



R Raghavan and E Ozkan

A method for computing unsteady flows in porous media



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ABOUT THIS VOLUME

This Research Note, self-contained and concise, provides a basis to study unsteady flow in saturated porous media. The note provides for the development of algorithms that examine three-dimensional flows subject to complicated boundary conditions that are a natural consequence of flow in geological systems. A new way to understand the flow in porous media is presented. The authors pay attention to computational considerations, and options for developing codes are addressed. The note consists of five chapters: the first is introductory; the second and third are devoted to showing how one arrives at the solutions of interest; the fourth chapter presents various reformulations to aid computations and presents a few illustrative examples; the fifth chapter is a natural progression of the first four chapters to more complicated visualizations of flow in porous media.

Readership: The book should be of interest to scientific researchers and graduate students in partial differential equations and diffusion phenomena, and to practitioners in soil science, ground water, oil recovery, geothermal systems, and underground disposal.

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∞ A method for computing unsteady flows in porous media

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Preface

We are stimulated to write this survey for at least two reasons. First, much of the information on flow through porous media appears in disparate journals and there is little or no interdisciplinary communication. Second, a number of researchers, particularly in petroleum engineering, have spent considerable effort over the past thirty years developing computationally-efficient algorithms to describe flow through porous media with a view to evaluating the properties of subterranean reservoirs. A compilation of recent advances should be of particular interest to those involved in the study of partial differential equations.

The issues we address arise in a number of scientific and engineering disciplines. Much of the impetus for the study of flow through porous media, however, derives as a consequence of flow in geologic media—the work of hydrologists, soil physicists, and agricultural engineers who work at shallow depths and petroleum engineers who, lured by the need to produce oil and gas more efficiently, operate at deeper levels. In chemical engineering, packed bed reactors are used to facilitate various reactions. Environmental concerns have also led to an increased interest in flow through porous media.

Briefly, these notes may be thought of as a survey of implicit evolution equations that arise in classical models of diffusion or convection in composite materials. Restricting our attention to linear systems, the mathematical description of the diffusion process in a composite medium consisting of two components leads to the following pair of equations:

$$c_1 u_t - a_1 \nabla^2 u + \frac{1}{\epsilon}(u - v) = 0, \quad (1)$$

and

$$c_2 v_t - a_2 \nabla^2 v + \frac{1}{\epsilon}(v - u) = 0. \quad (2)$$

Here, the dependent variables $u[(\mathbf{x}), t]$ and $v[(\mathbf{x}); t]$ denote temperatures, densities, or pressures in the respective components at $t > 0$, averaged at the point $\mathbf{x} \in R_n$ over a neighborhood containing both components. c_1 and c_2 are monotone functions that represent fluid storage-capacity or heat content, a_1 and a_2 are non-negative functions that correspond to permeability or conductivity, and ϵ models the exchange rate between the two components. Our principal focus is on cases wherein $a_2/c_2 \gg a_1/c_1$ which occur when $c_2 \approx 0$ —the fissured-medium equation, or the Barenblatt problem. The Barenblatt problem does not specify details regarding the exchange of fluid between the two media. We also consider models that examine fluid exchange between the matrix blocks and the fissures in greater detail.

In most geologic settings, it is not always possible to deal with the porous medium as if it were a single layer (even in the contexts noted above). Stratification cannot be

ignored. The extension of (1) and (2) to incorporate the layered nature of the porous medium is presented. When developing mathematical models to many of the settings noted above, it is essential to incorporate the existence of extraction points (wellbores). In many realistic situations the geometrical shapes of the extraction points can be quite complex. Thus, although we consider linear systems of the form noted in (1) and (2), procedures to obtain solutions when boundary conditions are incorporated can become rather formidable.

Simply stated, these notes provide a basis for the development of algorithms for the study of unsteady flow in saturated porous media. These algorithms provide for the examination of three-dimensional problems and complicated boundary conditions that are a natural consequence of flow in geologic media. Chapter I presents an overview of flow through porous media, previews notation and addresses a few basic issues for ease of understanding. Chapter II considers the fundamental solution for flow in porous media in Cartesian and cylindrical reference frames subject to Dirichlet, Neumann, and mixed boundary-conditions. Chapter III demonstrates how one arrives at the expressions for pressure distributions in porous media that account for the extraction points. For illustrative purposes, we assume that the extraction points may be represented by lines and discs (circles or rectangles). Chapter IV, the crux of this work, is intended to aid those who are interested in developing algorithms for computing pressure distributions. Solutions in Chapter III are reformulated with a view to aid computations. The observations noted here are based on our experience, and many of the solutions presented here have been computed with the numerical algorithm of Stehfest (Communications of the ACM, January 1970, page 47). Asymptotic forms of various solutions are also given here. Chapter V is a natural progression of the solutions in Chapter III to more complicated visualizations of flow in porous media.

It is our pleasure to thank four individuals who were indispensable to this work. This manuscript was typed in its entirety by Ms. Jan Want and we are grateful for her patience in going through the myriad revisions of this work. Ms. Kathleen Henzel of the Tulsa Public Schools helped us with editing the manuscript. Mr. Chih-Cheng Chen of Halliburton Energy Services verified many of the developments given here and provided us with valuable comments. We thank Ms. Laura Passiglia for drafting the figures. In spite of all the assistance we have received, we are responsible for the blemishes that remain. We thank the Society of Petroleum Engineers (SPE) for permission to reproduce Figure 5.1 in Chapter V of this book.

R.R.
E.O.

Nomenclature

B :	boundary
C :	ability of wellbore to store fluid per unit change in pressure
c :	compressibility (fluid)
C_D :	wellbore storage coefficient
C_{fD} :	dimensionless conductivity of fracture
C_{hD} :	dimensionless conductivity of horizontal well
c_m :	compressibility (porous medium)
c_t :	compressibility (total)
$ch(x)$:	hyperbolic cosine
D :	space-time domain, normalized (subscript)
$-Ei(-x)$:	exponential integral function
$\text{erf}(x)$:	error function
$\text{erfc}(x)$:	complementary error function
f :	source density, fanning friction factor, fissure system (subscript)
$f(M; t)$:	mass rate
$f(s)$:	function to incorporate fissured-medium characteristics
$\bar{f}(s)$:	Laplace transform of $F(t)$
f_t :	friction factor
h :	thickness
h_f :	length of a line source
$I_{m+1/2}(x)$:	modified spherical Bessel function
$I_\nu(x)$:	modified Bessel function
k :	permeability
k_f :	fracture permeability
k_i :	permeability in the i -direction
$Ki_n(x)$:	repeated integral of $K_0(x)$
$K_{m+1/2}(x)$:	modified spherical Bessel function
$K_\nu(x)$:	modified Bessel function
L :	diffusion operator
$\mathcal{L}$:	Laplace transform operator
l :	characteristic length
L_f :	half-length of a vertical fracture
L_h :	length of a horizontal well
$\mathbf{L}_\nu(x)$:	modified Struve function
$\bar{L}$:	symbolic Laplace transform of L
L^* :	formal adjoint of L
m :	blocks of fissured porous media (subscript)
M :	point, position vector

n :	outward normal to the boundary
p :	pressure
p_f :	fissure-system pressure
$p_h(x)$:	pressure in the horizontal well
p_i :	initial value of p
p_m :	matrix-system pressure
$P_m(x)$:	Legendre polynomial
$P_m(x)$:	Legendre polynomial
$P_m^k(x)$:	associated Legendre function
p_{wf} :	wellbore pressure
q :	withdrawal rate from porous medium
$Q(t)$:	volumetric rate
$q_{hc}(x)$:	flow rate at any point in the wellbore
Q_{pi} :	point-source density (instantaneous)
Q_{pc} :	point-source density (continuous)
q_{sf} :	influx into the wellbore
$\tilde{q}$:	volumetric production rate per unit volume
R :	subdomain, distance between points P and P'
r :	distance, r -coordinate
Re :	Reynolds number
r_e :	distance to the boundary
Re_t :	Reynolds number based on production rate
R_n :	n -dimensional Euclidean space
S :	normalized pressure drop in the skin region
s :	Laplace transform variable
$sh(x)$:	hyperbolic sine
T :	time interval; temperature
t :	time variable
$th(x)$:	hyperbolic tangent
u :	equals s or $sf(s)$
V :	volume, bulk volume
V_f :	fluid volume
V_j :	ratio of the volume of system j to total volume
$v(M; t)$:	fluid velocity
V_p :	pore volume
V_w :	wellbore volume
w_f :	width of a vertical fracture
x :	distance, x -coordinate
x_e :	distance to the boundary
y :	distance, y -coordinate
y_e :	distance to the boundary
z :	distance, z -coordinate

z_e :	distance to the boundary
α :	angle
Γ :	boundary
$\Gamma(x)$:	Gamma function
γ :	fundamental solution, Euler's constant (0.5772...)
$\delta(M; t)$:	Dirac-delta function
$1/\epsilon$:	degree of fissuring
η :	diffusivity
θ :	angle, coordinate
λ :	characteristic constant of the fissured system
μ :	viscosity
ν :	angle, order (subscript)
ρ :	distance, spherical coordinate
ρ_f :	radius of a disc source
$\rho(M; t)$:	density
ϕ :	porosity, angle, coordinate
$\phi(M; t)$:	test function
φ :	angle
ψ :	angle
Ω :	space domain
ω :	characteristic constant of the fissured system

