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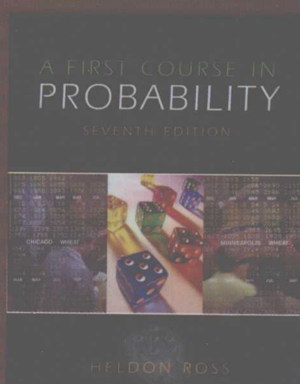
A First Course in Probability

概率论基础教程

（英文版 · 第 7 版）

[美] Sheldon M. Ross 著



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内 容 提 要

本书是全球高校广泛采用的概率论教材，通过大量的例子讲述了概率论的基础知识，主要内容有组合分析、概率论公理化、条件概率和独立性、离散和连续型随机变量、随机变量的联合分布、期望的性质、极限定理等．本书附有大量的练习，分为习题、理论习题和自检习题三大类，其中自检习题部分还给出全部解答．

本书作为概率论的入门书，适用于大专院校数学、统计、工程和相关专业（包括计算科学、生物、社会科学和管理科学）的学生阅读，也可供概率应用工作者参考．

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概率论基础教程（英文版 • 第 7 版）

◆ 著　[美] Sheldon M. Ross
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前　言

法国著名数学家和天文学家拉普拉斯侯爵（人称“法国的牛顿”）曾经说过：“我们发现概率论其实就是将常识问题归结为计算. 它使我们能够精确地评价凭某种直观感受到的、往往又不能解释清楚的见解……值得注意的是，概率论这门起源于机会游戏的科学，早就应该成为人类知识中最重要的组成部分……生活中那些最重要的问题绝大部分恰恰是概率论问题.”尽管许多人认为，这位对概率论的发展做出过重大贡献的著名侯爵说话有点过头，然而今日，概率论已经成为几乎所有的科学工作者、工程师、医务人员、法律工作者以及企业家们手中的基本工具，这是一个不争的事实. 事实上，现代人们不再问“是这样吗？”，而是问：“这件事发生的概率有多大？”

本书试图成为概率论的入门书. 读者对象是数学、统计、工程和其他专业（包括计算机科学、生物学、社会科学和管理科学）的学生. 他们的先修知识只是初等微积分. 本书试图介绍概率论的数学理论，同时通过大量例子说明这门学科的广泛的应用.

第1章介绍了组合分析的基本原理，它是计算概率的最有效的工具.

第2章介绍了概率论的公理体系，并且指出如何应用这些公理进行概率计算.

第3章讨论概率论中极为重要的概念，即事件的条件概率和事件间的独立性. 通过一系列例子说明当部分信息可利用时，条件概率就会发挥它的作用；即使在没有这部分信息时，条件概率也可以使概率的计算变得容易、可行. 利用“条件”计算概率这一极为重要的技巧还将出现在第7章，在那里我们用它来计算期望.

在第4章至第6章，我们引进随机变量的概念，第4章讨论离散随机变量，第5章讨论连续随机变量，而将随机变量的联合分布放在第6章. 在第4章和第5章中讨论了随机变量的期望和方差，并且对许多常见的随机变量，求出了相应的期望和方差.

第7章讨论了期望值和它的一些重要的性质. 书中引入了许多例子，解释如何利用随机变量和的期望等于随机变量期望的和这一重要规律来计算随机变量的期望，本章中还有几节介绍条件期望（包括它在预测方面的应用）和矩母函数等. 最后一节介绍了多元正态分布，同时给出了来自正态总体的样本均值和样本方差的联合分布的简单证明.

在第8章我们介绍了概率论的主要的理论结果. 特别地，我们证明了强大数定律和中心极限定理. 关于强大数定律的证明，我们假定随机变量具有有限的四阶矩. 在这种假定之下，证明十分简单. 在中心极限定理的证明中，我们假定了莱维（Lévy）的连续性定理成立. 在本章中，我们还介绍了若干概率不等式，如马尔可

夫不等式、切比雪夫不等式和切尔诺夫界. 在最后一节，我们给出用随机变量的相应概率去近似独立伯努利随机变量和的相关概率的误差界.

第9章介绍了一些附加课题，如马尔可夫链、泊松过程以及信息编码理论初步. 第10章介绍了统计模拟.

第7版将教材内容进一步扩充与调整，加入了很多新的习题和例子. 其中第3章例3h进一步展示了e的无处不在；第3章例5f讨论了信息的序贯修正；第4章例7d利用泊松近似的方法，证明了n次抛掷硬币试验中，正面朝上的最大游程的长度以0.86的概率落入$\log_2(n)\pm 2$的区域内. 同时引入了关于优惠收集的若干新的例子（第3章例4i，第7章例3d和例3f等）和多项分布的例子（第6章例4c）. 本版还加入了许多新的内容. 例如增加了第2章命题4.4的附注. 在关于事件和的概率等式中，若取前面若干项可依次得到事件和的概率的上下界. 7.3节是新编入的一节，它讨论一串事件发生次数的矩的计算方法. 作为例子，导出了二项、超几何、配对问题以及负几何随机变量的矩的公式. 关于二元正态分布，也加入了若干新材料，在第6章例5c，导出了它的条件分布和边缘分布. 第7章例5f中计算了两个变量的相关系数，并利用相关系数分析了第7章例8b中的贝叶斯统计的例子.

与前几版一样，每章后面附了三组练习题，它们分别命名为**习题**、**理论习题**和**自检习题**. 在附录B中提供了自检习题的全部解答，以供学生检验他们的理解能力.

本书前几版曾带有磁盘，包含有概率模型部分的材料，现在这些内容可从本书配套网站下载：http://www.prenhall.com/Ross.

学生利用网站可在以下6个方面快速计算和模拟：

- 有3个模块可进行二项、泊松和正态随机变量的计算.
- 另一个模块演示中心极限定理，考虑取0, 1, 2, 3, 4, 共5个值的随机变量，容许使用者输入相应的分布和样本量n. 模块将显示n个独立随机变量和的分布列，当n增加时，能“看”到其分布列收敛到正态分布的密度函数的形状.
- 其他2个模块演示强大数定律，使用者可以输入5个可能值的概率以及样本量n. 模块利用随机数模拟具有指定分布的一组样本. 模块将各个结果出现的次数用图形显示出来，同时给出样本均值. 两个模块在显示试验结果上稍有差别. 我们感谢下列对本书各个版本给出十分有价值的意见的人们：

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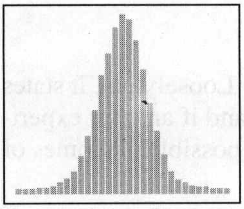

Chapter 1

Combinatorial Analysis

1.1 INTRODUCTION

Here is a typical problem of interest involving probability. A communication system is to consist of n seemingly identical antennas that are to be lined up in a linear order. The resulting system will then be able to receive all incoming signals—and will be called *functional*—as long as no two consecutive antennas are defective. If it turns out that exactly m of the n antennas are defective, what is the probability that the resulting system will be functional? For instance, in the special case where $n = 4$ and $m = 2$ there are 6 possible system configurations—namely,

0	1	1	0
0	1	0	1
1	0	1	0
0	0	1	1
1	0	0	1
1	1	0	0

where 1 means that the antenna is working and 0 that it is defective. As the resulting system will be functional in the first 3 arrangements and not functional in the remaining 3, it seems reasonable to take $\frac{3}{6} = \frac{1}{2}$ as the desired probability. In the case of general n and m, we could compute the probability that the system is functional in a similar fashion. That is, we could count the number of configurations that result in the system being functional and then divide by the total number of all possible configurations.

From the preceding we see that it would be useful to have an effective method for counting the number of ways that things can occur. In fact, many problems in probability theory can be solved simply by counting the number of different ways that a certain event can occur. The mathematical theory of counting is formally known as *combinatorial analysis*.

1.2 THE BASIC PRINCIPLE OF COUNTING

The following principle of counting will be basic to all our work. Loosely put, it states that if one experiment can result in any of m possible outcomes and if another experiment can result in any of n possible outcomes, then there are mn possible outcomes of the two experiments.

The basic principle of counting
Suppose that two experiments are to be performed. Then if experiment 1 can result in any one of m possible outcomes and if for each outcome of experiment 1 there are n possible outcomes of experiment 2, then together there are mn possible outcomes of the two experiments.

Proof of the Basic Principle: The basic principle may be proved by enumerating all the possible outcomes of the two experiments as follows:

$$\begin{matrix} (1,1), & (1,2), & \ldots, & (1,n) \\ (2,1), & (2,2), & \ldots, & (2,n) \\ & \vdots & & \\ (m,1), & (m,2), & \ldots, & (m,n) \end{matrix}$$

where we say that the outcome is (i, j) if experiment 1 results in its ith possible outcome and experiment 2 then results in the jth of its possible outcomes. Hence the set of possible outcomes consists of m rows, each row containing n elements, which proves the result.

EXAMPLE 2a

A small community consists of 10 women, each of whom has 3 children. If one woman and one of her children are to be chosen as mother and child of the year, how many different choices are possible?

Solution. By regarding the choice of the woman as the outcome of the first experiment and the subsequent choice of one of her children as the outcome of the second experiment, we see from the basic principle that there are $10 \times 3 = 30$ possible choices. ■

When there are more than two experiments to be performed, the basic principle can be generalized as follows.

The generalized basic principle of counting
If r experiments that are to be performed are such that the first one may result in any of n_1 possible outcomes, and if for each of these n_1 possible outcomes there are n_2 possible outcomes of the second experiment, and if for each of the

possible outcomes of the first two experiments there are n_3 possible outcomes of the third experiment, and if ..., then there is a total of $n_1 \cdot n_2 \cdots n_r$ possible outcomes of the r experiments.

EXAMPLE 2b

A college planning committee consists of 3 freshmen, 4 sophomores, 5 juniors, and 2 seniors. A subcommittee of 4, consisting of 1 person from each class, is to be chosen. How many different subcommittees are possible?

Solution. We may regard the choice of a subcommittee as the combined outcome of the four separate experiments of choosing a single representative from each of the classes. Hence it follows from the generalized version of the basic principle that there are $3 \times 4 \times 5 \times 2 = 120$ possible subcommittees. ■

EXAMPLE 2c

How many different 7-place license plates are possible if the first 3 places are to be occupied by letters and the final 4 by numbers?

Solution. By the generalized version of the basic principle the answer is $26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 175{,}760{,}000$. ■

EXAMPLE 2d

How many functions defined on n points are possible if each functional value is either 0 or 1?

Solution. Let the points be $1, 2, \ldots, n$. Since $f(i)$ must be either 0 or 1 for each $i = 1, 2, \ldots, n$, it follows that there are 2^n possible functions. ■

EXAMPLE 2e

In Example 2c, how many license plates would be possible if repetition among letters or numbers were prohibited?

Solution. In this case there would be $26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 \cdot 7 = 78{,}624{,}000$ possible license plates. ■

1.3 PERMUTATIONS

How many different ordered arrangements of the letters a, b, and c are possible? By direct enumeration we see that there are 6: namely, *abc*, *acb*, *bac*, *bca*, *cab*, and *cba*. Each arrangement is known as a *permutation*. Thus there are 6 possible permutations of a set of 3 objects. This result could also have been obtained from the basic principle, since the first object in the permutation can be any of the 3, the second object in the permutation can then be chosen from any of the remaining 2, and the third object in the permutation is then chosen from the remaining 1. Thus there are $3 \cdot 2 \cdot 1 = 6$ possible permutations.

Suppose now that we have n objects. Reasoning similar to that we have just used for the 3 letters shows that there are

$$n(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1 = n!$$

different permutations of the n objects.

EXAMPLE 3a

How many different batting orders are possible for a baseball team consisting of 9 players?

Solution. There are $9! = 362{,}880$ possible batting orders. ■

EXAMPLE 3b

A class in probability theory consists of 6 men and 4 women. An examination is given, and the students are ranked according to their performance. Assume that no two students obtain the same score.

(a) How many different rankings are possible?
(b) If the men are ranked just among themselves and the women among themselves, how many different rankings are possible?

Solution. (a) As each ranking corresponds to a particular ordered arrangement of the 10 people, we see that the answer to this part is $10! = 3{,}628{,}800$.

(b) As there are $6!$ possible rankings of the men among themselves and $4!$ possible rankings of the women among themselves, it follows from the basic principle that there are $(6!)(4!) = (720)(24) = 17{,}280$ possible rankings in this case. ■

EXAMPLE 3c

Ms. Jones has 10 books that she is going to put on her bookshelf. Of these, 4 are mathematics books, 3 are chemistry books, 2 are history books, and 1 is a language book. Jones wants to arrange her books so that all the books dealing with the same subject are together on the shelf. How many different arrangements are possible?

Solution. There are $4!\ 3!\ 2!\ 1!$ arrangements such that the mathematics books are first in line, then the chemistry books, then the history books, and then the language book. Similarly, for each possible ordering of the subjects, there are $4!\ 3!\ 2!\ 1!$ possible arrangements. Hence, as there are $4!$ possible orderings of the subjects, the desired answer is $4!\ 4!\ 3!\ 2!\ 1! = 6912$. ■

We shall now determine the number of permutations of a set of n objects when certain of the objects are indistinguishable from each other. To set this straight in our minds, consider the following example.

EXAMPLE 3d

How many different letter arrangements can be formed using the letters $PEPPER$?

Solution. We first note that there are 6! permutations of the letters $P_1E_1P_2P_3E_2R$ when the 3P's and the 2E's are distinguished from each other. However, consider any one of these permutations—for instance, $P_1P_2E_1P_3E_2R$. If we now permute the P's among themselves and the E's among themselves, then the resultant arrangement would still be of the form $PPEPER$. That is, all 3! 2! permutations

$$\begin{array}{ll} P_1P_2E_1P_3E_2R & P_1P_2E_2P_3E_1R \\ P_1P_3E_1P_2E_2R & P_1P_3E_2P_2E_1R \\ P_2P_1E_1P_3E_2R & P_2P_1E_2P_3E_1R \\ P_2P_3E_1P_1E_2R & P_2P_3E_2P_1E_1R \\ P_3P_1E_1P_2E_2R & P_3P_1E_2P_2E_1R \\ P_3P_2E_1P_1E_2R & P_3P_2E_2P_1E_1R \end{array}$$

are of the form $PPEPER$. Hence there are $6!/(3!\ 2!) = 60$ possible letter arrangements of the letters $PEPPER$. ■

In general, the same reasoning as that used in Example 3d shows that there are

$$\frac{n!}{n_1!\, n_2!\, \cdots\, n_r!}$$

different permutations of n objects, of which n_1 are alike, n_2 are alike, ..., n_r are alike.

EXAMPLE 3e

A chess tournament has 10 competitors of which 4 are Russian, 3 are from the United States, 2 from Great Britain, and 1 from Brazil. If the tournament result lists just the nationalities of the players in the order in which they placed, how many outcomes are possible?

Solution. There are

$$\frac{10!}{4!\ 3!\ 2!\ 1!} = 12{,}600$$

possible outcomes. ■

EXAMPLE 3f

How many different signals, each consisting of 9 flags hung in a line, can be made from a set of 4 white flags, 3 red flags, and 2 blue flags if all flags of the same color are identical?

Solution. There are

$$\frac{9!}{4!\ 3!\ 2!} = 1260$$

different signals. ■

1.4 COMBINATIONS

We are often interested in determining the number of different groups of r objects that could be formed from a total of n objects. For instance, how many different groups of 3 could be selected from the 5 items A, B, C, D, and E? To answer this, reason as follows: Since there are 5 ways to select the initial item, 4 ways to then select the next item, and 3 ways to select the final item, there are thus $5 \cdot 4 \cdot 3$ ways of selecting the group of 3 when the order in which the items are selected is relevant. However, since every group of 3, say, the group consisting of items A, B, and C, will be counted 6 times (that is, all of the permutations ABC, ACB, BAC, BCA, CAB, and CBA will be counted when the order of selection is relevant), it follows that the total number of groups that can be formed is

$$\frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} = 10$$

In general, as $n(n-1)\cdots(n-r+1)$ represents the number of different ways that a group of r items could be selected from n items when the order of selection is relevant, and as each group of r items will be counted $r!$ times in this count, it follows that the number of different groups of r items that could be formed from a set of n items is

$$\frac{n(n-1)\cdots(n-r+1)}{r!} = \frac{n!}{(n-r)!\,r!}$$

Notation and terminology

We define $\binom{n}{r}$, for $r \le n$, by

$$\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$$

and say that $\binom{n}{r}$ represents the number of possible combinations of n objects taken r at a time.†

Thus $\binom{n}{r}$ represents the number of different groups of size r that could be selected from a set of n objects when the order of selection is not considered relevant.

EXAMPLE 4a

A committee of 3 is to be formed from a group of 20 people. How many different committees are possible?

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†By convention, $0!$ is defined to be 1. Thus $\binom{n}{0} = \binom{n}{n} = 1$. We also take $\binom{n}{i}$ to be equal to 0 when either $i < 0$ or $i > n$.

Solution. There are $\binom{20}{3} = \dfrac{20 \cdot 19 \cdot 18}{3 \cdot 2 \cdot 1} = 1140$ possible committees. ■

EXAMPLE 4b

From a group of 5 women and 7 men, how many different committees consisting of 2 women and 3 men can be formed? What if 2 of the men are feuding and refuse to serve on the committee together?

Solution. As there are $\binom{5}{2}$ possible groups of 2 women, and $\binom{7}{3}$ possible groups of 3 men, it follows from the basic principle that there are $\binom{5}{2}\binom{7}{3} = \left(\dfrac{5 \cdot 4}{2 \cdot 1}\right)\dfrac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} =$ 350 possible committees consisting of 2 women and 3 men.

Now suppose that 2 of the men refuse to serve together. Because a total of $\binom{2}{2}\binom{5}{1} = 5$ out of the $\binom{7}{3} = 35$ possible groups of 3 men contain both of the feuding men, it follows that there $35 - 5 = 30$ that do not. Because there are still $\binom{5}{2} = 10$ ways to choose the 2 women, it follows that in this case that there are $30 \cdot 10 = 300$ possible committees. ■

EXAMPLE 4c

Consider a set of n antennas of which m are defective and $n - m$ are functional and assume that all of the defectives and all of the functionals are considered indistinguishable. How many linear orderings are there in which no two defectives are consecutive?

Solution. Imagine that the $n - m$ functional antennas are lined up among themselves. Now, if no two defectives are to be consecutive, then the spaces between the functional antennas must each contain at most one defective antenna. That is, in the $n - m + 1$ possible positions—represented in Figure 1.1 by carets—between the $n - m$ functional antennas, we must select m of these in which to put the defective antennas. Hence there are $\binom{n - m + 1}{m}$ possible orderings in which there is at least one functional antenna between any two defective ones. ■

$$\wedge\, 1 \wedge 1 \wedge 1 \ldots \wedge 1 \wedge 1 \wedge$$

1 = functional

$\wedge$ = place for at most one defective

Figure 1.1.