Astrophysical Quantities

BY

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PREFACE

THE intention of this book is to present the essential quantitative information of astrophysics in a form that can be readily used. Questions relating to the material included and the form adopted are discussed in the introductory chapter.

The information is as up to date as possible, but the approved values of astrophysical constants change from year to year and there can be no finality about the last digit of most of the values quoted. It is to be expected, then, that some users will wish to pencil in amended values to suit later results or to agree with their own opinions. The author hopes that readers will let him know of any errors or faulty values that are contained. From consideration of such advice, and from the use of new results, it should be possible to progress towards the ideal of recording an accurate value for every quantity.

It has not been possible to give adequate acknowledgment in the references to all the sources of information. The references quoted are mostly to recent papers, since earlier ones can be traced from these. Use has been made of many handbooks, text-books, and tabulations which are quoted in the references. The comprehensive Landolt-Börnstein tables became available during the late stages of preparation, and these were used for checking and filling in gaps. However, it is not thought that the existence of such tabulations restricts in any way the need for the present volume.

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CHAPTER 1

INTRODUCTION

§ 1. Requirements

PROGRESS in any of the physical sciences is very closely linked with a determination of the precise values of the quantities concerned. Extensive labour and very much care have been put into the measurement of some of these essential quantities, and in the end the user may obtain the advantage of all this effort simply by reading the number that represents the final value. Thus an enormous economy of expression can be effected by writing a final result and omitting mention of the long chapter of events that led up to it.

The present work is concerned with final results, and we have to consider how these can be most effectively extracted from the available information and then presented for use. It is found that the necessary procedure becomes fairly clearly defined once we have decided what are the more important of the various user requirements. These requirements are listed below together with the steps and policy found necessary to meet them.

Material to be included

The purpose of *Astrophysical Quantities* is to present the quantitative framework on which astrophysics is being built. To do this the book should contain all experimental and theoretical values, constants, and conversion factors that are fundamental to astrophysical arguments. The extent to which individual items should be described, e.g. individual stars or spectrum lines, depends on whether such description is necessary for an appreciation of the whole range of such items. It is generally found that a finite and quite small number of data is sufficient to put the ideas of any branch of astrophysics onto a quantitative basis. The following work is intended to be an assembly of such data.

Ready availability

First consideration has been given to presenting the data in a form in which they can be readily found, understood, and used. For this purpose it is essential that all individual results be reduced to one adopted 'best value', or to an averaged smooth curve. The detailed procedure used for weighting the individual results to obtain the adopted value cannot be described in full as this would take up too much space and detract from the systematic presentation of the numerical results themselves. Brief remarks on the averaging procedure are sometimes given, but more generally the reader must rely on the references for any more critical examination of the data.

It is not possible to quote values in all units and normally only one is given. In order to maintain general usefulness it is therefore essential to have conversion factors at hand, and some attention has been paid to this requirement. The conversions are often expressed as formulae and to this extent it has been necessary to insert a number of the more general formulae of astrophysics. However there is no attempt to set out a complete table of basic astrophysical formulae and those included are intended only as a reminder of the inter-relations between the quantities involved.

Avoidance of ambiguity

If one were to avoid ambiguity at all costs it would require a complete definition of every quantity involved. This would not be well suited to a work whose main aim is to give quantitative values, and it has been assumed instead that the quantities mentioned are already understood. Dangers of ambiguity, however, arise from the multiplicity of rather similar units and quantities, and efforts are directed mainly at resolving misunderstandings of this sort. In particular the numerical factors 2 and π are often troublesome, and the definitions given are intended to clear such points.

Another possible source of ambiguity is connected with the multiple meaning of symbols. To counter this difficulty the numbered sections are self-contained as regards terminology, and it should not be necessary to look outside a section for the explanation of a quantity mentioned in it. Certain well-known symbols, however, are used without repeated explanation, and these are collected in § 6.

Other questions of ambiguity in the meaning of symbols and of table and diagram headings are discussed in § 4.

Conciseness

Compactness of tabulation is quite essential in this work, not only to keep the size within reason, but also to allow a more useful presentation. For this purpose the intervals of the arguments are made fairly large, and simple graphical interpolation can be used for intermediate values. Empirical formulae are often used in preference to tables.

A set of reference numbers is used in each section and the references collected at the end.

Generality and completeness

So much progress in astrophysics is dependent on pressing beyond the present boundaries that it becomes necessary to give all data over as wide an argument as possible. Data for the extreme conditions are often not known accurately and must be regarded as provisional. The same applies to a great many quantities that are not directly observable, but they are included where possible. When various estimates differ greatly the results quoted are generally a compromise.

Selected examples are often given of those items that are too numerous for listing completely. When the value of a quantity varies considerably a mean or representative value is sometimes given.

Accuracy and probable errors

It would be useful if a probable error could be attached to every value quoted, but there is no consistent way of deriving such information for most of the data. Instead, probable errors are restricted to the more fundamental quantities, and some of these probable errors are dependent rather vaguely on the inter-agreement of different estimates. As a general policy the usual ± implies a probable error [p.e.], but in the cases where root-mean-square errors are used exclusively in the literature these are quoted and indicated by the addition of the expression [s.e.] (standard error).

Throughout the book some attempt has been made to give an indication of the error by quoting the correct number of figures. The intention is that the probable error would be between ½ and 5 in the last digit.

Versatility and consistency

The absolute values of astrophysical quantities constitute a live and ever-changing subject, and it is necessary to cater for numerical changes. For this reason tabulations are used in preference to diagrams which must be redrawn whenever a value is changed.

For some astronomical undertakings internal consistency among the constants is a matter of major importance [1]. However, it is only possible to produce a set of consistent data by an exhaustive analysis of all information available at a certain date. Once a new value of any constant is accepted an elaborate reshuffle of the values is generally necessary. A strict adherence to consistency would therefore cause a tendency to cling to old-established values and exclude new information. In the present work, on the other hand, the intention is to use new information wherever available. When such new information demands a clear-cut change in other constants the change has been made, but in other cases the change of dependent constants will await further analysis. The inconsistency errors so caused will not usually be greater than the probable error and therefore will not be serious. It is not expected that the values quoted here would be used without modification for an elaborate calculation in which internal consistency was vital.

There are a few constants that have been used so widely that they have achieved the status of conversion factors. These are sometimes quoted, even though they may not now be considered the best values.

Sources of information

There are several reasons for giving the references to the sources of information. They enable the reader to check any data as regards numerical value or meaning. This may be particularly necessary in the present case where the original information is frequently modified to conform to the plan of the tabulation. The references also enable the reader to extend the information to other details not catered for in the present tabulations. Finally, they give some credit to the original worker. Unfortunately it is not possible to give any adequate consideration to the last point since it would require references out of all proportion to the available space. Instead the main endeavour has been to refer to the more recent work on each topic so that through these the earlier work can be traced. More particularly the freest use has been made of summary articles in various branches of astrophysics and the references are often directed to these rather than to original work. In the physical sections many data have been obtained from handbooks and tabulations.

Calculation aids

There is no intention of supplying conversion tables for extended routine calculations. A few tables of this type are included (e.g. refraction, precession, and galactic coordinate tables), but they are intended rather as a means of indicating the values involved than for routine use.

[1] G. M. Clemence, *A.J.* **53**, 169 (1948).

§ 2. General Plan

The subdivisions of the book are almost independent. In any work that deals with a great number of varied concepts it becomes a problem to indicate where each one is defined, and this problem is accentuated when it may be required to extract isolated values as rapidly as possible. It is to cope with this situation that the work is divided into sections (§§) which are self-contained as regards symbols, definitions, and references. There is very little reading matter in any one section, and hence the search for an explanation should be rapid. The size of any section is governed by these considerations.

The tables and diagrams are not numbered separately, but each table or diagram is placed within the appropriate section. The symbols used at the head of a table are described within the section and not necessarily described again in the table. In this respect the script of a section may be regarded as an extended heading to the table.

The references are collected as near to the end of each section as allowed by the tabulation. Often some consideration of smoothness, consistency with other data, extension of material, accuracy, tabular interval, or unit used will effect some change in the figure quoted. A reference number in italics [*123*] implies that the data are taken exclusively from the reference without modification. A reference number not in italics [123] implies only that the reference has been consulted for obtaining the data published. In some sections it is found preferable to list references at the end without attempting to indicate how individual figures were obtained from them.

It is intended that names of chapters and sections should be sufficient for the location of most of the material. For more obscure quantities an index is available. A value may be quoted more than once if that is called for by the arrangement.

The sections §§ 6, 11, 22, 92, 93 may be consulted for symbols, contractions, etc. that are used frequently without redefinition.

§ 3. Quantitative Significance of Symbols

When symbols are to be used for expressing quantitative values it is essential to make it clear whether (*a*) the symbol represents a number which has still to be multiplied by the units before it equals the quantity, or (*b*) the symbol represents the quantity itself. Both systems have some disadvantages.

The difficulty with the first system (*a*) is that a new symbol must be defined for every set of units used. We would not be able to say, for example,

$$\text{density } \rho = 5{\cdot}2\ \mathscr{M}_\odot\ \text{parsec}^{-3} = 3{\cdot}5 \times 10^{-22}\ \text{g cm}^{-3},$$

but would have to define two symbols for the density. Furthermore, the definition would have to be made in the text, which means that the equation would not be self-contained. The system would therefore lead to clumsy expression which could not be tolerated in the present work.

Instead we adopt the second system (*b*), sometimes known as the Stroud system [1], and we may write, for example,

$$s = 12\ \text{cm} = 0{\cdot}12\ \text{m} = 0{\cdot}394\ \text{ft}.$$

In simple cases this leads to no mathematical or dimensional ambiguity. For example, in the familiar acceleration equation

$$s = ut + \tfrac{1}{2}at^2$$

we might have the numerical values

$$72\ \text{cm} = 6\ \text{cm s}^{-1}\ 3\ \text{s} + \tfrac{1}{2} \times 12\ \text{cm s}^{-2}\ 9\ \text{s}^2,$$

and we find the dimensions cancel, leaving a clear numerical relation.

However there are some difficulties which should be faced before the Stroud system is adopted completely. Consider the case of the Stefan equation

$$\mathscr{F} = \sigma T^4.$$

In order to make the dimensions cancel it is necessary to give the constant σ the dimensions erg cm^{-2} s^{-1} $degree^{-4}$. One could be quite logical and always give the constants those dimensions that would reduce the quantity equation to a numerical equation. However, this would only add to the confusion, as can be seen if we consider the empirical form of the Rayleigh extinction equation

$$a_\lambda = 0{\cdot}0082\ \lambda^{-4{\cdot}05}.$$

To make this complete the numerical constant 0·0082 must be multiplied by the dimensions cm^{-1} $\mu^{4{\cdot}05}$ (where μ = microns). Besides being a very confusing dimension, there is no practical place to write the dimension in the equation. Furthermore, the idea is too unconventional to be useful here. A better method, which we adopt in the present work, is to add the dimensions of those quantities that require them; thus

$$a_\lambda = 0{\cdot}0082\ \lambda^{-4{\cdot}05} \quad [a_\lambda \text{ in cm}^{-1},\ \lambda \text{ in } \mu].$$

The information in the square brackets then converts the quantities a_λ and λ into numbers. It will be noticed that the algebraic significance of the word 'in' within the square brackets is that the quantity should be 'divided by' the unit. Sometimes, however, the bracket expressions are only a helpful guide. For instance [e in E.S.U.] means that the electrostatic system of units is to be used for the charge e and it must imply (unless otherwise mentioned) that the C.G.S. system is used for the other quantities. Where any expression still appears to be ambiguous it might be assumed that the C.G.S. system is intended.

[1] J. B. HENDERSON, *Engineering*, Sept. (1923).

§ 4. Headings

Since astrophysics deals with some very large and very small numbers, great use must be made of the powers of 10 in expressing the values. For this purpose it is important to avoid any ambiguity of the sign of the index, and therefore the relation between the heading, the power of ten, the units, and the tabular number must be clearly understood.

The following is an example of a common fault:

Tabular heading $v \times 10^{-8}$ cm/sec

Tabulated numbers T

In this case v is a velocity, but it is ambiguous from the heading whether

$$\text{velocity} = \text{T} \times 10^{-8}\ \text{cm/sec}$$

or

$$\text{velocity} = \text{T} \times 10^{8}\ \text{cm/sec}.$$

It is also ambiguous whether v is intended to represent the actual velocity or a number which has to be multiplied by a unit to give the velocity.

In order to use a table (or diagram) quantitatively one has always to construct an equation of the type

$$\text{quantity} = (\text{tabular value}) \times (\text{power of } 10) \times (\text{unit}).$$

As in any other equation, it is essential that we know on which side of the equation each factor falls, and our headings should be constructed in a way that makes this perfectly clear. In the tabulations that follow we keep as close to this equation as possible by putting the heading or symbol that describes the quantity above the line, and all the factors of the right-hand side of the equation below the line. The line separating the heading from the table is then analogous to the = sign. However, it will not be necessary to read this explanation in order to use the tables without risk of ambiguity.

This procedure has the natural advantage that large numbers have positive indices of 10 and small numbers negative indices. We make an occasional exception to this rule when the quantity concerned is in wide use or has the status of a standard unit. For example, we use the heading $10^6 W/\lambda$ [W = equivalent width, λ = wavelength] which is not ambiguous.

On the borders of diagrams it is sometimes even more difficult to avoid the same type of ambiguity. As an actual example we quote the diagram border

$$T_e\ (^\circ\text{K} \times 10^{-6}).$$

This leaves it uncertain whether the temperatures plotted are of the order 10^{-6} °K or 10^6 °K. The following forms, however, are unambiguous and satisfactory:

$$T_e\ (\text{unit} = 10^6\ ^\circ\text{K}),$$

$$v\ (\text{in } 10^8\ \text{cm/s}),$$

$$\log \rho\ (\rho \text{ in g cm}^{-3}).$$

§ 5. Logarithmic Quantities

In astrophysics great use is made of coefficients of diminution of intensity. These may be expressed in magnitude, exponential, or decadic scales, and these coefficients have the character of dimensions in the equations that arise. There is a danger of not knowing which scale is used unless it is stated with the units. We therefore adopt the notations

mag = magnitude interval,

exp = exponential interval,

dex = interval in powers of 10,

decibel = interval in 0·1 powers of 10,

which are inter-related as follows:

$$1{\cdot}0857\,\text{mag} = 1{\cdot}0000\,\text{exp} = 0{\cdot}43429\,\text{dex} = 4{\cdot}3429\,\text{decibel},$$

$$2{\cdot}5000\,\text{mag} = 2{\cdot}3026\,\text{exp} = 1{\cdot}00000\,\text{dex} = 10{\cdot}0000\,\text{decibel}.$$

We could, for example, express the absorption a of S.T.P. ozone at 6000 A as

$$a = 0{\cdot}068\,\text{dex/cm} = 0{\cdot}157\,\text{exp/cm} = 0{\cdot}170\,\text{mag/cm}.$$

This would follow the policy of putting as much information as possible into the equation. In this form it is preferable to speak of the absorption rather than the absorption coefficient. When the word coefficient is mentioned it immediately raises the question whether the coefficient is exponential or decadic.

The term *dex* in the above notation has been introduced to meet the needs of convenience [1]. Dex converts the number before it into its 10-based antilogarithm. The term can be used for a typographically convenient method of expressing large numbers, as in the example $10^{39} = 39$ dex. It can also be used to introduce verbal simplicity into statements on probable errors, ranges, and variations. The following hypothetical statements illustrate its use: (*a*) the probable error of the density of matter in the universe is $\pm 1{\cdot}5$ dex, (*b*) the frequency range of useful radio-astronomy observations is 3·2 dex, and (*c*) the increase in the distance scale of extragalactic nebulae as a result of recent researches is 0·28 dex.

[1] C. W. Allen, *Observatory*, **71**, 157 (1951).

§ 6. Notation

As far as possible the notation used is in agreement with that recommended by the International Astronomical Union [1], and the International Union of Pure and Applied Physics [2]. The notation is usually described within each section, but a number of symbols are of such generality that it is not thought necessary to define them repeatedly. The notation in this section will be used without further definition when it is thought no ambiguity can arise. Sections (§§) 11, 22, 92, and 93 also give notation in wide use.

Signs

$\simeq$ approximately equal	$\rightarrow$ leads to
$\propto$ varies as	∞ infinity
$\equiv$ equivalent to, means	Ø diameter

$\bar{x}$, $\bar{2}{\cdot}34$ mean value of x, $0{\cdot}34 - 2{\cdot}00$

$\int_{4\pi} \ldots d\omega$ integration in all directions over solid angle 4π

$\oint$ integration around a closed curve

Astronomical symbols

	✱ Star	
☉ Sun	♁ ⊕ Earth	♅ ♅ Uranus
☾ Moon	♂ Mars	♆ Neptune
☿ Mercury	♃ Jupiter	♇ Pluto
♀ Venus	♄ Saturn	☄ Comet

♈ Aries	0°	♌ Leo	120°	♐ Sagittarius	240°
♉ Taurus	30°	♍ Virgo	150°	♑ Capricornus	270°
♊ Gemini	60°	♎ Libra	180°	♒ Aquarius	300°
♋ Cancer	90°	♏ Scorpio	210°	♓ Pisces	330°

☌ Conjunction, having the same longitude or right ascension
□ Quadrature, differing by 90° in longitude or right ascension
☍ Opposition, differing by 180° in longitude or right ascension
☊ Ascending node (longitude of)
☋ Descending node (longitude of)
♈ First point of Aries

⊕ Globular cluster
○ Planetary nebula
⊙ Annular nebula

Symbols in frequent use

Italic, Greek, and special type letters are used for symbols.

π	ratio circumference/diameter, parallax (in seconds of arc)
e	exponential base, electron charge (E.S.U. implied)
ν, λ	frequency, wavelength
ω	solid angle, angular frequency ($= 2\pi\nu$)
c	velocity of light
t	time
$d\omega$, dV, ds, dt	element of solid angle, volume, length, time
T	temperature
m	mass of electron, apparent magnitude
m_{pv}, m_{pg}, m_{bol}	photovisual, photographic, and bolometric magnitudes
M	absolute magnitude (standardized to 10 pc). Subscripts often added
$\mathcal{M}$	mass of an astronomical object
R	radius, Rydberg wave-number, gas constant, angle of refraction
k	Boltzmann constant, Gaussian gravitational constant
μ	microns, proper motion (in seconds of arc per year)
ρ	density
h	height, altitude, Planck constant ($= 2\pi\hbar$)
N	number of objects (usually per unit volume)
I_ν, I_λ	spectral intensity
g	acceleration of gravity
α, δ	right ascension and declination
l, b	galactic longitude and latitude
σ	radiation constant ($\mathcal{F} = \sigma T^4$), standard deviation