国外数学名著系列

（影印版）17

E.Hairer S.P.Nørsett G.Wanner

Solving Ordinary Differential Equations I

Nonstiff Problems

Second Edition

常微分方程的解法 I

非刚性问题

（第二版）

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图字:01-2005-6739

E. Hairer, S. P. Nørsett, G. Wanner: Solving Ordinary Differential Equations Ⅰ: Nonstiff Problems

图书在版编目(CIP)数据

常微分方程的解法Ⅰ:非刚性问题:第2版=Solving Ordinary Differential Equations Ⅰ: Nonstiff Problems (Second Edition)/(瑞士)海尔(Hairer, E.)等著. —影印版. —北京:科学出版社, 2006

(国外数学名著系列)

ISBN 7-03-016680-9

Ⅰ. 常… Ⅱ. 海… Ⅲ. ①常微分方程-数值计算-英文②非刚性常微分方程-英文 Ⅳ. 0241.81

中国版本图书馆 CIP 数据核字(2005)第 154397 号

科学出版社 出版
北京东黄城根北街16号
邮政编码:100717
http://www.sciencep.com
中国科学院印刷厂 印刷
科学出版社发行 各地新华书店经销
*
2006年1月第 一 版 开本:B5(720×1000)
2006年1月第一次印刷 印张:34 1/4
印数:1—2 500 字数:647 000

定价:78.00 元

(如有印装质量问题,我社负责调换〈科印〉)

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《国外数学名著系列》(影印版)序

要使我国的数学事业更好地发展起来，需要数学家淡泊名利并付出更艰苦地努力。另一方面，我们也要从客观上为数学家创造更有利的发展数学事业的外部环境，这主要是加强对数学事业的支持与投资力度，使数学家有较好的工作与生活条件，其中也包括改善与加强数学的出版工作。

从出版方面来讲，除了较好较快地出版我们自己的成果外，引进国外的先进出版物无疑也是十分重要与必不可少的。从数学来说，施普林格（Springer）出版社至今仍然是世界上最具权威的出版社。科学出版社影印一批他们出版的好的新书，使我国广大数学家能以较低的价格购买，特别是在边远地区工作的数学家能普遍见到这些书，无疑是对推动我国数学的科研与教学十分有益的事。

这次科学出版社购买了版权，一次影印了 23 本施普林格出版社出版的数学书，就是一件好事，也是值得继续做下去的事情。大体上分一下，这 23 本书中，包括基础数学书 5 本，应用数学书 6 本与计算数学书 12 本，其中有些书也具有交叉性质。这些书都是很新的，2000 年以后出版的占绝大部分，共计 16 本，其余的也是 1990 年以后出版的。这些书可以使读者较快地了解数学某方面的前沿，例如基础数学中的数论、代数与拓扑三本，都是由该领域大数学家编著的“数学百科全书”的分册。对从事这方面研究的数学家了解该领域的前沿与全貌很有帮助。按照学科的特点，基础数学类的书以“经典”为主，应用和计算数学类的书以“前沿”为主。这些书的作者多数是国际知名的大数学家，例如《拓扑学》一书的作者诺维科夫是俄罗斯科学院的院士，曾获“菲尔兹奖”和“沃尔夫数学奖”。这些大数学家的著作无疑将会对我国的科研人员起到非常好的指导作用。

当然，23 本书只能涵盖数学的一部分，所以，这项工作还应该继续做下去。更进一步，有些读者面较广的好书还应该翻译成中文出版，使之有更大的读者群。

总之，我对科学出版社影印施普林格出版社的部分数学著作这一举措表示热烈的支持，并盼望这一工作取得更大的成绩。

王　元

2005 年 12 月 3 日

This edition is dedicated to
Professor John Butcher
on the occasion of his 60th birthday

His unforgettable lectures on Runge-Kutta methods, given in June 1970 at the University of Innsbruck, introduced us to this subject which, since then, we have never ceased to love and to develop with all our humble abilities.

From the Preface to the First Edition

> So far as I remember, I have never seen an Author's Preface which had any purpose but one — to furnish reasons for the publication of the Book. (Mark Twain)

> Gauss' dictum, "when a building is completed no one should be able to see any trace of the scaffolding," is often used by mathematicians as an excuse for neglecting the motivation behind their own work and the history of their field. Fortunately, the opposite sentiment is gaining strength, and numerous asides in this Essay show to which side go my sympathies. (B.B. Mandelbrot 1982)

> This gives us a good occasion to work out most of the book until the next year. (the Authors in a letter, dated Oct. 29, 1980, to Springer-Verlag)

There are two volumes, one on non-stiff equations, ..., the second on stiff equations, ... The first volume has three chapters, one on classical mathematical theory, one on Runge-Kutta and extrapolation methods, and one on multistep methods. There is an Appendix containing some Fortran codes which we have written for our numerical examples.

Each chapter is divided into sections. Numbers of formulas, theorems, tables and figures are consecutive in each section and indicate, in addition, the section number, but not the chapter number. Cross references to other chapters are rare and are stated explicitly. ... References to the Bibliography are by "Author" plus "year" in parentheses. The Bibliography makes no attempt at being complete; we have listed mainly the papers which are discussed in the text.

Finally, we want to thank all those who have helped and encouraged us to prepare this book. The marvellous "Minisymposium" which G. Dahlquist organized in Stockholm in 1979 gave us the first impulse for writing this book. J. Steinig and Chr. Lubich have read the whole manuscript very carefully and have made extremely valuable mathematical and linguistical suggestions. We also thank J.P. Eckmann for his troff software with the help of which the whole manuscript has been printed. For preliminary versions we had used textprocessing programs written by R. Menk. Thanks also to the staff of the Geneva computing center for their help. All computer plots have been done on their beautiful HP plotter. Last but not least, we would like to acknowledge the agreable collaboration with the planning and production group of Springer-Verlag.

October 29, 1986 The Authors

Preface to the Second Edition

The preparation of the second edition has presented a welcome opportunity to improve the first edition by rewriting many sections and by eliminating errors and misprints. In particular we have included new material on

- Hamiltonian systems (I.14) and symplectic Runge-Kutta methods (II.16);
- dense output for Runge-Kutta (II.6) and extrapolation methods (II.9);
- a new Dormand & Prince method of order 8 with dense output (II.5);
- parallel Runge-Kutta methods (II.11);
- numerical tests for first- and second order systems (II.10 and III.7).

Our sincere thanks go to many persons who have helped us with our work:

- all readers who kindly drew our attention to several errors and misprints in the first edition;
- those who read preliminary versions of the new parts of this edition for their invaluable suggestions: D.J. Higham, L. Jay, P. Kaps, Chr. Lubich, B. Moesli, A. Ostermann, D. Pfenniger, P.J. Prince, and J.M. Sanz-Serna.
- our colleague J. Steinig, who read the entire manuscript, for his numerous mathematical suggestions and corrections of English (and Latin!) grammar;
- our colleague J.P. Eckmann for his great skill in manipulating Apollo workstations, font tables, and the like;
- the staff of the Geneva computing center and of the mathematics library for their constant help;
- the planning and production group of Springer-Verlag for numerous suggestions on presentation and style.

This second edition now also benefits, as did Volume II, from the marvels of TEXnology. All figures have been recomputed and printed, together with the text, in Postscript. Nearly all computations and text processings were done on the Apollo DN4000 workstation of the Mathematics Department of the University of Geneva; for some long-time and high-precision runs we used a VAX 8700 computer and a Sun IPX workstation.

November 29, 1992 The Authors

Contents

Chapter I. Classical Mathematical Theory

Chapter II. Runge-Kutta and Extrapolation Methods

Chapter III. Multistep Methods and General Linear Methods

Chapter I. Classical Mathematical Theory

> ... halte ich es immer für besser, nicht mit dem Anfang anzufangen, der immer das Schwerste ist.
>
> (*B. Riemann copied this from F. Schiller into his notebook*)

This first chapter contains the classical theory of differential equations, which we judge useful and important for a profound understanding of numerical processes and phenomena. It will also be the occasion of presenting interesting examples of differential equations and their properties.

We first retrace in Sections I.2-I.6 the historical development of classical integration methods by series expansions, quadrature and elementary functions, from the beginning (Newton and Leibniz) to the era of Euler, Lagrange and Hamilton. The next part (Sections I.7-I.14) deals with theoretical properties of the solutions (existence, uniqueness, stability and differentiability with respect to initial values and parameters) and the corresponding flow (increase of volume, preservation of symplectic structure). This theory was initiated by Cauchy in 1824 and then brought to perfection mainly during the next 100 years. We close with a brief account of boundary value problems, periodic solutions, limit cycles and strange attractors (Sections I.15 and I.16).

I.1 Terminology

A *differential equation of first order* is an equation of the form

$$y' = f(x,y) \tag{1.1}$$

with a given function $f(x,y)$. A function $y(x)$ is called a *solution* of this equation if for all x,

$$y'(x) = f\big(x, y(x)\big). \tag{1.2}$$

It was observed very early by Newton, Leibniz and Euler that the solution usually contains a free parameter, so that it is uniquely determined only when an *initial value*

$$y(x_0) = y_0 \tag{1.3}$$

is prescribed. Cauchy's existence and uniqueness proof of this fact will be discussed in Section I.7. Differential equations arise in many applications. We shall see the first examples of such equations in Section I.2, and in Section I.3 how some of them can be solved explicitly.

A *differential equation of second order* for y is of the form

$$y'' = f(x, y, y'). \tag{1.4}$$

Here, the solution usually contains *two* parameters and is only uniquely determined by *two* initial values

$$y(x_0) = y_0, \qquad y'(x_0) = y_0'. \tag{1.5}$$

Equations of second order can rarely be solved explicitly (see I.3). For their numerical solution, as well as for theoretical investigations, one usually sets $y_1(x) := y(x)$, $y_2(x) := y'(x)$, so that equation (1.4) becomes

$$\begin{aligned} y_1' &= y_2 & \qquad y_1(x_0) &= y_0 \\ y_2' &= f(x, y_1, y_2) & \qquad y_2(x_0) &= y_0'. \end{aligned} \tag{1.4'}$$

This is an example of a *first order system of differential equations,* of dimension n (see Sections I.6 and I.9),

$$\begin{aligned} y_1' &= f_1(x, y_1, \ldots, y_n) & \qquad y_1(x_0) &= y_{10} \\ &\ldots & &\ldots \\ y_n' &= f_n(x, y_1, \ldots, y_n) & \qquad y_n(x_0) &= y_{n0}. \end{aligned} \tag{1.6}$$