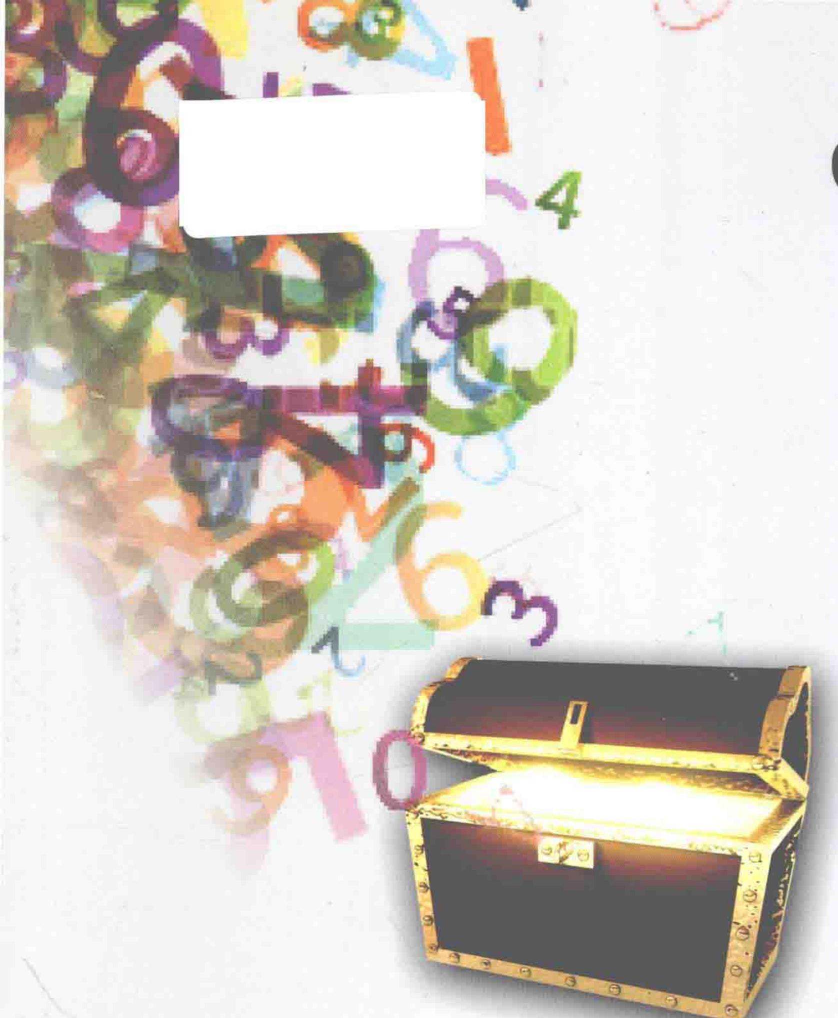

Third Edition

Number Treasury^3

Investigations, Facts and Conjectures about More than 100 Number Families

Margaret J Kenney • Stanley J Bezuszka

World Scientific

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Number Treasury^3 is dedicated to my co-author Stanley J. Bezuszka, longtime Professor of Mathematics and Director of the Mathematics Institute at Boston College, who was a source of inspiration and encouragement to so many in the world of mathematics education.

Margaret J. Kenney

Foreword

The gift of numbers like the gift of fire has made the world much brighter.
Stanley J. Bezuszka (1914–2008)

Introduction

Time and numbers were born together. Time is a measure of change, and numbers express that measure. There is an awesome mystery that surrounds the all-pervasive dimension of time. Likewise, there is an awesome mystery and fascination about numbers that attract distinguished mathematics researchers as well as imaginative amateurs the world over. Teachers who pursue numbers and the rich history of numbers with their classes can provide students with an understanding that mathematics is a collaborative effort that has been nurtured by individuals and groups representing many cultures and periods of history. Students can learn to become accomplished investigators, to make discoveries, and contribute to a branch of mathematics that is vibrant and motivating. **Number Treasury**^{3} has evolved in order to serve as a catalyst for those who ascribe to this point of view.

Details

Number Treasury^3 is a broadening and update of **Number Treasury**^{2}. The book contains information about more than 100 families of positive integers. Brief historical notes often accompany the descriptions and examples of the number families. Exercises for each major family are provided to stimulate insight. Some exercises contain problems that are thought provokers to be resolved simply with paper and pencil; others should be tackled with calculator in hand so that lengthier computations can be managed with ease and take the results to a higher level of understanding. Still other problems are intended for more extensive exploration with the use of computer software. In some instances it is helpful to model problems with hands-on materials.

The emphasis in **Number Treasury**^{3} is on doing rather than proving. However, the reader is urged to think critically about situations, to provide reasoned explanations, to make generalizations and to formulate conjectures. The book begins with a chapter of Investigations. These are principally stand-alone activities that represent content drawn from the Chapters 2 through 7 of the book. Their purpose is to set the tone of the book and to stimulate student reflection and research in a variety of areas. In fact, throughout the book, the reader will find numerous open-ended problems. This book also contains detailed solutions to the Exercises and Investigations. A Glossary and Index are provided for quick access to information. References and recommended readings are supplied so that teachers and students can use this book as a stepping stone to more concentrated study.

Who Uses Number Treasury^3

This book is written for teachers and students. For teachers **Number Treasury**^{3} is a resource for instructional preparation and problems, together with snapshots of mathematical history intended for teachable moments. For students who are engaged in learning about number families and who are assigned problems, projects and papers, **Number Treasury**^{3} is a useful source of ideas and topics. The mix of discussion with examples and illustrations is intended to serve as a writing model for the student. Both audiences should think critically about the content, provide carefully reasoned explanations, make generalizations, and form conjectures.

Who Is Involved

The first edition was completed with the able assistance of six Boston College graduates and undergraduates: Jeanne Cavanaugh, James Cavanaugh, Claudia Katze, Stephen Kokoska, Jill Nille, and Jonathan Smith.

Seven Boston College graduates and undergraduates were indispensable in the production of the second edition. Special thanks and grateful appreciation go to Joan Martin for her thoughtful content and style suggestions, editorial advice and word processing skills; to Cynthia Tahlmore, Geraldine Mele, and Erin Mitchell for computer graphics and word processing assistance; to Allyson Russo, Shannon Toomey, and Megan Mazzara for problem solutions.

The third edition has been completed by the surviving original author with the invaluable assistance and perseverance of Geraldine Mele who offered not only content suggestions but who also especially contributed word processing, computer graphics and style expertise. Sincere gratitude and appreciation is also extended to Joan Martin for her careful review of the manuscript.

Contents

Chapter 1

A Perfect Number of Investigations $28 = 1 + 2 + 4 + 7 + 14$

A GREAT discovery solves a great problem but there is a grain of discovery in any problem.
George Polya (1887–1985)

What are They?

The Investigations that follow are a set of stand-alone activities. Each Investigation focuses on at least one number family or topic relating to numbers. All but six of the Investigations are described on one page. Share with students that the problems in an Investigation are intended to be challenging in many ways:

- The time needed to complete an Investigation may vary and exceed the time required to finish a typical homework assignment.
- The computation necessary to bring closure may be lengthy and demanding — even with the use of technology.
- The amount of writing, discussing, explaining and illustrating may be more than anticipated.

Teacher Tips

The Investigations are listed in ascending order of difficulty in the table on the next two pages. There are three levels of difficulty represented in the 28 Investigations that can be assigned individually or adapted for group work. The lowest level consists of the first eight Investigations that have the least prerequisites. The middle level consists of the next nine Investigations and requires more use of abstract reasoning and familiarity with algebraic expressions. The final 11 Investigations challenge the student to persist and probe more deeply in order to complete the work.

There is also a column in the table naming the most significant prerequisite(s) needed by the student to understand and carry out the work in each Investigation. The teacher may choose to provide additional content background for some specific Investigations prior to assigning them. Assign the Investigations as extended homework or as in-class work. Some Investigations call for the preparation of reports. Thus, students may need further directions, especially about the kind of resources available for them to use. Students should know the Internet is an excellent resource, and that it should be used appropriately as they compile their reports.

Finally, pages noted in the Prerequisites column refer to related material contained in Chapters 2 through 7.

Footsteps of Lagrange

Develop the first 20 terms of the sequence s_n, where s_n is the number of ways n can be written as a sum of at most 4 squares. Note one term will be counted as a sum.

EXAMPLE

How many ways can 5 be written as a sum of at most 4 squares?
1 way since $5 = 4 + 1 = 2^2 + 1^2$.

EXAMPLE

How many ways can 9 be written as a sum of at most 4 squares?
2 ways since $9 = 2^2 + 2^2 + 1^2$ and $9 = 3^2$.

1. Fill in the following table.

n	Ways to Write n as Sum of at Most 4 Squares	Number of Ways
1		
2		
3		
4		
5	$2^2 + 1^2$	1
6		
7		
8		
9	3^2; $2^2 + 2^2 + 1^2$	2
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		

2. Describe in a few sentences some patterns you observe in the table.
3. Find a number and verify that it can be written as a sum of at most 4 squares in exactly
   a) three ways   b) four ways