D. M. Young A. D. Crowell

# Physical Adsorption of Gases

# PHYSICAL ADSORPTION OF GASES

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### PREFACE

STRINGENT limitations have to be imposed on the scope of a book covering, in a single volume, the literature on physical adsorption since 1930. The present treatment deals only with the adsorption process proper, excluding additional complications such as the capillary phenomena peculiar to porous adsorbents. Furthermore, no attempt has been made to catalogue the myriad gas-solid systems whose properties are reported in the literature. Rather, individual systems have been mentioned only where they illustrate general principles. Only in this way has it been possible to give a coherent account of the subject, supported by an adequate but not irritatingly profuse number of references. Happily, the formidable task of classifying the detailed information on gas-solid systems in the literature has been ably discharged by Deitz in the two editions of Bibliography of Solid Adsorbents, leaving writers of books free to devote their attention to the general characteristics of the physical adsorption process.

We are greatly indebted to Professor F. C. Tompkins, F.R.S., who suggested the book, and whose counsel and encouragement have been invaluable. Thanks are also due to Professor R. A. Beebe and Dr. J. A. Morrison for reviewing parts of the chapter on calorimetry, to Professor R. L. McIntosh for reviewing the section on dielectric properties and to Dr. G. J. Young for advice on heats of immersion. One of us (D. M. Y.) wishes to thank his colleague Dr. D. N. Glew for many helpful discussions during the preparation of the manuscript.

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### ROPETER

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### INTRODUCTION

### GENERAL

When a gas\* is allowed to come to equilibrium with a solid or liquid surface, the concentration of gas molecules is always found to be greater in the immediate vicinity of the surface than in the free gas phase, regardless of the nature of the gas or surface. The process by which this surface excess is formed is termed adsorption. In any solid or liquid, the atoms at the surface are subject to unbalanced forces of attraction normal to the surface plane; the balance of forces is partially restored by the adsorption of gas molecules.

Adsorption is to be distinguished from absorption, which involves bulk penetration of the gas into the structure of the solid or liquid by some process of diffusion. Since absorption is governed by the laws of diffusion, it can usually be differentiated from adsorption. The term sorption is applied to cases where both the above processes may

be occurring simultaneously, obnoo solid moregroups [golevn]

Adsorption of a gas on a solid is a spontaneous process and is therefore accompanied by a decrease in the free energy of the system. Since the process involves loss of degrees of freedom of the gas, in passing from the free gas to the adsorbed film, there is also a decrease in entropy. It follows from the equation

(1) sure, at the temperati 
$$2\Delta T - H\Delta = \Delta H$$
 tion took place, although

that the adsorption process must always be exothermic. This is true regardless of the nature of the forces involved.

### PHYSICAL AND CHEMICAL ADSORPTION

Adsorption processes may be classified as physical or chemical, depending on the nature of the forces involved. *Physical adsorption*, also termed van der Waals adsorption, is caused by molecular interaction forces; the formation of a physically adsorbed layer may be likened to the condensation of a vapour to form a liquid. Not only is the heat of physical adsorption of the same order of magnitude as that of

<sup>\*</sup> The term 'gas' will serve to denote gas or vapour. When it is necessary to distinguish between gases and vapours, the terms 'permanent gas' and 'condensible vapour' will be used.

liquefaction, but physically adsorbed layers, particularly those many molecular diameters thick, behave in many respects like two-dimensional liquids. On the other hand, chemical adsorption, usually abbreviated to *chemisorption*, involves transfer of electrons between the solid (or *adsorbent*) and the gas (or *adsorbate*). The process essentially involves the formation of a chemical compound between the adsorbate and the outermost layer of adsorbent atoms.

The distinction between physical adsorption and chemisorption is usually clear cut, but where there is doubt a decision can be made on

the basis of one or more of the following criteria:

(a) The heat of physical adsorption is of the same order of magnitude as the heat of liquefaction of the adsorbate and is rarely more than twice or three times as large, whereas the heat of chemisorption is of the same order as that of the corresponding bulk chemical reaction. In some cases, however, exceptionally low heats of chemisorption are found. In making such comparisons it should be noted that in both types of adsorption, because of surface heterogeneity and lateral interaction effects, the heat of adsorption may vary considerably with surface coverage. This effect is particularly marked in chemisorption where the lateral interaction forces, being invariably repulsive, reinforce the effects of heterogeneity.

(b) Physical adsorption, like condensation, is a general phenomenon and will occur with any gas-solid system provided only that the conditions of temperature and pressure are suitable. On the other hand, chemisorption will take place only if the gas is capable

of forming a chemical bond with the surface atoms.

(c) A physically adsorbed layer may be removed by reducing the pressure, at the temperature at which adsorption took place, although the process may be slow on account of diffusion effects. The removal of a chemisorbed layer, however, often requires much more rugged conditions, especially on metal surfaces where very high temperatures or positive ion bombardment are needed. An exceptional case is the system oxygen on charcoal, in which the chemisorbed layer is so strongly held that high temperature desorption yields not oxygen but a mixture of carbon monoxide and dioxide.

(d) Under suitable conditions of temperature and pressure, physically adsorbed layers several molecular diameters in thickness are frequently found. In contrast, chemisorption is complete once a monomolecular layer is built up, although physical adsorption may

occur on top of the chemisorbed monolayer. sodgrosha learning do

(e) Since physical adsorption is related to the process of liquefaction, it only occurs to an appreciable extent at pressures and temperatures close to those required for liquefaction. Thus if p is the

### THE DATA OF ADSORPTION

equilibrium pressure of the adsorbed film and  $p_0$  is the vapour pressure of the bulk liquid at the temperature of the experiment, it is generally found that below  $p/p_0 = 0.01$  no significant adsorption takes place. There are some exceptions, notably with adsorbents having very fine pores. On the other hand, chemisorption often proceeds at much lower pressures and much higher temperatures.

(f) Physical adsorption and chemisorption may sometimes be distinguished by their different rates of approach to equilibrium. Physical adsorption per se is instantaneous but, with highly porous or finely powdered adsorbents, diffusion of the gas into the adsorbent mass is often slow, particularly at low pressures. Chemisorption may be instantaneous but there are many systems where chemisorption involves an activation energy. In both physical and chemical adsorption, precise measurements may be hampered by the establishment of a pseudo-equilibrium. Thus the outer strata of adsorbent are more heavily covered with adsorbate than the centre of the solid mass; subsequent redistribution of the adsorbed film to give uniform coverage at all points in the solid mass is sometimes an extremely slow process. Clearly, the use of a rate criterion to distinguish physical from chemical adsorption is fraught with complications.

### THE DATA OF ADSORPTION

Experimental measurements of the amount adsorbed, v, as a function of pressure and temperature may conveniently be plotted in the form of adsorption isotherms

 $v = f(p)_T \tag{2}$ 

Isotherms are essentially plots of the free energy change as a function of amount adsorbed. Their shape can also yield qualitative information about the adsorption process and a semi-quantitative measure of the fraction of the surface covered by adsorbate (and hence, with assumptions, the surface area of the adsorbent). For these reasons and because they can be measured directly, isotherms are the most commonly used  $p\!-\!v\!-\!T$  plot in adsorption studies.

Although adsorption isotherms with shapes ranging from the monotonous to the fantastic have been reported in the literature, the classification introduced by Brunauer et al.<sup>2</sup>, for systems at temperatures below the critical temperature of the gas, is nonetheless valuable. Their five basic isotherm shapes are shown in Figure 1.1. Isotherms of Type I are associated with systems where adsorption does not proceed beyond a monomolecular layer. The remainder all involve the multilayer formation. Types IV and V are characteristic of

### INTRODUCTION

multilayer adsorption on highly porous adsorbents, the flattening of the isotherms at the highest pressures being attributed to capillary phenomena. Since capillary effects lie outside the scope of this book, reference will only be made to isotherms of Types I, II and III.

Further information can be secured from a study of adsorption

isosteres

nonque 
$$p = f(T)$$
 nonque la labert  $f(T)$ 

These cannot be measured directly because it is impractical to hold v constant. Instead, values of p and T corresponding to fixed values of v are interpolated from a family of adsorption isotherms. Provided

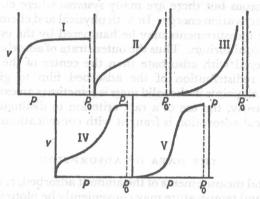


Figure 1.1. The five types of physical adsorption isotherm.  $p_0$  is the saturation vapour pressure. (From Brunauer, The Adsorption of Gases and Vapours (1945); by courtesy of the Clarendon Press and Princeton University Press.)

the heat of adsorption does not vary significantly over the temperature range studied, plotting the isosteres in the familiar form  $\log p$  vs. 1/T will yield a family of straight lines, each corresponding to a fixed value of the amount adsorbed. The linearity of the isosteres, incidentally, provides a useful check on the internal consistency of the isotherms. The heats of adsorption at each amount adsorbed may be calculated from the slopes of the isosteres, using the Clausius—Clapeyron equation in the form

$$\frac{\mathrm{d}\ln p}{\mathrm{d}(1/T)} = -\frac{q_{st}}{R} \tag{4}$$

Here  $q_{st}$  is the *isosteric* heat of adsorption, a differential quantity which varies with the degree of surface coverage, hence also with the amount

### REFERENCES

adsorbed v. Plots of  $q_{st}$  vs. v, commonly referred to as 'heat curves', are useful in determining the thermodynamic functions of the system, in characterizing the nature of the adsorbent surface and in estimating the degree of lateral interaction.

Isobars, plots of amount adsorbed at constant pressure as a function of temperature, may be interpolated from isotherms or measured directly but they have little utility in physical adsorption studies.

The heat of adsorption may also be measured calorimetrically and, since the amount adsorbed is also required, the adsorption isotherm is conveniently measured at the same time. In calorimetric work a finite amount of gas is admitted to the adsorbent in the calorimeter and so the heat measured is an integral quantity. In practice, however, the increments of gas admitted may be made so small that the heat measured closely approximates the differential quantity. Good agreement between isosteric and calorimetric heats of adsorption has been secured, particularly in recent work, giving grounds for confidence in both types of measurement.

Given the free energy change as a function of v (i.e. an isotherm) and the heat of adsorption as a function of v, the change in entropy accompanying adsorption may also be obtained as a function of the amount adsorbed (equation 1). These thermodynamic quantities may be complemented by calorimetrically determined values of the heat capacity of the adsorbed film.

heat capacity of the adsorbed film. The party granter and the order of the adsorbed film. The company of the adsorbed film.

### fisch od jonnes amelden REFERENCES\*

- <sup>1</sup>Trappell, B. M. W. Chemisorption, Chapter I. Butterworths, London (1955) <sup>2</sup>Brunauer, S., Deming, L. S., Deming, W. E. and Teller, E. J. Amer. chem. Soc. 62, 1723 (1940)
- \* The publishers have used journal abbreviations as given in World List of Scientific Periodicals: Butterworths, London, 1962.

adsorbed v. Plots of  $q_n$  vs.  $v_n$  commonly referred to as 'heat curves', are useful in determining the thermodynamic functions of the system, in characterizing the nature of the  $q_n$  is sorbent surface and in estimating the degree of lateral interaction.

# PHYSICAL ADSORPTION FORCES

### The heat of adsorption northugogrammed calorimetrically and,

directly but they have little utility in physical adsorption studies.

since the amount adsorbed is also required, the adsorption isotium. THE forces giving rise to adsorption are no different from those involved in any other interatomic or intermolecular interaction phenomenon, but there are problems of special interest because the atoms of the solid are affected by the fact that they participate in the structure of the solid. The interactions between an atom or molecule and a solid surface are electromagnetic in origin, involving the electrons and nuclei of the system, the state of which is determined by quantum mechanics. When the equilibrium charge distribution is such that there is no transfer or sharing of electrons among the participating atoms and the individuality of the interacting species is thus maintained, the forces are said to be physical. Such forces are associated with physical or van der Waals adsorption. In principle then. the interaction between an atom or molecule and a solid surface can be found by determining the quantum mechanical state of the system of an atom or molecule and a solid, and then calculating the electromagnetic interactions. Since far simpler problems cannot be dealt with exactly, it is necessary to resort to various approximations.

The physical interaction between an atom or molecule and a solid surface is due to the attractive van der Waals forces which, following Margenau<sup>1</sup>, may be defined simply as the forces which give rise to the constant a in van der Waals' equation, and the repulsive forces which arise when atoms come close enough together to allow inter-

penetration of the electron clouds.

Attractive forces pertinent to physical adsorption may be divided into several categories. If the adsorbed atom or molecule possesses no permanent dipole or multipole moment, then the attractive interaction with the solid surface is due to non-polar dispersion forces only, unless the solid itself has an external electric field, as, for example, in the case of an ionic crystal. In this latter case the field of the adsorbent will induce electric moments in the adatom\*, producing an interaction in addition to the dispersion forces. If the adsorbed atom or molecule has multipole moments of its own, there will be additional

<sup>\*</sup> The term adatom or admolecule is used to designate an adsorbed atom or molecule.

### DISPERSION FORCES

interactions with the adsorbent due to: (a) charge distributions induced in the adsorbent, and (b) interactions of these moments with any permanent field of the solid. Although it is undoubtedly artificial to separate the above interactions, it is convenient to do so, both for ease in conception and simplicity in computation.

# oscillators with a single frequency corresponding to the characteristic optical dispersion fre Sadarof Noisaaqsid conditions, London

# Dispersion Forces between Pairs of Molecules

For a better understanding of the general procedures involved, it is profitable to examine first the nature of dispersion forces between a pair of atoms or molecules. Consider an inert gas atom in the ground state. The charge distribution is spherically symmetrical and therefore the atom possesses no permanent dipole or multipole moment and thus no external field. The kinetic energy of this state is not zero, as the atom has zero-point energy and possesses instantaneous dipole and multipole moments. The instantaneous moments induce in any neighbouring atom resonant moments which are in phase with those of the first atom and there is thus a force of attraction between the atoms. This resonance has been elegantly described by Lennard-Jones<sup>2</sup> as due to 'a sympathetic fluctuation of the electron space clouds of the two atoms, which produces in the atoms effective dipoles tending to move more in phase than out of phase'. These forces are known as dispersion forces on account of their relationship, noted by London<sup>3</sup>, to optical dispersion.

The usual quantum mechanical calculation of the dispersion energy makes use of a straightforward perturbation or variational technique. In all but the simplest cases the wave function of the individual atoms is taken as that for a set of isotropic harmonic oscillators or, as London 4 has put it, an 'orchestra of periodic dipoles'. It is significant that the unperturbed wave function is usually not anti-symmetrized in the electron coordinates, which means that the results are valid only as long as the atoms are far apart, usually farther apart than the equilibrium separation, the distance of principal interest in adsorption studies. If the individual atoms possess no resultant angular momentum and the perturbation is taken as the interaction between instantaneous dipoles, the first-order perturbation is zero and it is the second-order perturbation which is the dispersion energy. The result under these conditions is the well known expression

culations of interactions of molecules with solid suffices, is the 
$$E = \frac{1}{2} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} = \frac{1}{2}$$

### PHYSICAL ADSORPTION FORCES

where r is the distance between the atoms and C is a constant. E is

often referred to simply as the dispersion energy.

The value of C in the perturbation approach outlined above depends on the energy levels of the atoms and the quantum mechanical oscillator strengths, which in turn depend on the transition probabilities between the electronic states. Actually, as far as interactions of this type are concerned, atoms frequently behave as isotropic oscillators with a single frequency corresponding to the characteristic optical dispersion frequency. Under these conditions, London derived the expression

si ii , boylovni sounbood 
$$C_{\rm L}=\frac{3}{2}\alpha_1\alpha_2\frac{h\nu_1\nu_2}{\nu_1+\nu_2}$$
 , as sound noise qual to (2)

where  $G_{\rm L}$  refers to this particular expression for C,  $\alpha_1$  and  $\alpha_2$  are the polarizabilities of the two atoms,  $\nu_1$  and  $\nu_2$  are the corresponding characteristic frequencies, and h is Planck's constant. Since  $h\nu_1$  and  $h\nu_2$  are often approximately equal to the ionization energies  $I_1$  and  $I_2$  of the atoms,  $C_{\rm L}$  is sometimes written as

and nearest definition of the contract of 
$$C_L = \frac{3}{2} \alpha_1 \alpha_2 \frac{I_1 I_2}{I_1 + I_2}$$
 and the moin standard contract of the contract of the

By applying a variational method Slater and Kirkwood<sup>5</sup> obtained the expression

$$C_{\rm SK} = \frac{3eh}{4\pi m^{1/2}} \frac{\alpha_1 \alpha_2}{(\alpha_1/N_1)^{1/2} + (\alpha_2/N_2)^{1/2}} \tag{4}$$

where e and m are the charge and mass of the electron, and  $N_1$  and  $N_2$  represent the number of electrons in the outer shells of the atoms. This treatment neglected the inner electrons. Kirkwood<sup>6</sup> let  $N_1$  and  $N_2$  be the total number of electrons in the atoms, but this approach clearly overweighted the inner electrons. Hellmann<sup>7</sup> and Buckingham<sup>8</sup> introduced summations over the sub-shells; unfortunately the information needed to make use of these results is rarely available. Perhaps the most reasonable interpretation, suggested by Margenau<sup>1</sup>, is that the values of N play the same role as quantum mechanical oscillator strengths and might best be regarded as empirical constants.

Another expression for C which has been used, particularly in calculations of interactions of molecules with solid surfaces, is the

Kirkwood–Müller<sup>9</sup> equation which involves the magnetic susceptibilities  $\chi_1$  and  $\chi_2$  of the atoms:

$$C_{\text{KM}} = 6mc^2 \frac{\alpha_1 \alpha_2}{(\alpha_1/\chi_1) + (\alpha_2/\chi_2)}$$
 (5)

where m is the mass of the electron and c is the velocity of light. Actually this equation cannot be expected to be precise, as Margenau<sup>1</sup> has noted, since the mechanism which gives rise to dispersion forces is quite different from that which is significant in the interaction of electrons with magnetic fields. A refined form of equation (5) which gives closer agreement with experimental results and includes electron correlation terms, has been developed by Salem <sup>10</sup>.

Derivations of equations (1) to (5) may be found in Margenau<sup>1</sup>. Other summaries of particular value have been given by London<sup>4</sup>, and more recently by Hirschfelder, Curtiss and Bird<sup>11</sup> and by

Pitzer 12.

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Equation (1) arises from dipole-dipole interactions only. If dipole-quadrupole and quadrupole-quadrupole interactions are included, it is found that

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$$E = -C/r^6 - C'/r^8 - C''/r^{10}$$
 because (6)

where the constants C' and C" can in principle be calculated from the wave functions of the interacting species. Calculations have been carried out only for a few atoms (see Hirschfelder, Curtiss and Bird 11 for a review) and for harmonic oscillators 13,14,15 giving expressions analogous to equation (2). Kiselev and co-workers 16 have obtained equations for C' and C" analogous to the Kirkwood-Müller equation (5). The contribution to E from the terms in equation (6) clearly depends on the separation, r, between the atoms in question. At the van der Waals minimum separation between two hydrogen atoms, Margenau estimated that the dipole-quadrupole term amounts to about one-half, and the quadrupole-quadrupole term one-seventh, of the dipole-dipole term. For helium at the van der Waals minimum the dipole-quadrupole term was about one-quarter the dipoledipole term. Using equation (6) to calculate heats of adsorption of a variety of gases on graphite, Kiselev and collaborators 16 estimated that dipole-quadrupole term contributed of the order of 10 per cent and the quadrupole-quadrupole term of the order of 1 per cent to the total value of the dispersion interaction.

A problem of some importance is to find a value for  $C_{12}$  between unlike atoms 1 and 2, if values of  $C_{11}$  and  $C_{22}$ , the appropriate constants