

Michel
Remoissenet

WAVES CALLED SOLITONS

Concepts and
Experiments

Second Revised and
Enlarged Edition

孤子波

第2版



Springer-Verlag

世界图书出版公司

M. Remoissenet

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With 135 Figures

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北京·广州·上海·西安

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Library of Congress Cataloging-in-Publication Data

Remoissenet, M.
Waves called solitons : concepts and experiments / Michel
Remoissenet. -- 2nd ed.
p. cm.
Includes bibliographical references and index.
ISBN 3-540-60502-9 (softcover : alk. paper)
1. Solitons. I. Title.
QC174.26.W28R46 1996
530.1'55353--dc20
95-49021
CIP

ISBN 3-540-60502-9 2nd Edition
Springer-Verlag Berlin Heidelberg New York

ISBN 3-540-57000-4 1st Edition
Springer-Verlag Berlin Heidelberg New York

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© Springer-Verlag Berlin Heidelberg 1994, 1996
Printed in Germany

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Reprinted in China by Beijing World Publishing Corporation, 1999

Cover Design: Struve & Partner, Atelier für Grafik-Design, Heidelberg
Typesetting: Data conversion by Springer-Verlag
SPIN 10516388 55/3144 - 5 4 3 2 1 0 - Printed on acid-free paper

书 名: Waves Called Solitons 2nd ed.
作 者: M. Remoissenet
中 译 名: 孤子波
出 版 者: 世界图书出版公司北京公司
印 刷 者: 北京中西印刷厂
发 行: 世界图书出版公司北京公司 (北京市朝阳门内大街 137 号 100010)
开 本: 大 32 开 850×1168 印 张: 8.75
版 次: 1999 年 4 月第 1 版 1999 年 4 月第 1 次印刷
书 号: 7-5062-4105-6/O·245
版权登记: 图字 01-98-1785
定 价: 40.00 元

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独家重印发行。

Preface to the Second Edition

Encouraged by the friendly reception given to the first edition, I have preserved its basic form and most of the details. Apart from some corrections, minor changes, and addition of references where it was necessary, I have made the following changes.

Chapter 1 was expanded by a discussion of the discovery of solitons in the field of electromagnetic waves and optics. A new section devoted to nonlinear transmission lines and their applications in the microwave range has been added to Chap. 3. It seems to me that it was important to describe laboratory experiments on modulational instability, and subsequent generation of solitons, both in electrical transmission lines and in deep water in Chaps. 4 and 5. A description of a very simple experimental pocket version of the mechanical transmission line has been included in Chap. 6. Such a versatile and useful device should stimulate a practical approach to soliton physics. Chapter 7 was completed by a short presentation of some recent experimental results on discrete Josephson transmission lines. A discussion of the experimental modulational instability of coupled optical waves and a simple look at quantum solitons were added to Chap. 8 in order to introduce the reader to such remarkable topics.

Of the many people who made valuable comments on the first edition, I am particularly grateful to M. Dragoman, Y.S. Kivshar and A.W. Snyder.

I would like to thank R.S. MacKay whose corrections and suggestions helped refine the manuscript of this second edition.

I also take pleasure in thanking my Dijon colleague J.M. Bilbault and ex-student P. Marquié whose research efforts are responsible for some part of the new material added to this edition. I also wish to extend my appreciation to M. Pauty for a useful comment on the historical background.

Again, for this edition I have benefited from the technical assistance of B. Michaux and D. Arnoult in designing and performing new experiments, and in preparing a number of new diagrams and photographs.

Dijon
November 1995

Michel Remoissenet

Preface to the First Edition

Nonlinearity is a fascinating element of nature whose importance has been appreciated for many years when considering large-amplitude wave motions observed in various fields ranging from fluids and plasmas to solid-state, chemical, biological, and geological systems. Localized large-amplitude waves called *solitons*, which propagate without spreading and have particle-like properties, represent one of the most striking aspects of nonlinear phenomena. Although a wealth of literature on the subject, including theoretical and numerical studies, is available in good recent books and research journals, very little material has found its way into introductory textbooks and curricula. This is perhaps due to a belief that nonlinear physics is difficult and cannot be taught at an introductory level to undergraduate students and practitioners. Consequently, there is considerable interest in developing practical material suitable for students, at the lowest introductory level.

This book is intended to be an elementary introduction to the physics of solitons, for students, physicists, engineers and practitioners. We present the modeling of nonlinear phenomena where soliton-like waves are involved, together with applications to a wide variety of concrete systems and experiments. This book is designed as a book of physical ideas and basic methods and not as an up-to-the-minute book concerned with the latest research results. The background in physics and the amount of mathematical knowledge assumed of the reader is within that usually accumulated by junior or senior students in physics.

Much of the text of this book is an enlargement of a set of notes and descriptions of laboratory experiments developed over a period of years to supplement lectures on various aspects of wave motion. In spite of the diversity of the material, the book is not a collection of disconnected topics, written for specialists. Instead, I have tried to supply the practical and fundamental background in soliton physics, and to plan the book in order that it should be as much as possible a *self-contained and readable interdisciplinary whole*. Often, the important ideas or results are repeated several times, in different contexts. *Many of my choices of emphasis and examples have been made with experimental aspects in mind. Several experiments described in this book can be performed by the reader.* Although numerical studies play an important role in nonlinear science, I will not consider them in this book because they are described in a considerable body of literature.

In order to facilitate the use of this book, many illustrations have been included in the text and the details of theoretical calculations are relegated to

appendices at the end of each chapter. A number of basic references are given as well as references intended to document the historical development of the subject. The referencing is not systematic; the bibliography listed at the end of the book serves only to advise the reader which sources could be used to fill in gaps in his or her basic knowledge and where he or she could turn for further reading.

The text is organized as follows. Our introduction in Chap.1 is devoted to the beautiful historical path of the soliton. The fundamental ideas of wave motion are then set forth in Chap.2 using simple electrical transmission lines and electrical networks as examples. At an elementary level, we review and illustrate the main properties of linear nondispersive and dispersive waves propagating in one spatial dimension. In Chap.3, we consider waves in transmission lines with nonlinearity. These simple physical systems are very useful for a pedagogical introduction to the soliton concept, and they are easy to construct and to model, allowing one to become quickly familiar with the essential aspects of solitary waves and solitons, and their properties. Specifically, we first examine the effect of nonlinearity on the shape of a wave propagating along a nonlinear dispersionless transmission line. Then we consider the remarkable case where dispersion and nonlinearity can balance to produce a pulse-like wave with a permanent profile. We describe simple experiments on pulse solitons, which illustrate the important features of such remarkable waves. In Chap.4 we consider the *lattice solitons*, which can propagate on an electrical network; then we examine periodic wavetrains, and modulated waves such as *envelope or hole solitons*, which can travel along electrical transmission lines.

Chapter 5 concentrates on such spectacular waves as the hydrodynamic pulse soliton, which was first observed in the nineteenth century, and the hydrodynamic envelope soliton. Simple water-tank experiments are described.

In Chap. 6, by using a chain of coupled pendulums, that is, a mechanical transmission line, we introduce a new class of large amplitude waves, known as *kink solitons* and *breather solitons* which present remarkable particle-like properties. Simple experiments that allow one to study qualitatively the properties of these solitons are presented.

Chapter 7 deals with a more sophisticated device: the superconductive Josephson junction. Here the physical quantity of interest is a quantum of magnetic flux, or *fluxon*, which behaves like a kink soliton and has properties remarkably similar to the mechanical solitons of Chap.5. In Chap.8 *bright and dark solitons* emerge, which correspond to the optical envelope or optical hole solitons, respectively. They can be observed in optical fibers where exploitation of the typical dispersive and nonlinear effects has stimulated theoretical and experimental studies on nonlinear guided waves.

Whereas the previous chapters are concerned with solitons in the macroworld, Chap.9 deals with nonlinear excitations in the microworld. Specifically, we consider the soliton concept in the study of nonlinear atomic lattices. The nonlinear equations that are encountered in the soliton story and models of several systems described in the text can be solved by using remarkable and powerful mathematical techniques, the main steps of which are given in the last chapter.

If a substantial fraction of users of this book feel that it helped them to approach the fascinating world of nonlinear waves or enlarge their outlook, its purpose will have been fulfilled. I hope the reader will feel encouraged to bring to my notice any remaining errors and other suggestions.

I have greatly benefited from frequent discussions with my colleagues and students. I am particularly grateful to Jean Marie Bilbault, who went over the entire manuscript and gave me invaluable comments.

I would also like to thank Alwyn Scott whose criticism and suggestions helped refine the manuscript.

I also wish to extend my appreciation to Patrick Marquié, Guy Millot, Jean François Paquerot, Michel Peyrard, and Claudine and Gérard Pierre for their comments on various chapters. Special thanks go to Bernard Michaux for his assistance in designing and performing experiments, and improving numerous illustrations throughout the book. Finally, it is a pleasure for me to acknowledge the technical assistance I have received from Dominique Arnoult and Claudine Jonon.

Dijon
October 1993

Michel Remoissenet

Solitons Blues

Music: Michel Remoissenet
Arrangement and transcription:
Michel Thibault

The musical score for "Solitons Blues" is presented in five systems, each consisting of a treble and bass staff joined by a brace. The key signature is one sharp (F#), and the tempo is marked as quarter note = 72. The notation includes various musical symbols such as eighth and sixteenth notes, rests, and chords. The piece features a complex interplay between the two staves, with the bass line often providing a harmonic foundation while the treble line carries the melodic and rhythmic motifs. The score is written in a clear, professional style suitable for a music book.

*This book is dedicated to all the scientists
who have made the soliton concept a reality*



On the occasion of a Conference on Nonlinear Coherent Structures in Physics and Biology organized by J. C. Eilbeck and D. B. Duncan, physicists and mathematicians solemnly gathered, on 13 July 1995, by an aqueduct of the Union Canal (see *Nature*, **376**, 373, 1995). This aqueduct, which was named *Scott Russell aqueduct* after a ceremony carried out by Alwyn Scott, is located at Hermiston near the site of the present Heriott-Watt University (Edinburgh). Martin Kruskal was one of the gathering, attempting, with the help of a borrowed boat to recreate *the solitary wave* of John Scott Russell (see Sect.1.2) which can be observed propagating a few meters in front of the boat. (Photo by K. Paterson, reproduced by kind permission of Heriot-Watt University).

Contents

1	Basic Concepts and the Discovery of Solitons.....	1
1.1	A look at linear and nonlinear signatures.....	1
1.2	Discovery of the solitary wave.....	3
1.3	Discovery of the soliton.....	6
1.4	The soliton concept in physics.....	10
2	Linear Waves in Electrical Transmission Lines.....	12
2.1	Linear nondispersive waves.....	12
2.2	Sinusoidal-wave characteristics.....	15
2.2.1	Wave energy density and power.....	18
2.3	The group-velocity concept.....	19
2.4	Linear dispersive waves.....	21
2.4.1	Dispersive transmission lines.....	21
2.4.2	Electrical network.....	23
2.4.3	The weakly dispersive limit.....	26
2.5	Evolution of a wavepacket envelope.....	27
2.6	Dispersion-induced wavepacket broadening.....	31
	Appendix 2A. General solution for the envelope evolution.....	34
	Appendix 2B. Evolution of the envelope of a Gaussian wavepacket...	35
3	Solitons in Nonlinear Transmission Lines.....	37
3.1	Nonlinear and dispersionless transmission lines.....	37
3.2	Combined effects of dispersion and nonlinearity.....	41
3.3	Electrical solitary waves and pulse solitons.....	42
3.4	Laboratory experiments on pulse solitons.....	46
3.4.1	Experimental arrangement.....	46
3.4.2	Series of experiments.....	48
3.5	Experiments with a pocket version of the electrical network.....	52

3.6	Nonlinear transmission lines in the microwave range	56
Appendix 3A.	Calculation of the effect of nonlinearity on wave propagation	58
Appendix 3B.	Derivation of the solitary-wave solution	60
Appendix 3C.	Derivation of the KdV equation and its soliton solution	62
Appendix 3D.	Details of the electronics: switch driver and pulse generator	64
4	More on Transmission-Line Solitons	65
4.1	Lattice solitons in the electrical Toda network	65
4.1.1	Lattice solitons.....	67
4.2	Experiments on lattice solitons	68
4.2.1	Collisions of two lattice solitons moving in opposite directions.....	70
4.2.2	The Fermi-Pasta-Ulam recurrence phenomenon.....	70
4.3	Periodic wavetrains in transmission lines.....	71
4.3.1	The solitary wave limit and sinusoidal limit of the cnoidal wave.....	72
4.4	Modulated waves and the nonlinear dispersion relation	72
4.5	Envelope and hole solitons	74
4.5.1	Experiments on envelope and hole solitons	76
4.6	Modulational instability.....	77
4.7	Laboratory experiments on modulational instability	82
4.7.1	Model equations	82
4.7.2	Experiments.....	84
4.8	Modulational instability of two coupled waves.....	86
Appendix 4A.	Periodic wavetrain solutions	88
Appendix 4B.	The Jacobi elliptic functions.....	90
4B.1	Asymptotic limits.....	91
4B.2	Derivatives and integrals.....	93
Appendix 4C.	Envelope and hole soliton solutions.....	93
5	Hydrodynamic Solitons	98
5.1	Equations for surface water waves.....	98
5.1.1	Reduced fluid equations	99
5.2	Small-amplitude surface gravity waves.....	100

5.3	Linear shallow- and deep-water waves.....	103
5.3.1	Shallow-water waves.....	103
5.3.2	Deep-water waves	104
5.4	Surface-tension effects: capillary waves	105
5.5	Solitons in shallow water	107
5.6	Experiments on solitons in shallow water	110
5.6.1	Experimental arrangement.....	111
5.6.2	Experiments.....	111
5.7	Stokes waves and soliton wavepackets in deep water	115
5.7.1	Stokes waves	115
5.7.2	Soliton wavepackets	116
5.7.3	Experiments on solitons in deep water	117
5.8	Experiments on modulational instability in deep water	118
Appendix 5A.	Basic equations of fluid mechanics.....	121
5A.1	Conservation of mass.....	121
5A.2	Conservation of momentum.....	123
5A.3	Conservation of entropy.....	124
Appendix 5B.	Basic definitions and approximations.....	124
5B.1	Streamline	124
5B.2	Irrotational and incompressible flow	125
5B.3	Two-dimensional flow: the stream function.....	126
5B.4	Boundary conditions.....	128
5B.5	Surface tension	129
Appendix 5C.	Derivation of the KdV equation: the perturbative approach.....	130
Appendix 5D.	Derivation of the nonlinear dispersion relation.....	133
Appendix 5E.	Details of the probes and the electronics.....	136
6	Mechanical Solitons	137
6.1	An experimental mechanical transmission line	137
6.1.1	General description of the line	137
6.1.2	Construction of the line.....	139
6.2	Mechanical kink solitons	139
6.2.1	Linear waves in the low-amplitude limit.....	140
6.2.2	Large amplitude waves: kink solitons.....	141
6.2.3	Lorentz contraction of the kink solitons	143

6.3	Particle properties of the kink solitons.....	145
6.4	Kink-kink and kink-antikink collisions.....	146
6.5	Breather solitons	148
6.6	Experiments on kinks and breathers	150
6.7	Helical waves, or kink array.....	151
6.8	Dissipative effects.....	153
6.9	Envelope solitons	155
6.10	Pocket version of the pendulum chain, lattice effects.....	157
Appendix 6A.	Kink soliton and antikink soliton solutions	159
Appendix 6B.	Calculation of the energy and the mass of a kink soliton	160
Appendix 6C.	Solutions for kink-kink and kink-antikink collisions, and breathers.....	161
6C.1	Kink solutions	163
6C.2	Kink-kink collisions.....	163
6C.3	Breather solitons.....	164
6C.4	Kink-antikink collision.....	165
Appendix 6D.	Solutions for helical waves.....	166
7	Fluxons in Josephson Transmission Lines.....	168
7.1	The Josephson effect in a short junction	168
7.1.1	The small Josephson junction	169
7.2	The long Josephson junction as a transmission line	171
7.3	Dissipative effects	175
7.4	Experimental observations of fluxons.....	177
7.4.1	Indirect observation	177
7.4.2	Direct observation	178
7.4.3	Lattice effects	180
Appendix 7A.	Josephson equations	180
8	Solitons in Optical Fibers	182
8.1	Optical-fiber characteristics	182
8.1.1	Linear dispersive effects.....	183
8.1.2	Nonlinear effects	185
8.1.3	Effect of losses	186
8.2	Wave-envelope propagation	187

8.3	Bright and dark solitons	189
8.3.1	Bright solitons	190
8.3.2	Dark solitons	192
8.4	Experiments on optical solitons	193
8.5	Perturbations and soliton communications	195
8.5.1	Effect of losses	195
8.5.2	Soliton communications	196
8.6	Modulational instability of coupled waves	197
8.7	A look at quantum optical solitons	198
	Appendix 8A. Electromagnetic equations in a nonlinear medium	199
9	The Soliton Concept in Lattice Dynamics	202
9.1	The one-dimensional lattice in the continuum approximation	202
9.2	The quasi-continuum approximation for the monatomic lattice	207
9.3	The Toda lattice	209
9.4	Envelope solitons and localized modes	210
9.5	The one-dimensional lattice with transverse nonlinear modes	212
9.6	Motion of dislocations in a one-dimensional crystal	215
9.7	The one-dimensional lattice model for structural phase transitions	216
9.7.1	The order-disorder transition	218
9.7.2	The displacive transition	219
	Appendix 9A. Solutions for transverse displacements	221
	Appendix 9B. Kink soliton or domain-wall solutions	223
10	A Look at Some Remarkable Mathematical Techniques	225
10.1	Lax equations and the inverse scattering transform method	225
10.1.1	The Fourier-transform method for linear equations	226
10.1.2	The Lax pair for nonlinear evolution equations	227
10.2	The KdV equation and the spectral problem	229
10.3	Time evolution of the scattering data	230
10.3.1	Discrete eigenvalues	230
10.3.2	Continuous spectrum	232
10.4	The inverse scattering problem	233
10.4.1	Discrete spectrum only: soliton solution	234
10.5	Response of the KdV model to an initial disturbance	236

10.5.1	The delta function potential	236
10.5.2	The rectangular potential well	237
10.5.3	The sech-squared potential well	237
10.6	The inverse scattering transform for the NLS equation.....	238
10.7	The Hirota method for the KdV equation	239
10.8	The Hirota method for the NLS equation	243
References		247
Subject Index		259

1 Basic Concepts and the Discovery of Solitons

Today, many scientists see nonlinear science as the most important frontier for the fundamental understanding of Nature. The soliton concept is now firmly established after a gestation period of about one hundred and fifty years. Since then, different kinds of solitons have been observed experimentally in various real systems, and today they have captured the imagination of scientists in most physical discipline. They are widely accepted as a structural basis for viewing and understanding the dynamic behavior of complex nonlinear systems. Before introducing the soliton concept via its remarkable and beautiful historical path we compare briefly the linear and nonlinear behavior of a system.

1.1 A look at linear and nonlinear signatures

First, let us consider at time t the response R_1 of a linear system, an amplifier for example, to an input signal $E_1 = A \sin \omega t$ of angular frequency ω , as sketched in Fig. 1.1. In the low amplitude limit the output signal or the response of the system is linear, in other words it is proportional to the excitation

$$R_1 = a_1 E_1. \quad (1.1)$$

Here a_1 is a quantity that we assume to be constant (time independent) to simplify matters. If we double the amplitude of the input signal, the amplitude of the output signal is doubled and so on. The sum of two input signals E_1 and E_2 yields a response which is the superposition of the two output signals,

$$R = a_1(E_1 + E_2) = R_1 + R_2, \quad (1.2)$$

and a similar result holds for the superposition of several signals.

Next, if the amplitude of the input signal gets very large, distortion occurs as a manifestation of overloading. In this case, the response is no longer proportional to the excitation; one has

$$R = a_1 E_1 + a_2 E_1^2 + a_3 E_1^3 + \dots = a_1 E_1 \left(1 + \frac{a_2}{a_1} E_1 + \frac{a_3}{a_1} E_1^2 + \dots \right) \quad (1.3)$$

and signals at frequencies 2ω , 3ω , and so on, that is, *harmonics of the input signal are generated*. In some cases a chaotic response can occur: this phenomenon will not be considered in this book. Moreover, the sum of two signals at the input results not only in the sum of responses at the output but also in the product of sums and so on. *The superposition of states is no longer valid.*