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ELEMENTARY REAL ANALYSIS

实分析基础

Brian S. Thomson
Judith B. Bruckner 原著
Andrew M. Bruckner
钟承奎 赵 敦 邱春雨 改编

高等教育出版社
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Brian S.Thomson
Simon Fraser University
Judith B.Bruckner **原著**
Andrew M.Bruckner
University of California, Santa Barbara

钟承奎 赵 敦 邱春雨 改编
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高等教育出版社

2005 年 5 月

改编说明

由Brian S. Thomson等人编写的“Elementary Real Analysis”是一本非常富有特色的教材，该书的材料显然是作者深思熟虑后精心选取的。

原书的目的是在严格的逻辑体系下讲授实分析的基本理论。阅读该书要求读者对于微积分理论已有初步了解，程度相当于学习过欧美国家的基本微积分课程。

原书有以下几个突出特点：一是有翔实的背景资料，对每个重要问题的历史背景及发展都有简明的阐述，这对读者理解所介绍的内容是非常有好处的；二是内容丰富，书中的不少方法和结论在其他同类书籍中是难以找到系统介绍的；三是主线清晰，语言简洁流畅，易于阅读；第四个突出特点体现在习题方面，在每节内容后面，作者精心选择了相当数量的习题，同时，在每章后面，作者又特别设计了一些作者所称的挑战性习题，其中部分题目选自美国的Putnam数学竞赛和著名期刊“American Mathematical Monthly”，这部分习题对于提高学生能力是非常有好处的。同时，作者在附录中对部分习题给出了提示，因此使得该书也很适合于自学的读者。

高等教育出版社委托我对该书进行适当改编，使之能与国内现行教材配套使用，这是一项很有难度和挑战性的工作。为做好这项工作，我组织几位教师和研究生仔细阅读了原书，进行了讨论，确定了下面的改编原则：

1. 保持原书的风格，让作者创作该书的指导思想原汁原味地体现出来；

2. 尽量使内容编排与国内的教材保持一致，使之不仅是参考书，也可以直接在国内作为教材使用。

与国内教材相比，原书的大部分(前12章)内容相当于数学专业“数学分析”课程标准内容除去多元函数积分学以外的内容，其中有少量内容为国内“实变函数”课程中集合论的内容，其最后一章(第13章)关于度量空间的内容通常出现在国内的“泛函分析”课程中。

我们认为，如果在原书中添加一元函数不定积分、含参变量积分和多元函数积分的内容，则可以使该书内容自成体系，并且在内容上覆盖国内高等学校数学专业“数学分析”课程的内容，这样只需稍作处理，就可使该书直接作为国内“数学分析”课程的一本极具特色的教材。

由于上面的原因，我们希望原书在改编后能成为国内数学专业一年级学生“数学分析”课程的有价值的参考书，或作为国内数学专业“数学分析”课程的双语教材的重要参考书，或作为非数学专业学生学完一年的“高等数学”课程后的提高教材。

本书的起点仍然是假定读者对于微积分理论已有初步掌握，因此书中内容的展开有一定的深度，并且仍然依照原书的指导思想，略去一元函数简单积分的内容。由于我国在高中数学教材中已增加了关于微积分的基本内容，并且在高考中有所要求，我们认为上述的假定对于数学专业即将学习“数学分析”课程的学生也是成立的。我们曾经打算在原书中加入多元函数积分学的内容，这样就可使其内容与国内现行“数学分析”课程的标准内容相一致。但经仔细斟酌后认为，这样做一方面要增加很多篇幅，另一方面也可能会破坏原作者创作本书的意图和风格。再者，关于多元函数微积分国内也已有很好的专著和译本。所以我们最终还是放弃了这种打算，仅在最小的程度上对原书内容进行调整。

我们所作的调整主要有以下几个地方：

1. 将原书第3章调至原书第9章之前，将数项级数与函数项级数放在一起作为一个板块，这符合国内现行教材的讲法，这样似乎比国外教材中通行的将数项级数作为数列提前处理的讲法要好一些。

2. 取消原书第6章，各章序号依次改动。将原书6.2、6.3节放入改编后的第3章，变为3.7和3.8节；将原书6.5节中的6.5.3删去，剩余部分作为改编后的3.9节；将原书6.8节变为改编后的3.10节；将原书6.7.1的内容作为改编后的4.10节的内容，并将此节的标题取为原书6.7节的标题。删掉原

书第6章中的其他内容，这些内容属国内“实变函数”课程中的标准内容，将会在“实变函数”课程中学到，这样处理的目的是降低难度。

3. 删掉原书中的9.8节。

4. 删掉原书中的13.13、13.14节。虽然第13章的内容为国内“泛函分析”课程的标准内容，但我们仍保留了13.1～13.12节，以在整体上突出从1维空间到一般有限维空间、再到无穷维空间的研究方法之间的联系和差异。

5. 对涉及的习题及附录中的习题提示作相应调整。

应该说，要对原书作成功的改编，需要熟读原书，并已取得使用原书进行教学的实践经验。由于时间的关系，我们在这些方面有所欠缺。虽然我们尽了很大努力想去做好这件事，但对原书所作的上述改编，仍难免有不妥之处，我们诚恳地希望广大读者及同行专家提出宝贵建议。

兰州大学数学与统计学院

钟承奎

2005年6月

PREFACE

University mathematics departments have for many years offered courses with titles such as *Advanced Calculus* or *Introductory Real Analysis*. These courses are taken by a variety of students, serve a number of purposes, and are written at various levels of sophistication. The students range from ones who have just completed a course in elementary calculus to beginning graduate students in mathematics. The purposes are multifold:

1. To present familiar concepts from calculus at a more rigorous level.
2. To introduce concepts that are not studied in elementary calculus but that are needed in more advanced undergraduate courses. This would include such topics as point set theory, uniform continuity of functions, and uniform convergence of sequences of functions.
3. To provide students with a level of mathematical sophistication that will prepare them for graduate work in mathematical analysis, or for graduate work in several applied fields such as engineering or economics.
4. To develop many of the topics that the authors feel all students of mathematics should know.

There are now many texts that address some or all of these objectives. These books range from ones that do little more than address objective (1) to ones that try to address all four objectives. The books of the first extreme are generally aimed at one-term courses for students with minimal background. Books at the other extreme often contain substantially more material than can be covered in a one-year course.

The level of rigor varies considerably from one book to another, as does the style of presentation. Some books endeavor to give a very efficient streamlined development; others try to be more user friendly. We have opted for the user-friendly approach. We feel this approach makes the concepts more meaningful to the student.

Our experience with students at various levels has shown that most students have difficulties when topics that are entirely new to them first appear. For some students that might occur almost immediately when rigorous proofs are required, for example, ones needing ε-δ arguments. For others, the difficulties begin with elementary point set theory, compactness arguments, and the like.

To help students with the transition from elementary calculus to a more rigorous course, we have included motivation for concepts most students have not seen before and provided more details in proofs when we introduce new methods. In addition, we have tried to give students ample opportunity to see the new tools in action.

For example, students often feel uneasy when they first encounter the various compactness arguments (Heine-Borel theorem, Bolzano-Weierstrass theorem, Cousin's lemma, introduced in Section 3.5). To help the student see why such theorems are useful, we pose the problem of determining circumstances under which local boundedness of a function f on a set E implies global boundedness of f on E. We show by example that some conditions on E are needed, namely that E be closed and bounded, and then show how each of several theorems could be used to show that closed and boundedness of the set E suffices. Thus we introduce students to the theorems by showing how the theorems can be used in natural ways to solve a problem.

We have also included some optional material, marked as "Advanced" or "Enrichment" and flagged with the symbol ✂.

Enrichment

We have indicated as "Enrichment"‘ some relatively elementary material that could be added to a longer course to provide enrichment and additional examples. For example, in Chapter 7 we have added to the study of series a section on infinite products. While such a topic plays an important role in the representation of analytic functions, it is presented here to allow the instructor to explore ideas that are closely related to the study of series and that help illustrate and review many of the fundamental ideas that have played a role in the study of series.

Advanced

We have indicated as "Advanced" material of a more mathematically sophisticated nature that can be omitted without loss of continuity. These topics might be needed in more advanced courses in real analysis or in certain of the marked sections or exercises that appear later in this book. For example, in Chapter 2 we have added to the study of sequence limits a section on

lim sups and lim infs. For an elementary first course this can be considered somewhat advanced and skipped. Later problems and text material that require these concepts are carefully indicated. Thus, even though the text carries on to relatively advanced undergraduate analysis, a first course can be presented by avoiding these advanced sections.

We apply these markings to some entire chapters as well as to some sections within chapters and even to certain exercises. We do not view these markings as absolute. They can simply be interpreted in the following ways. Any unmarked material will not depend, in any substantial way, on earlier marked sections. In addition, if a section has been flagged and will be used in a much later section of this book, we indicate where it will be required.

The material marked "Advanced" is in line with goals (2) and (3). We resist the temptation to address objective (4). There are simply too many additional topics that one might feel every student should know (e.g., functions of bounded variation, Riemann-Stieltjes and Lebesgue integrals). To cover these topics in the manner we cover other material would render the book more like a reference book than a text that could reasonably be covered in a year. Students who have completed this book will be in a good position to study such topics at rigorous levels.

We include, however, a chapter on metric spaces. We do this for two reasons: to offer a more general framework for viewing concepts treated in earlier chapters, and to illustrate how the abstract viewpoint can be applied to solving concrete problems. The metric space presentation in Chapter 12 can be considered more advanced as the reader would require a reasonable level of preparation. Even so, it is more readable and accessible than many other presentations of metric space theory, as we have prepared it with the assumption that the student has just the minimal background. For example, it is easier than the corresponding chapter in our graduate level text (*Real Analysis*, Prentice Hall, 1997) in which the student is expected to have studied the Lebesgue integral and to be at an appropriately sophisticated level.

The Exercises

The exercises form an integral part of the book. Many of these exercises are routine in nature. Others are more demanding. A few provide examples that are not usually presented in books of this type but that students have found challenging, interesting, and instructive.

Some exercises have been flagged with the ✂ symbol to indicate that they maybe require material from a flagged section. For example, a first course is likely to skip over the section on lim sups and lim infs of sequences. Exercises that require those concepts are flagged so that the instructor can

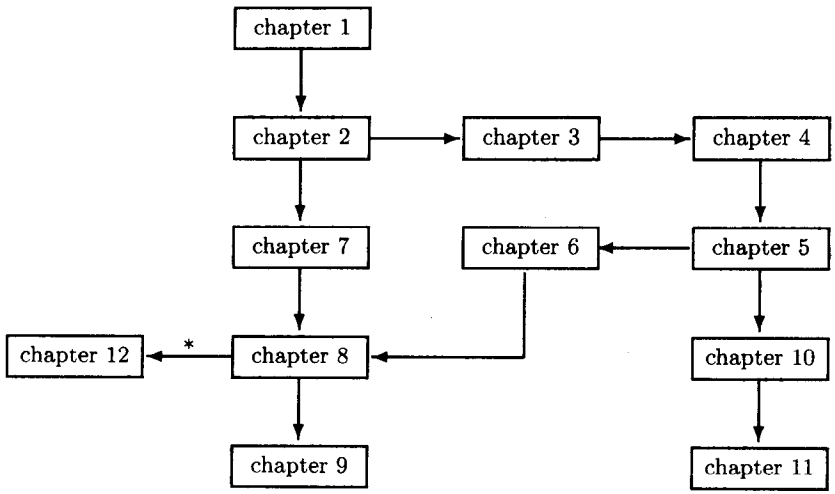

Figure 0.1. Chapter Dependencies (Unmarked Sections).

decide whether they can be used or not. Generally, that symbol on an exercise warns that it might not be suitable for routine assignments.

The exercises at the end of some of the chapters can be considered more challenging. They include some Putnam problems and some problems from the journal *American Mathematical Monthly*. They do not require more knowledge than is in the text material but often need a bit more persistence and some clever ideas. Students should be made aware that solutions to Putnam problems can be found on various Web sites and that solutions to *Monthly* problems are published; even so, the fun in such problems is in the attempt rather than in seeing someone else's solution.

Designing a Course

We have attempted to write this book in a manner sufficiently flexible to make it possible to use the book for courses of various lengths and a variety of levels of mathematical sophistication.

Much of the material in the book involves rigorous development of topics of a relatively elementary nature, topics that most students have studied at a nonrigorous level in a calculus course. A short course of moderate mathematical sophistication intended for students of minimal background can be based entirely on this material. Such a course might meet objective (1).

We have written this book in a leisurely style. This allows us to provide motivational discussions and historical perspective in a number of places. Even though the book is relatively large (in terms of number of pages), we can comfortably cover most of the the main sections in a full-year course, including many of the interesting exercises.

Instructors teaching a short course have several options. They can base a course entirely on the unmarked material of Chapters 1, 2, 3, 4, and 5. As time permits, they can add the early parts of Chapters 6 and 7 or parts of Chapters 10 and 11 and some of the enrichment material.

Background

We should make one more point about this book. We do assume that students are familiar with nonrigorous calculus. In particular, we assume familiarity with the elementary functions and their elementary properties. We also assume some familiarity with computing derivatives and integrals. This allows us to illustrate various concepts using examples familiar to the students. For example, we begin Chapter 2, on sequences, with a discussion of approximating $\sqrt{2}$ using Newton's method. This is merely a motivational discussion, so we are not bothered by the fact that that we don't treat the derivative formally until Chapter 5 and haven't yet proved that $\frac{d}{dx}(x^2 - 2) = 2x$. For students with minimal background we provide an appendix that informally covers such topics as notation, elementary set theory, functions, and proofs.

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A. M. B.
J. B. B.
B. S. T.

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目 录

[1]选读章节用†表示“Enrichment”，*表示“Advanced”.