

# **Marine Control Systems**

Guidance, Navigation, and Control  
of Ships, Rigs and Underwater Vehicles



**Thor I. Fossen**

# **Marine Control Systems**

Guidance, Navigation and Control  
of Ships, Rigs and Underwater Vehicles

**Thor I. Fossen**

*Norwegian University of Science and Technology  
Trondheim, Norway*

Copyright © 2002 by Marine Cybernetics AS.

All rights reserved.

**For ordering** see URL: <http://www.marinecybernetics.com>. The book can also be ordered by sending an e-mail to:

info@marinecybernetics.com

or via fax:

MARINE CYBERNETICS AS  
P. O. Box 4607  
NO-7451 Trondheim, Norway  
fax: [+47] 72 81 00 18

No parts of this publication may be reproduced by any means, transmitted, or translated into machine language without the written permission of the author. Requests for permission to reproduce parts of the book should be addressed directly to Professor Thor I. Fossen, Department of Engineering Cybernetics, Norwegian University of Science and Technology, NO-7491 Trondheim, Norway; E-mail: tif@itk.ntnu.no, fax: [+47] 73594399.

ISBN 82-92356-00-2 1st ed. 3rd printing

Produced from camera-ready copy supplied by the author using *Scientific WorkPlace*.

Printed and bound by Tapir Trykkeri, Trondheim, Norway.

This book is dedicated

to

Professor Jens G. Balchen

who introduced me to the fantastic  
world of feedback control.



# Preface

The main motivation for writing this book was to collect new results on nonlinear control of marine craft that have appeared since I published my first book: “*Guidance and Control of Ocean Vehicles*” (John Wiley & Sons Ltd. 1994). Most of these results have been developed in the Department of Engineering Cybernetics at Norwegian University of Science and Technology (NTNU) in close cooperation with my doctoral students; *Ola-Erik Fjellstad, Trygve Lauvdal, Jann Peter Strand, Jan Fredrik Hansen, Bjørnar Vik, Svein Peder Berge, Mehrdad P. Fard, Karl-Petter Lindegaard, Ole Morten Aamo, and Roger Skjetne* in the period 1991–2002. We have all been a great team, producing more than one hundred international publications in this period. These have resulted in several patents and industrial implementations.

In particular, I want to express my gratitude to *Dr. Roger Skjetne* and *Dr. Jann Peter Strand* for suggestions, case studies and comments to the manuscript. They have been instrumental in most of the work presented in the book. United European Car Carriers (UECC) and SeaLaunch LLC should also be thanked for contributing with full scale experimental results and case studies to the book. *Dr. Svein Peder Berge* and *Lars Ove Sæther* at Marintek AS have contributed with experimental results from the Ocean Basin in which model ships have been tested. *Adjunct Professor Svein I. Sagatun* and Norsk Hydro should be thanked for supporting our research projects on inertial navigation systems and marine operations.

*Dr. Bjørnar Vik* has been invaluable as a research fellow and colleague in the period 1998–2002. His expertise in the rapid prototyping of ship control systems, control theory and navigation systems has been most valuable. This expertise has been one of the keystones in the development of the GNC Lab (Guidance, Navigation and Control Laboratory), MCLAB (Marine Cybernetics Laboratory), and GPS/INS Laboratory at NTNU.

I would like to thank my family, *Heidi, Sindre, and Lone* for supporting this book project. Without their support it would have been impossible to accomplish the task of writing 586 pages with equations in less than two years.

*Professor Asgeir J. Sørensen* and *Professor Tor Arne Johansen* should be thanked for their careful proofreading and comments on the final manuscript. *Professor Olav Egeland* should be thanked for our mutual discussions on modeling and control. I am also grateful to *Andrew Ross* at the University of Glasgow for his assistance with the English language. The book also greatly benefits from students who took the course in Guidance, Navigation and Control at NTNU in 2002. They have all helped me to keep the number of typographical errors to an acceptable level.

Finally, I would like to thank *Professor Miroslav Krstic* for inviting me on a sabbatical at the University of California, San Diego (UCSD) in 2001, so that I could escape the office in Norway and finish the book in a reasonable time.

**Thor I. Fossen**  
**November 2002**

# Contents

<b>Preface</b>	<b>xi</b>
<b>Tables</b>	<b>xiii</b>
<b>1 Introduction</b>	<b>1</b>
1.1 From the Invention of the Gyroscope to Model Based Ship Control . . . . .	3
1.1.1 The Gyroscope and its Contributions to Ship Control . . . . .	4
1.1.2 Autopilots . . . . .	5
1.1.3 Dynamic Positioning and Position Mooring Systems . . . . .	6
1.1.4 Way-Point Tracking Control Systems . . . . .	7
1.1.5 The Sea Launch System . . . . .	7
1.2 Model Representations for Marine Vessels . . . . .	9
1.2.1 The Classical Model in Naval Architecture . . . . .	9
1.2.2 The Vectorial Model Representation of Fossen (1991) . . . . .	10
1.3 The Principle of Guidance, Navigation and Control . . . . .	11
1.3.1 Definitions of Guidance, Navigation and Control . . . . .	11
1.3.2 Set-Point Regulation versus Trajectory Tracking Control . . . . .	12
1.4 Organization of Book . . . . .	12
<b>I Modeling of Marine Vessels</b>	<b>15</b>
<b>2 Kinematics</b>	<b>17</b>
2.1 Reference Frames . . . . .	19
2.2 Transformations between BODY and NED . . . . .	21
2.2.1 Euler Angle Transformation . . . . .	23
2.2.2 Unit Quaternions . . . . .	29
2.2.3 Quaternions from Euler Angles . . . . .	33
2.2.4 Euler Angles from Quaternions . . . . .	35
2.2.5 QUEST Algorithm for Position and Attitude Determination . . . . .	36
2.3 Transformation between ECEF and NED . . . . .	38
2.3.1 Longitude and Latitude Transformations . . . . .	38
2.3.2 Longitude and Latitude from ECEF Coordinates . . . . .	41
2.3.3 ECEF Coordinates from Longitude and Latitude . . . . .	43
2.4 Transformations for Stability and Flow Axes . . . . .	44
2.5 Exercises . . . . .	47

<b>3 Dynamics of Marine Vessels</b>	<b>49</b>
3.1 Rigid-Body Dynamics . . . . .	50
3.1.1 Translational Motion . . . . .	51
3.1.2 Rotational Motion (Attitude Dynamics) . . . . .	53
3.1.3 Rigid-Body Equations of Motion . . . . .	57
3.2 Hydrodynamic Forces and Moments . . . . .	62
3.2.1 Added Mass and Inertia . . . . .	64
3.2.2 Hydrodynamic Damping . . . . .	71
3.2.3 Restoring Forces and Moments . . . . .	75
3.2.4 Ballast Systems . . . . .	82
3.3 6 DOF Equations of Motion . . . . .	88
3.3.1 Nonlinear Equations of Motion . . . . .	88
3.3.2 Linearized Equations of Motion . . . . .	91
3.4 Model Transformations using Matlab . . . . .	94
3.4.1 System Transformation Matrix . . . . .	94
3.4.2 Computation of the System Inertia Matrix . . . . .	96
3.4.3 Computation of the Coriolis-Centrifugal Matrix . . . . .	100
3.4.4 Computation of the Damping Matrix . . . . .	100
3.4.5 Computation of the Restoring Forces and Moments . . . . .	102
3.5 Standard Models for Marine Vessels . . . . .	103
3.5.1 3 DOF Horizontal Model . . . . .	104
3.5.2 Decoupled Models for Forward Speed/Maneuvering . . . . .	107
3.5.3 Longitudinal and Lateral Models . . . . .	109
3.6 Exercises . . . . .	113
<b>4 Models for Wind, Waves and Ocean Currents</b>	<b>115</b>
4.1 Wind Models . . . . .	116
4.1.1 Wind Forces and Moments . . . . .	116
4.1.2 Wind Resistance of Merchant Ships (Isherwood 1972) . . . . .	117
4.1.3 Wind Resistance of Very Large Crude Carriers (OCIMF 1977) . . . . .	120
4.1.4 Wind Resistance of Large Tankers and Medium Sized Ships . . . . .	123
4.1.5 Wind Resistance of Moored Ships and Floating Structures . . . . .	123
4.2 Models for Wind Generated Waves . . . . .	123
4.2.1 Nonlinear Models of Wave Spectra . . . . .	123
4.2.2 Linear Wave Response Models . . . . .	130
4.2.3 Frequency of Encounter . . . . .	136
4.2.4 Wave Forces and Moments . . . . .	137
4.3 Models for Ocean Currents . . . . .	138
4.3.1 3D Irrotational Current Model . . . . .	139
4.3.2 2D Irrotational Current Model (Horizontal-Plane Model) . . . . .	139
4.4 Exercises . . . . .	140
<b>II Guidance, Navigation and Control Fundamentals</b>	<b>143</b>
<b>5 Maritime Guidance Systems</b>	<b>145</b>
5.1 Reference Models . . . . .	146
5.1.1 Velocity Reference Model . . . . .	147

5.1.2	Position and Attitude Reference Models . . . . .	147
5.1.3	Saturating Elements . . . . .	148
5.1.4	Nonlinear Damping . . . . .	148
5.2	Way-Point Guidance Systems . . . . .	149
5.2.1	Trajectory Tracking and Maneuvering Control . . . . .	149
5.2.2	Way-Point Representation . . . . .	152
5.2.3	Trajectory Generation using a Vessel Simulator . . . . .	154
5.2.4	Path and Trajectory Generation using Interpolation . . . . .	156
5.2.5	Weather Routing . . . . .	165
5.3	Line-of-Sight Guidance . . . . .	167
5.3.1	2-Dimensional LOS Guidance System for Surface Vessel . . . . .	168
5.3.2	3-Dimensional LOS Guidance System for Underwater Vehicles . . . . .	169
5.4	Exercises . . . . .	169
<b>6</b>	<b>Estimator Based Navigation Systems</b>	<b>171</b>
6.1	Observers for Heading Autopilots . . . . .	172
6.1.1	Magnetic and Gyroscopic Compasses . . . . .	172
6.1.2	Low-Pass and Notch Filtering of Wave Frequency Motions . . . . .	173
6.1.3	Fixed Gain Observers using only Compass Measurements . . . . .	177
6.1.4	Kalman Filter Based Wave Filter Design using only Compass Measurements . . . . .	185
6.1.5	Observer and Wave Filter Design using both Compass and Rate Measurements . . . . .	189
6.2	Observers for Dynamic Positioning Systems . . . . .	191
6.2.1	Navigation Systems . . . . .	191
6.2.2	Inertial Measurement Systems . . . . .	194
6.2.3	Kalman Filter for Velocity and Wave Frequency Motion . . . . .	196
6.2.4	Passive Nonlinear Observer for Velocity and Wave Frequency Motion	201
6.3	6 DOF Integration Filter for IMU and Satellite Navigation Systems . . . . .	213
6.3.1	Integration Filter for Position and Linear Velocity . . . . .	214
6.3.2	Attitude Observer . . . . .	217
6.4	Exercises . . . . .	221
<b>7</b>	<b>Control Methods for Marine Vessels</b>	<b>223</b>
7.1	PID-Control and Acceleration Feedback . . . . .	224
7.1.1	Linear Mass-Damper-Spring Systems . . . . .	224
7.1.2	Acceleration Feedback . . . . .	228
7.1.3	Acceleration Feedback + PID Control . . . . .	230
7.1.4	MIMO Acceleration Feedback and Nonlinear PID Control . . . . .	233
7.1.5	Inertia Shaping Techniques using Acceleration Feedback . . . . .	235
7.2	Linear Quadratic Optimal Control . . . . .	237
7.2.1	Linear Quadratic Regulator . . . . .	239
7.2.2	Extensions to Trajectory Tracking and Integral Action . . . . .	240
7.2.3	General Solution of the LQ Trajectory Tracking Problem . . . . .	242
7.3	State Feedback Linearization . . . . .	250
7.3.1	Decoupling in the b-Frame (Velocity) . . . . .	250
7.3.2	Decoupling in the n-Frame (Position and Attitude) . . . . .	252

7.3.3	Adaptive Feedback Linearization . . . . .	254
7.4	Integrator Backstepping . . . . .	256
7.4.1	A Brief History of Backstepping . . . . .	257
7.4.2	The Main Idea of Integrator Backstepping . . . . .	257
7.4.3	Backstepping of SISO Mass-Damper-Spring Systems . . . . .	264
7.4.4	Integral Action by Constant Parameter Adaptation . . . . .	268
7.4.5	Integrator Augmentation Technique . . . . .	272
7.4.6	Backstepping of MIMO Mass-Damper-Spring Systems . . . . .	276
7.4.7	MIMO Backstepping of Ships . . . . .	280
7.4.8	MIMO Backstepping Design with Acceleration Feedback . . . . .	284
7.5	Control Allocation . . . . .	288
7.5.1	Actuator Models . . . . .	288
7.5.2	Unconstrained Control Allocation (Nonrotatable Actuators) . . . . .	291
7.5.3	Constrained Control Allocation (Nonrotatable Actuators) . . . . .	293
7.5.4	Constrained Control Allocation (Azimuthing Thrusters) . . . . .	295
7.6	Exercises . . . . .	298

### III Ship and Rig Applications 301

<b>8 Course Autopilots</b>	<b>303</b>
8.1 Autopilot Models . . . . .	304
8.1.1 Rigid-Body Ship Dynamics . . . . .	304
8.1.2 The Linear Ship Steering Equations . . . . .	307
8.1.3 Non-Dimensional Autopilot Models . . . . .	311
8.1.4 Nonlinear Models for Autopilot Design . . . . .	315
8.2 Open-Loop Stability Analysis of Ships . . . . .	320
8.2.1 Stability Considerations for Ship Steering and Positioning . . . . .	320
8.2.2 Criteria for Straight-Line Stability . . . . .	324
8.2.3 Criteria for Directional Stability . . . . .	327
8.3 Maneuverability . . . . .	328
8.3.1 Turning Circle . . . . .	330
8.3.2 Kempf's Zig-Zag Maneuver . . . . .	334
8.3.3 Pull-Out Maneuver . . . . .	336
8.3.4 Dieudonné's Spiral Maneuver . . . . .	338
8.3.5 Bech's Reverse Spiral Maneuver . . . . .	338
8.4 Course-Keeping Autopilots and Turning Control . . . . .	340
8.4.1 Autopilot Reference Model . . . . .	340
8.4.2 Conventional PID-Control . . . . .	342
8.4.3 PID Control including Acceleration Feedback . . . . .	347
8.4.4 PID Control including Wind Feedforward . . . . .	349
8.4.5 Linear Quadratic Optimal Control . . . . .	350
8.4.6 State Feedback Linearization . . . . .	355
8.4.7 Adaptive Feedback Linearization and Optimality . . . . .	356
8.4.8 Nonlinear Backstepping . . . . .	358
8.4.9 SISO Sliding Mode Control . . . . .	359
8.4.10 Output Feedback . . . . .	363

---

8.5 Exercises . . . . .	365
<b>9 Autopilots with Roll Damping</b>	<b>367</b>
9.1 Autopilot Models for Steering and Roll Damping . . . . .	368
9.1.1 The Linear Model of Van Amerongen and Van Cappelle (1981) . . . . .	368
9.1.2 The Nonlinear Model of Son and Nomoto (1981) . . . . .	373
9.1.3 The Nonlinear Model of Christensen and Blanke (1986) . . . . .	374
9.2 Rudder-Roll Damping (RRD) Control Systems . . . . .	374
9.2.1 Linear Quadratic Optimal RRD Control System . . . . .	375
9.2.2 Performance Criterion for RRD . . . . .	380
9.3 Fin Stabilization Control Systems and RRD . . . . .	380
9.3.1 Linear Quadratic Energy Optimal Autopilot with Roll Damping . . . . .	381
9.4 Operability and Motion Sickness Incidence Criteria . . . . .	384
9.4.1 Human Operability Limiting Criteria in Roll . . . . .	384
9.4.2 Criterion for Motion Sickness Incidence (MSI) . . . . .	384
9.5 Exercises . . . . .	387
<b>10 Trajectory Tracking and Maneuvering Control</b>	<b>389</b>
10.1 Trajectory Tracking Control . . . . .	389
10.1.1 Conventional PID Cross-Tracking System . . . . .	390
10.1.2 Line of Sight Cross-Tracking System . . . . .	391
10.1.3 Linear Quadratic Optimal Cross-Tracking System . . . . .	392
10.1.4 Underactuated Trajectory Tracking Control . . . . .	394
10.2 Maneuvering Control . . . . .	394
10.2.1 Robust Output Maneuvering . . . . .	396
10.2.2 Adaptive Output Maneuvering . . . . .	404
10.2.3 Maneuvering Control of Underactuated Ships . . . . .	415
10.3 Exercises . . . . .	416
<b>11 Positioning Systems</b>	<b>417</b>
11.1 Models for Station-Keeping . . . . .	417
11.1.1 Vessel Kinematics and Dynamics . . . . .	417
11.1.2 DP and PM Thrust Models . . . . .	418
11.1.3 Environmental Disturbances . . . . .	422
11.2 Dynamic Positioning (DP) Systems . . . . .	423
11.2.1 Thrust Allocation in DP Systems . . . . .	424
11.2.2 Linear Quadratic Optimal Control . . . . .	425
11.2.3 Nonlinear PID Control . . . . .	427
11.2.4 Nonlinear Separation Principle for PD-Control/Observer Design . . . . .	428
11.2.5 Nonlinear Observer Backstepping . . . . .	436
11.2.6 Nonlinear Inverse Optimal Control . . . . .	447
11.2.7 Underactuated Stabilization . . . . .	448
11.3 Position Mooring (PM) Systems . . . . .	449
11.4 Weather Optimal Positioning Control (WOPC) . . . . .	450
11.4.1 3 DOF Equations of Motion using Polar Coordinates . . . . .	451
11.4.2 Weather Optimal Control Objectives . . . . .	454
11.4.3 Nonlinear and Adaptive Control Design . . . . .	456
11.4.4 Experiments and Simulations . . . . .	462

11.5 Exercises . . . . .	467
<b>IV Underwater Vehicle Applications</b>	<b>469</b>
<b>12 Propeller Control System Design</b>	<b>471</b>
12.1 Models for Propeller Shaft Speed and Motors . . . . .	471
12.1.1 Propeller Shaft Speed Models . . . . .	471
12.1.2 Unified Representation of DC-Motor Controllers . . . . .	473
12.1.3 Propeller Losses . . . . .	475
12.2 Propeller Thrust and Torque Modelling . . . . .	475
12.2.1 Quasi-Steady Thrust and Torque . . . . .	476
12.3 Nonlinear Observer for Estimation of Propeller Axial Velocity . . . . .	479
12.3.1 Vehicle Speed and Propeller Axial Flow Dynamics . . . . .	479
12.3.2 Observer Equations . . . . .	480
12.3.3 Lyapunov Analysis . . . . .	481
12.4 Nonlinear Output Feedback Control Design . . . . .	484
12.4.1 Nonlinear Model for Propeller Shaft Speed Control . . . . .	485
12.4.2 Lyapunov Analysis . . . . .	485
12.4.3 Extensions to Integral Control . . . . .	487
<b>13 Decoupled Autopilot Design</b>	<b>489</b>
13.1 Course Autopilot . . . . .	491
13.1.1 PID, Optimal Control and $\mathcal{H}_\infty$ -Control . . . . .	491
13.1.2 Nonlinear Control . . . . .	491
13.1.3 Sliding Mode Control using the Eigenvalue Decomposition . . . . .	492
13.2 Depth Autopilot . . . . .	495
13.2.1 Optimal Control . . . . .	497
13.2.2 Sliding Mode Control using the Eigenvalue Decomposition . . . . .	497
13.3 Speed Control System . . . . .	498
13.4 Exercises . . . . .	498
<b>14 6 DOF Position and Attitude Control</b>	<b>501</b>
14.1 Nonlinear PID Control . . . . .	501
14.1.1 Set-Point Regulation . . . . .	503
14.1.2 Trajectory Tracking Control . . . . .	504
14.2 State Feedback Linearization . . . . .	507
14.2.1 Trajectory Tracking Control . . . . .	507
14.2.2 Adaptive Feedback Linearization . . . . .	509
14.3 Exercises . . . . .	511
<b>V Appendices</b>	<b>513</b>
<b>A Nonlinear Stability Theory</b>	<b>515</b>
A.1 Lyapunov Stability for Autonomous Systems . . . . .	515
A.1.1 Stability and Convergence . . . . .	515
A.1.2 Lyapunov's Direct Method . . . . .	517

A.1.3	Krasovskii–LaSalle’s Theorem . . . . .	518
A.1.4	Global Exponential Stability . . . . .	519
A.2	Lyapunov Stability of Nonautonomous Systems . . . . .	520
A.2.1	Barbalat’s Lemma . . . . .	520
A.2.2	LaSalle–Yoshizawa’s Theorem . . . . .	520
A.2.3	Matrosov’s Theorem . . . . .	521
A.2.4	UGAS when Backstepping with Integral Action . . . . .	522
<b>B</b>	<b>Numerical Methods</b>	<b>525</b>
B.1	Discretization of Continuous-Time Systems . . . . .	525
B.1.1	Linear State-Space Models . . . . .	525
B.1.2	Nonlinear State-Space Models . . . . .	527
B.2	Numerical Integration Methods . . . . .	529
B.2.1	Euler’s Method . . . . .	529
B.2.2	Adams–Bashforth’s 2nd-Order Method . . . . .	530
B.2.3	Runge–Kutta 2nd-Order Method (Heun’s Method) . . . . .	531
B.2.4	Runge–Kutta 4th-Order Method . . . . .	531
B.3	Numerical Differentiation . . . . .	531
<b>C</b>	<b>Matlab GNC Toolbox</b>	<b>533</b>
C.1	M-File Library . . . . .	534
C.2	Simulink Library . . . . .	535



# List of Tables

2.1	The notation of SNAME (1950) for marine vessels. . . . .	18
2.2	WGS-84 parameters. . . . .	42
4.1	Wind force parameters in surge, sway, and yaw (Isherwood 1972). . . . .	118
4.2	Definition of Sea State (SS) codes (Price and Bishop, 1974). Notice that the percentage probability for SS codes 0, 1, and 2 is summarized. . . . .	125
4.3	Definition of Beaufort numbers (Price and Bishop, 1974). . . . .	126
6.1	Continuous-Time Kalman Filter. . . . .	186
6.2	Discrete-Time Kalman Filter. . . . .	189
6.3	Performance characteristics for different types of gyros. . . . .	195
6.4	Performance characteristics for different types of accelerometers. . . . .	196
6.5	Discrete-Time Extended Kalman Filter (EKF) . . . . .	199
6.6	Alternative choices of attitude update laws. The first alternative is GAS while the other two are (local) asymptotically stable due to unstable equilibria. . . . .	220
7.1	Definition of actuators and control variables. . . . .	290
8.1	Parameters for a cargo ship and a fully loaded oil tanker. . . . .	311
8.2	Normalization variables used for the Prime-system and Bis-system. . . . .	313
8.3	6 DOF normalization variables. . . . .	314
8.4	Routh array. . . . .	326
9.1	Eigenvalues, damping ratios and frequencies for RRD control systems. . . . .	379
9.2	Criteria for effectiveness of the crew (Faltinsen 1990). . . . .	384
10.1	Robust Maneuvering: Steps $i = 3, \dots, n$ . . . . .	400
10.2	Adaptive Maneuvering: Step 2. . . . .	406
10.3	Adaptive Maneuvering: Steps $i=3, \dots, n$ . . . . .	407
12.1	DC-motor control model. . . . .	473
A.1	Classification of theorems for stability and convergence. . . . .	516

