thus an entire curve of decrement versus amplitude of vibration may be obtained from a single decay curve if the precision of the data is sufficiently good.

Equations (21) and (25) also show that the natural frequency in free vibration and the resonant frequency  $\omega_r$  in forced vibration are the same. This resonant frequency is affected by the existence of a  $\Delta M$  effect; under conditions where non-elastic behaviour does not occur, its value would be  $\omega' = (k'/I)^{1/2}$ , where  $k'$  is the force constant corresponding to the modulus  $M'$ . Because of the  $\Delta M$  effect, there will be a decrease in resonant frequency and, if we define  $\Delta\omega = \omega' - \omega_r$ , we find that

$$\Delta\omega/\omega_r = \frac{1}{2}\Delta M/M \quad . . . . (26)$$

Thus, the existence of a  $\Delta M$  effect is synonymous with a decrease in the resonant frequency or in the frequency of free vibration. Also, as already noted, amplitude dependent internal friction is usually accompanied by an amplitude dependent dynamic modulus. This effect is detected by a change of resonant frequency, or of the frequency of free vibration, with a change in the amplitude of vibration.

Another measure of internal friction is the fractional decrease in vibrational energy per cycle. Inasmuch as  $\delta$  is the fractional decrease



in amplitude per cycle, and the vibrational energy is proportional to the square of the amplitude, it follows that

$$\delta = \Delta W / 2W$$

where  $W$  is the vibrational energy and  $\Delta W$  is the loss of vibrational energy in one cycle. A closely related quantity which is widely used in engineering applications is the *damping capacity* which is the per cent decrease in vibrational energy per cycle, so that: damping capacity =  $200\delta$ .

The relation between the various measures of internal friction are summarized as follows:

$$\phi = \delta / \pi = \Delta W / 2\pi W = Q^{-1} \quad . . . . (27)$$

The last equality applies only when internal friction is amplitude independent. These simple relationships between the various measures of internal friction are derived under the assumption of small damping and break down when this assumption is no longer valid. For large damping, the relations between different measures of internal friction depend on the mechanism of the damping and must be derived separately for each case.<sup>22</sup>

The measured values of  $\phi$  and  $\Delta M / M$  will not only depend upon the frequency and possibly the amplitude of vibration, but will also depend on the type of stress employed. The relation between values obtained in shear and in longitudinal vibration, for example, will depend on the mechanism of the non-elastic behaviour.

#### PHENOMENOLOGICAL THEORY: NON-HOMOGENEOUS STRAIN\*

In practice, damping is rarely, if ever, studied under conditions where the mass is all concentrated in an auxiliary inertia member, and the stress and strain are homogeneous throughout the "elastic" member. Very often, the material whose elastic and non-elastic properties are to be studied, will be in the form of a bar which is vibrated in a longitudinal, transverse, or torsional mode of oscillation. Under these circumstances the stress amplitude is not constant throughout the specimen. It will be shown in this section, however, that the interpretation of vibration experiments on these more realistic specimens is completely analogous to the case of homogeneous strain, except that special attention is required when the internal friction is amplitude dependent.

It is well known<sup>23</sup> that a perfectly elastic system will vibrate in certain normal modes of vibration characteristic of the system. Once it is vibrating in one normal mode it will continue to do so indefinitely.

\* The reader who is not interested in the details of this section may omit the section without loss of continuity.