Fractal, scaling and growth far from equilibrium

分形、标度及远离平衡态的增长

Paul Meakin 著

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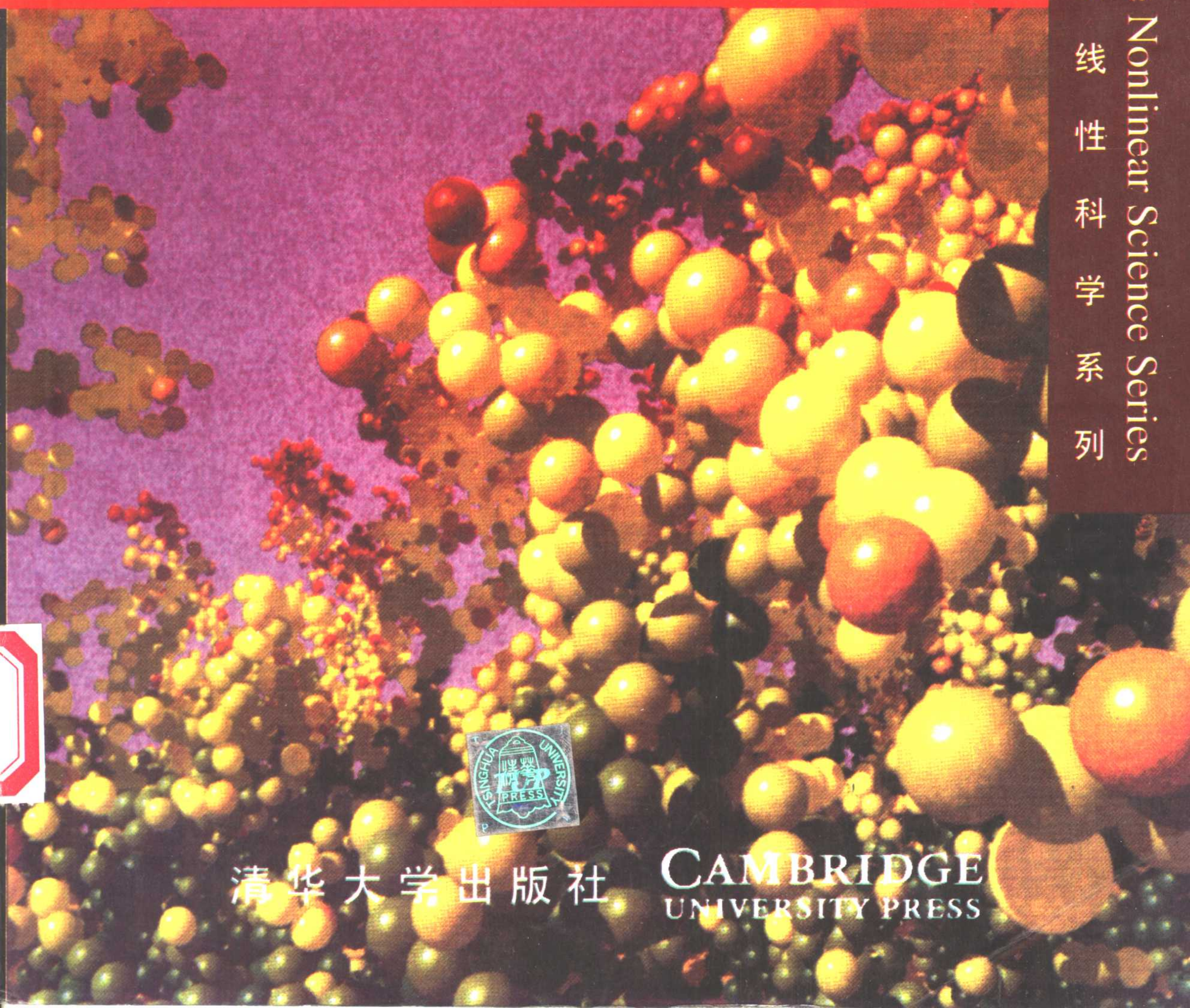

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This book describes the progress that has been made towards the development of a comprehensive understanding of the formation of complex, disorderly patterns under far-from-equilibrium conditions.

The application of fractal geometry and scaling concepts to the quantitative description and understanding of structure formed under non-equilibrium conditions is described. Self-similar fractals, self-affine fractals, multifractals and scaling methods are discussed, with examples, to facilitate applications in the physical sciences. Computer simulations and experimental studies are emphasised, but the author also includes a discussion of theoretical advances in the subject. Much of the book deals with diffusion-limited growth processes and the evolution of rough surfaces, although a broad range of other applications is also included. The book concludes with an extensive reference list and guide to additional sources of information.

This book will be of interest to graduate students and researchers in physics, chemistry, materials science, engineering and the earth sciences, and especially those interested in applying the ideas of fractals and scaling to their work or those who have an interest in non-equilibrium phenomena.

Preface

The development of a full understanding of the universe around us, in terms of the basic properties of fundamental "particles" and their interactions, has long been a dream of the physicist. Mindful of the difficulties encountered when this approach is used to calculate the behavior of very simple systems, such as molecules containing just a few atoms, the problem of understanding the nature of much more complex systems, such as snowflakes, soot aggregates and rough surfaces produced by processes such as vapor deposition or erosion, might seem to be a daunting prospect. However, during the past one or two decades, substantial progress has been made, based on statistical physics concepts such as scaling and the independent development of fractal geometry, based on late 19th century and early 20th century mathematics, by Benoit Mandelbrot. To a large extent, this progress has been made by giving up the idea that an understanding of complex systems can be based on an ever more detailed knowledge of their microscopic components and focusing instead on the "universal" properties that all materials possess in common, irrespective of their atomic and molecular structure, and the manner in which properties on one length scale relate to those on other length scales. The connection between microscopic and macroscopic behavior is still important, and the theoretical justification for much of the work described in this book is based on models that contain microscopic components and interactions, at least on an abstract level. However, one of the objectives of this book is to illustrate that scaling symmetries can be used, in much the same way as other symmetries, to study a wide variety of systems and phenomena, without taking into account the underlying microscopic physics on a detailed level.

One of the main objectives of this book is to show how a surprisingly wide range of complex, disorderly systems can be quantitatively understood using simple statistical physics concepts and simple mathematical tools. This book contains many equations, but it should be accessible to anyone with a good undergraduate education in the physical sciences. My original idea was to write a single volume on the basics of fractals and scaling and applications in various areas of science and technology. It soon became apparent that I wanted to say more than could reasonably be contained in one book. Consequently this book concentrates on some of the more fundamental aspects of pattern formation, fractals and scaling. I am in the process of writing a second book, focusing on colloidal fractals and aggregation kinetics, and a third monograph on the applications of fractals and scaling in selected areas of science and technology.

My own interest in this area was first stimulated by the work of Thomas Witten and Leonard Sander, more than ten years ago, on diffusion-limited aggregation. The diffusion-limited aggregation model has become one of the most important paradigms for disorderly growth, far from equilibrium, and plays a central role in this book.

Much of the work on this book was carried out during a one year visit to the Center for Advanced Studies at the Norwegian Academy of Science and Letters. The remainder of the work was carried out in the Physics Department at the University of Oslo. I would like to thank Jens Feder and especially Torstein Jøssang for hospitality at the University of Oslo, and Torstein Jøssang for making my stay at the Center for Advanced Studies possible. I have also benefited considerably from stimulating interactions with a quite large number of graduate students and post-doctoral associates at the University of Oslo.

I would like to thank Fereydoon Family, Joachim Krug, Leonard Sander, Lorraine Siperko, Tamás Vicsek and Stephanie Wunder for making valuable comments on a draft of this book and for making suggestions that have led to substantial improvements. I am also grateful to many colleagues and collaborators who have contributed figures; they are acknowledged in the figure captions. Many of the figures have been provided by graduate students in the Cooperative Phenomena Group at the University of Oslo and illustrate various aspects of their own research.

Paul Meakin

Oslo, Norway

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Color plates are between pp. 242 and 243.

Chapter 1

Pattern Formation Far From Equilibrium

The diversity of the natural shapes that surround us has a profound impact on the quality of our lives. For this reason alone, it is not surprising that the origins of these shapes have been the subject of serious study since antiquity.[1] It has long been believed that a quantitative characterization of natural forms is an important step towards understanding their origins and behavior. Unfortunately, there have, until recently, been relatively few general approaches towards the quantitative description of the complex, disorderly patterns that are characteristic of most natural phenomena. The essentially non-equilibrium nature of most pattern-formation processes has also contributed to the comparatively slow development of this field. The systematic and generally well understood techniques of equilibrium statistical mechanics cannot be applied to the majority of pattern-formation processes.

In the past one to two decades, the outlook has improved substantially. The pioneering, interdisciplinary work of B. Mandelbrot [2] has demonstrated that mathematical concepts, once believed to be of no possible relevance to the real world, can provide us with new ways of describing and thinking about an amazingly broad range of structures and phenomena. In addition, the scaling concepts that were originally applied to a relatively narrow range of problems such as critical phenomena [3] and the structure of macromolecules [4] have been successfully applied to a very much broader range of problems. In many cases, the fractal geometry approach developed by Man-

1. For example, references to the six-fold form of snowflakes, that go back many centuries, can be found in the commentary by Mason in Hardie's translation of Kepler's work on snowflakes [1].

delbrot can be used to provide a more intuitive, geometric interpretation of scaling behavior. This has brought a measure of intellectual democracy to previously arcane areas of physics and has enabled physicists to contribute to a wide range of important problems outside of the traditional confines of physics.

Many isolated early applications of the fractal approach to physical phenomena can be found. For example, in 1926 Richardson [5] asked the question "*Does the wind possess a velocity?* " and suggested that the distance x traveled by an air "particle" in time t may have to be described "*by something rather like Weierstrass's function*

$$x = kt + \sum_n (1/2)^n \cos(5^n \pi t)." \qquad (1.1)$$

The Weierstrass–Mandelbrot function, a generalization of equation 1.1, described in chapter 2, is now widely recognized as an example of a self-affine fractal function. Richardson went on to describe studies of the dispersion of tracers in the atmosphere and suggested that this process could be described in terms of a length-dependent diffusion coefficient $\mathscr{D}(\ell) \sim \ell^{4/3}$. Richardson argued that the projection of the density of the dispersing tracers onto a straight line $\rho(\ell, t)$ could then be described by the non-Fickian diffusion equation

$$\partial\rho(\ell,t)/\partial t = \partial[\mathscr{D}(\ell)\partial\rho(\ell,t)/\partial\ell]/\partial\ell. \qquad (1.2)$$

Richardson showed that the solution to this equation, starting with a delta function distribution at time $t = 0$, has the form

$$\rho(\ell,t) = t^{-3/2} f(\ell^{2/3}/t), \qquad (1.3)$$

where $f(x)$ is an exponentially decaying function. It follows, from equation 1.3, that $< \ell^2(t) >^{1/2} \sim t^{\nu} \sim t^{3/2}$. For a particle moving with a constant velocity, the exponent $1/\nu$ can be interpreted as a fractal dimensionality. However, for the problem studied by Richardson, space and time are "mixed" so that the exponent relating the distance traveled to the elapsed time cannot be given a simple interpretation in terms of a fractal particle trajectory [6].

Similarly, the development of scaling ideas [3, 4, 7, 8], during the last three decades, has provided new ways of quantitatively describing and better understanding the growth kinetics of both fractal and non-fractal objects. During the same period, a new understanding of non-linear phenomena was developed. The work on non-linear systems demonstrated that apparently complex processes could have simple origins and provided paradigms that helped to motivate the work described in this book.

Fractal geometry has been shown to provide a basis for describing objects as small as polymer molecules and as large as the coastlines of continents.

Mandelbrot [2, 9] has also discussed the application of fractals to the distribution of visible matter in the universe. For some time, it has been accepted that the distribution of visible matter is inhomogeneous and can be described in terms of a fractal dimensionality of $D \approx 1.25$ on "short" length scales, less than the "galaxy correlation length" of about 5 Mpc.[2] The ideas that there may be no galaxy correlation length and that the distribution might be fractal out to much longer length scales have been the subject of heated controversy. Some aspects of this problem are discussed in chapter 2.

During the past 15 years, the "big bang" theory for the creation of the universe has been seriously challenged by an inflationary model [10, 11, 12] with an inherently fractal nature. This scenario suggests a fractal universe of almost unlimited size.

Most of the applications discussed in this book are much more "down to Earth". However, it is interesting that theoretical and modeling approaches, similar to those used to simulate non-equilibrium growth and aggregation processes (chapters 3 and 4) and surface growth (chapter 5), have been proposed for the evolution of galaxies [13, 14, 15] and galaxy distributions [16, 17].

While the major applications of fractal geometry have been in the physical and life sciences, many interesting examples have been found that are related to human activities. Examples include the form of urban centers [18, 19], the distribution of weather monitoring stations [20, 21, 22], gravity stations [23, 24] and railroad networks. These fractal distributions did not come about as a result of a design process, but, in some cases, a fractal distribution may be advantageous [25].

It has also been suggested that the distributions of rapidity,[3] characterizing the particles generated by high energy collisions, including hadron–hadron, hadron–nucleus and nucleus–nucleus collisions, and cosmic shower events may have a multifractal character [26, 27, 28, 29, 30, 31, 32]. This has been interpreted in terms of a random cascade model [33].

Fractal geometry has also been used to describe the distribution of events in time. In the case of discrete events, the fractal dimensionality D of the set of times $\{t_i\}$ can be measured. Examples of cases in which this approach appears to be of value include the distribution of reversals in the Earth's magnetic field ($D \approx 0.89$ [34]) and the temporal distribution of earthquakes in a region of limited size ($0.12 < D < 0.26$ [35]).

2. 5 Mpc is 5 megaparsecs, where 1 parsec is the distance at which the mean radius of the Earth's orbit subtends an angle of 1 second, 1 pc $\approx$ 3.2 light years $\approx 3 \times 10^{16}$ m.
3. The rapidity y is defined as $y = (1/2)\ln[(E + p_L)/(E - p_L)]$, where E is the energy of an emitted particle and p_L is the component of its momentum along the collision axis. In practice, the pseudo-rapidity, $\eta = -\ln\tan(\theta/2)$, is usually measured, where θ is the emission angle.

1.1 Power Laws and Scaling

Power law relationships play a central role in the study of fractals and scaling. The power law function $y(x)$, given by

$$y(x) = cx^a, \tag{1.4}$$

has an important symmetry that can be expressed as

$$y(\lambda x) = c(\lambda x)^a = c\lambda^a x^a = const. \cdot y(x). \tag{1.5}$$

This describes the scale invariance of $y(x)$ (the power law function $y(x)$ in equation 1.4 has the same shape on all scales). A trivial, but important, consequence of this scale invariant symmetry is that the exponent a does not depend on the units in which x or y are measured. Functions that satisfy the relationship $y(\lambda x) = \lambda^a y(x)$ are said to be homogeneous. The function

$$y(x) = c_1 x^{a_1} + c_2 x^{a_2} \tag{1.6}$$

does not satisfy equation 1.5 and is an example of an inhomogeneous power law. In practice, the analysis of data from simulations or experiments, in terms of power law exponents, is based on the logarithmic version of equation 1.4

$$\log y(x) = \log c + a \log x, \tag{1.7}$$

so that the exponent a and the amplitude c can be obtained by plotting $\log y(x)$ against $\log x$. The observation of a linear relationship between the logarithms of two quantities over a sufficiently large range of scales is often considered to provide *prima facie* evidence for a power law relationship between these quantities.

In practice, this simple procedure is fraught with hazards. There is no consensus on the standards required for establishing power law relationships from experimental or numerical data. In some areas of physics, it has been possible to observe linear behavior, on a log–log plot, covering more than four orders of magnitude (powers of ten), in both the related quantities. However, data of this quality are quite rare. To observe power law behavior over four decades, from analysis of a recorded image of a physical object, it would be necessary to have a digitized representation with a resolution of better than one part in 10^5 (10^{10} pixels in a 2-dimensional image!). Such images are not routinely available. It would, of course, be possible to extend the range of observation by using images of parts of the structure recorded under different magnifications. If this approach is used, care must be taken to avoid bias towards the selection of "interesting" or even "typical" parts of the pattern.

An important example of a power law relationship is that between the mass M and the characteristic size (average diameter for example) L for a self-similar fractal aggregate, composed of particles with a diameter ϵ. In this case, the mass of the aggregate is given by

$$M(L,\epsilon) \approx C_{o}m(L/\epsilon)^{D}, \qquad (1.8)$$

where m is the mass of a single particle ($m \approx \rho\epsilon^{d}$, where d is the Euclidean dimensionality of the particle and ρ is the particle density) and C_{o} is a geometrical constant of order 1. The exponent D, in equation 1.8, is the fractal dimensionality. Very often, the dependence of the mass on L, when all other quantities are held fixed, is the main focus of interest. In this case, a "shorthand" version of equation 1.8

$$M \sim L^{D} \qquad (1.9)$$

is used. However, it should always be remembered that this equation "stands in" for the dimensionally balanced, or dimensionally homogeneous, equation 1.8. In equation 1.9, and others like it, the symbol "$\sim$" should be interpreted as meaning "scales as".

This book focuses attention on the power law part of equations like 1.8. In practice, the "amplitude" (c in equation 1.4) is important and embodies the "real physics" behind power law relationships. In many phenomena, the exponents are universal (invariant to small changes in the physical process or model). Under these conditions, the amplitudes provide the only means to control physical properties and behavior. However, important insights can be obtained from the scaling relationships, described by equations such as 1.9. The amplitudes are omitted (some would say perversely) from almost all of the equations in this book, even when they are well known and/or easily calculated. In most cases, it is much more difficult to calculate the amplitude than the exponent. In many practical situations, the scaling relationship $y(x) \sim x^{a}$ implied by equations such as 1.9 is all that is required. By ignoring the amplitudes, a much broader range of phenomena can be discussed, so that the power and simplicity of the scaling approach is emphasized. The situation here is similar to that encountered in applications of the quantum theory of angular momentum. In this case, the matrix elements needed to calculate properties of physical interest can be divided into two parts, according to the Wigner–Eckart theorem [36, 37]: a part called the reduced matrix element, analogous to the amplitude c in equation 1.4, that depends on the physical details and that is, in most cases, difficult to calculate, and a part that depends only on the angular momentum quantum numbers, which can easily be calculated using group theory.

Phenomena that require the description of the properties of a large ensemble of similar structures, rather than a single sample, are frequently encountered. In this case, equation 1.8 should be replaced by

$$< M(L,\epsilon) > \approx C_{o}m(L/\epsilon)^{D}, \qquad (1.10)$$