Textbooks for University

Fundamentals of Advanced Mathematics (II)

Zhien Ma Miansen Wang
Fred Brauer

高等教育出版社
Higher Education Press

Fundamentals of Advanced Mathematics (I)

Fundamentals of Advanced Mathematics (II)

ISBN 7-04-017794-3

9 787040 177947 >

定价 36.20 元

高等学校教材

高等数学基础(Ⅱ)

马知恩　王绵森
西安交通大学数学系

Fred Brauer
Department of Mathematics
The University of British Columbia

高 等 教 育 出 版 社
Higher Education Press

图书在版编目(CIP)数据

高等数学基础(Ⅱ). 2=Fundamentals of Advanced Mathematics(Ⅱ)/马知恩,王绵森,(加)布劳尔(Brauer, F.)编.—北京:高等教育出版社,2006.1

ISBN 7-04-017794-3

Ⅰ.高...　Ⅱ.①马...②王...③布...　Ⅲ.高等数学—高等学校—教材—英文　Ⅳ.O13

中国版本图书馆 CIP 数据核字(2005)第 149498 号

策划编辑 徐　刚　　**责任编辑** 董达英　　**封面设计** 刘晓翔
责任绘图 朱　静　　**版式设计** 范晓红　　**责任校对** 王效珍
责任印制 陈伟光

出版发行	高等教育出版社	**购书热线**	010-58581118
社　　址	北京市西城区德外大街 4 号	**免费咨询**	800-810-0598
邮政编码	100011	**网　　址**	http://www.hep.edu.cn
总　　机	010-58581000		http://www.hep.com.cn
		网上订购	http://www.landraco.com
经　　销	蓝色畅想图书发行有限公司		http://www.landraco.com.cn
印　　刷	北京外文印刷厂	**畅想教育**	http://www.widedu.com
开　　本	787×960　1/16	**版　　次**	2006 年 1 月第 1 版
印　　张	29.25	**印　　次**	2006 年 1 月第 1 次印刷
字　　数	520 000	**定　　价**	36.20 元

本书如有缺页、倒页、脱页等质量问题,请到所购图书销售部门联系调换。

物料号　17794-00

郑重声明

About the book

This is the second volume of the textbook "Fundamentals of Advanced Mathematics" written by the same authors. It includes vector algebra and analytic geometry in space, multivariable calculus, and linear ordinary differential equations. The intentions and features are as introduced in the preface to the first volume. We repeat here the important advice to students in the first volume, as it is equally important for this second volume.

In order to learn calculus, it is not enough to read the textbook as if it were a newspaper. Learning requires careful reading, working through examples step by step, and solving problems. Solving problems requires more than imitation of examples. It is necessary to think about what the problem really asks and to develop a method for that particular problem.

If something is still not clear after you have tried to understand it, you should ask a classmate, a more advanced student, or your teacher. If a classmate asks you a question, you may learn a great deal from explaining the answer.

The following two additional remarks might be helpful to readers in using the second volume.

(1) The material on linear systems of ordinary differential equations (Section 9.2) is not included in the fundamental requirements. Before studying it, readers will need some basic knowledge of linear algebra.

(2) Some of the material in this volume has been stated in terms of matrices and determinants. For readers who are not yet familiar with the basic concepts and operations for matrices and determinants we have included a brief outline in Appendix A.

Contents

Chapter 5
Vector Algebra and Analytic Geometry in Space

This chapter introduces the fundamental concepts of vector algebra and analytic geometry in space. These concepts are very important not only for studying the calculus of functions of several variables in the next chapter, but also for use in applications in physics, mechanics, other sciences, and engineering.

5.1 Vectors and Their Linear Operations

5.1.1 The concept of vector

Some of the quantities in nature are determined completely by their magnitudes. For example, to record length, area, mass, temperature, etc., we can represent them by means of real numbers if an appropriate unit of measure is given. These quantities are called **scalar quantities**. But there are also some quantities in nature, such as displacement, velocity, and force, for which we need more information to describe them. To describe a displacement of a body we have to know how far it moves and in what direction. To describe the velocity of a body, we have to know where the body is headed as well as how fast it is going. To describe a force, we need to record the direction in which it acts as well as how large it is. These quantities that have both direction and magnitude, are called **vectors**. A vector is usually represented by a line segment with an arrow, a **directed line segment**. The length of the directed line segment represents the magnitude of the vector and the arrow points in the direction of the vector. The vector defined by the directed line segment from the initial point A to the terminal point B is written as $\overrightarrow{AB}$. In print, for convenience, vectors are also represented by small boldface letters such as $\boldsymbol{a}$, $\boldsymbol{\alpha}$, and $\boldsymbol{x}$ (Fig. 5.1.1). The magnitude of a vector is called the **length** (or **norm**) of the vector. The length of the vectors $\overrightarrow{AB}$ and $\boldsymbol{a}$ are written as $\| \overrightarrow{AB} \|$ and $\| \boldsymbol{a} \|$, respectively.

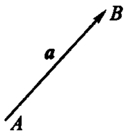

Figure 5.1.1

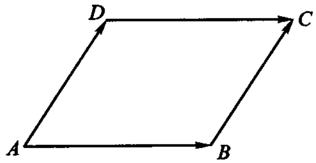

Figure 5.1.2

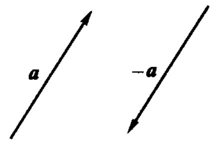

Figure 5.1.3

A vector whose length is 1 is called a **unit vector**; a unit vector whose direction is the same as that of $\boldsymbol{a}$ is written as $\boldsymbol{a}^{\circ}$. A vector whose length is 0 is called the **zero vector** and is written as $\boldsymbol{0}$. The initial point of the zero vector coincides with its terminal point so that it represents a point, and it is also the only vector with no specific direction. It is seen from the definition of vector that a vector is determined completely by its length and direction and is independent of the location of its initial point and terminal point. Therefore, two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are said to be **equal** if they have the same length and direction, denoted by $\boldsymbol{a} = \boldsymbol{b}$. For example, for the parallelogram $ABCD$ in Fig. 5.1.2, we have $\overrightarrow{AB} = \overrightarrow{DC}$, $\overrightarrow{AD} = \overrightarrow{BC}$.

A vector is called **the negative vector** of a vector $\boldsymbol{a}$, if it has the same length as $\boldsymbol{a}$ but its direction is opposite to that of $\boldsymbol{a}$, denoted by $-\boldsymbol{a}$ (Fig. 5.1.3). Obviously, we have $\overrightarrow{AB} = -\overrightarrow{BA}$.

Let $\boldsymbol{a}$ and $\boldsymbol{b}$ be two nonzero vectors. Then $\boldsymbol{a}$ and $\boldsymbol{b}$ are said to be **parallel** or **collinear**, denoted by $\boldsymbol{a} // \boldsymbol{b}$, if their directions are the same or opposite, because in this case, $\boldsymbol{a}$ and $\boldsymbol{b}$ can be moved to the same line by means of parallel translation. The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are said to be **orthogonal** or **perpendicular**, denoted by $\boldsymbol{a} \perp \boldsymbol{b}$ if the directions of $\boldsymbol{a}$ and $\boldsymbol{b}$ are orthogonal.

Suppose that $\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_k$ $(k \geqslant 3)$ are k vectors with a common initial point. If they lie in the same plane, then we say that these vectors are **coplanar**. It is easy to see that any two vectors are coplanar.

5.1.2 Linear operations on vectors

Addition of vectors

In physics, we often need to find the composition of displacements or forces. For instance, consider a particle moving from the point O to the point A, and then moving from the point A to the point B (Fig. 5.1.4). The displacement of the particle is equivalent to the one moving from the point O to the point B. Hence, if we take a vector $\overrightarrow{OA}$ from the initial point O to the terminal point A, and then take a vector $\overrightarrow{AB}$ from the initial point A to the terminal point B, then the total displacement $\overrightarrow{OB}$ is equal to the composition of the two displacements; that is, $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$.

In general, we have the following definition.

Definition 5.1.1 (Triangle law of addition of vectors) Suppose that $\boldsymbol{a}$ and $\boldsymbol{b}$ are any two vectors and O is any point. If we draw a vector $\overrightarrow{OA} = \boldsymbol{a}$ from O to A and then draw the vector $\overrightarrow{AB} = \boldsymbol{b}$ starting from the terminal point A of $\boldsymbol{a}$, then the vector $\overrightarrow{OB}$ is called the **sum** of $\boldsymbol{a}$ and $\boldsymbol{b}$, denoted by $\boldsymbol{a} + \boldsymbol{b}$, that is,

$$\boldsymbol{a} + \boldsymbol{b} = \overrightarrow{OB} \quad \text{or} \quad \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB} \text{ (Fig. 5.1.4).}$$

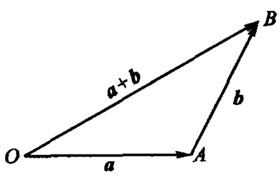

Figure 5.1.4

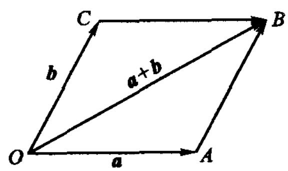

Figure 5.1.5

If the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are not parallel, then we can also find their sum according to the following **parallelogram law**. We take an arbitrary point O, draw $\overrightarrow{OA} = \boldsymbol{a}$, $\overrightarrow{OC} = \boldsymbol{b}$, and take OA and OC as the adjoining sides of a parallelogram $OABC$ (Fig. 5.1.5). Then $\boldsymbol{a} + \boldsymbol{b} = \overrightarrow{OB}$.

The addition of vectors satisfies the following laws:

(1) Commutative law $\quad \boldsymbol{a} + \boldsymbol{b} = \boldsymbol{b} + \boldsymbol{a}$;

(2) Associative law $\quad (\boldsymbol{a} + \boldsymbol{b}) + \boldsymbol{c} = \boldsymbol{a} + (\boldsymbol{b} + \boldsymbol{c})$;

(3) $\boldsymbol{a} + \boldsymbol{0} = \boldsymbol{a}$;

(4) $\boldsymbol{a} + (-\boldsymbol{a}) = \boldsymbol{0}$.

Here, (3) and (4) are obvious, and (1) and (2) are illustrated geometrically in Fig. 5.1.6 and Fig. 5.1.7 respectively.

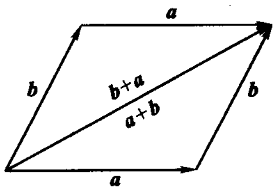

Figure 5.1.6

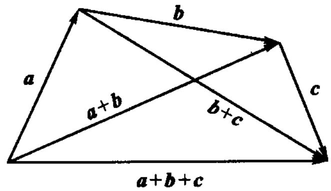

Figure 5.1.7

Multiplication by a scalar

Definition 5.1.2 Let λ be a nonzero scalar and $\boldsymbol{a}$ a nonzero vector. Then the **product** (or scalar multiple) of λ and $\boldsymbol{a}$ is a vector, denoted by $\lambda\boldsymbol{a}$. Its length is $\| \lambda\boldsymbol{a} \| = |\lambda| \, \| \boldsymbol{a} \|$, its direction is the same as that of $\boldsymbol{a}$ if $\lambda > 0$ or is opposite to that of $\boldsymbol{a}$ if $\lambda < 0$ (Fig. 5.1.8). If $\lambda = 0$ or $\boldsymbol{a} = \boldsymbol{0}$, we define $\lambda\boldsymbol{a} = \boldsymbol{0}$.

From definition 5.1.2 we have $(-1)\boldsymbol{a} = -\boldsymbol{a}$, and $\boldsymbol{a} = \| \boldsymbol{a} \| \boldsymbol{a}^\circ$, where $\boldsymbol{a}^\circ$ is the unit vector with the direction of $\boldsymbol{a}$, so that we have

$$\boldsymbol{a}^\circ = \frac{\boldsymbol{a}}{\| \boldsymbol{a} \|} \tag{5.1.1}$$

provided $\boldsymbol{a} \neq \boldsymbol{0}$. Moreover, we define the **difference** of two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ by

$$\boldsymbol{a} - \boldsymbol{b} = \boldsymbol{a} + (-\boldsymbol{b}).$$

According to the triangle law of addition of vectors, if the initial points of $\boldsymbol{a}$ and $\boldsymbol{b}$ are the same, then the vector from the terminal point of $\boldsymbol{b}$ to the terminal point of $\boldsymbol{a}$ is just the difference $\boldsymbol{a} - \boldsymbol{b}$ (Fig. 5.1.9).

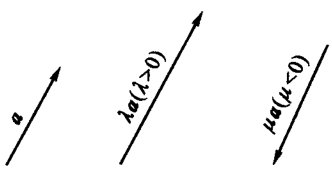

Figure 5.1.8

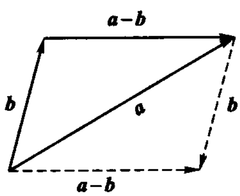

Figure 5.1.9

Products of scalars and vectors satisfy the following laws:

(1) Associative law $\lambda(\mu\boldsymbol{a}) = (\lambda\mu)\boldsymbol{a}$;

(2) Distributive law $\lambda(\boldsymbol{a}+\boldsymbol{b})=\lambda\boldsymbol{a}+\lambda\boldsymbol{b}$,

$(\lambda+\mu)\boldsymbol{a}=\lambda\boldsymbol{a}+\mu\boldsymbol{a}$;

(3) $1\boldsymbol{a}=\boldsymbol{a}$.

We prove only the distributive law $\lambda(\boldsymbol{a}+\boldsymbol{b})=\lambda\boldsymbol{a}+\lambda\boldsymbol{b}$, leaving the others to the reader. If $\lambda=0$, the equality holds obviously. Let $\lambda>0$ and draw $\overrightarrow{OA}=\boldsymbol{a}$, $\overrightarrow{OB}=\lambda\boldsymbol{a}$, $\overrightarrow{AD}=\boldsymbol{b}$, $\overrightarrow{BE}=\lambda\boldsymbol{b}$ (Fig. 5.1.10). Then the three points O, A, B are collinear, and $\overrightarrow{AD}\ /\!/\ \overrightarrow{BE}$. Therefore, $\dfrac{\|\overrightarrow{OE}\|}{\|\overrightarrow{OD}\|}=\dfrac{\|\overrightarrow{OB}\|}{\|\overrightarrow{OA}\|}=\lambda$ and the points O, D, E are also collinear, $\overrightarrow{OE}=\lambda\overrightarrow{OD}$. According to the triangle law, we have $\overrightarrow{OE}=\lambda\boldsymbol{a}+\lambda\boldsymbol{b}$, $\overrightarrow{OD}=\boldsymbol{a}+\boldsymbol{b}$, and hence $\lambda(\boldsymbol{a}+\boldsymbol{b})=\lambda\boldsymbol{a}+\lambda\boldsymbol{b}$. If $\lambda<0$, the proof is similar.

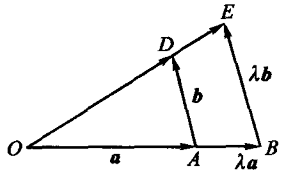

Figure 5.1.10

Addition and multiplication by a scalar are called by a joint name **linear operator** on vectors.

From the above discussion we know that the length of a vector has the following basic properties:

(1) Nonnegativity $\|\boldsymbol{a}\|\geqslant 0$, and $\|\boldsymbol{a}\|=0 \Leftrightarrow \boldsymbol{a}=\boldsymbol{0}$;

(2) Absolute homogeneity $\|\lambda\boldsymbol{a}\|=|\lambda|\,\|\boldsymbol{a}\|$;

(3) Triangle inequality $\|\boldsymbol{a}+\boldsymbol{b}\|\leqslant\|\boldsymbol{a}\|+\|\boldsymbol{b}\|$, where the sign of equality holds $\Leftrightarrow$ $\boldsymbol{a}$ and $\boldsymbol{b}$ have the same direction.

The geometric meaning of the triangle inequality is that the sum of the lengths of two adjoining sides of a triangle is greater than or equal to the length of the third side of the triangle.

5.1.3 Projection of vectors

To prepare for applications in the next section, we introduce the concept of projection of a vector onto another vector. Let us begin with the included angle between two vectors. Suppose that $\boldsymbol{a}$ and $\boldsymbol{b}$ are any two nonzero vectors. Taking any point M in the space, we draw vectors $\overrightarrow{MA}=\boldsymbol{a}$, $\overrightarrow{MB}=\boldsymbol{b}$ (Fig. 5.1. 11). Then the angle $\angle AMB$ (not greater than π) is called the **included angle** between the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, denoted by $(\boldsymbol{a}, \boldsymbol{b})$. If the included angle be-