

INTRODUCTION TO
QUANTITATIVE METHODS
A MANAGERIAL EMPHASIS

Harish L. Verma

Charles W. Gross

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INTRODUCTION TO QUANTITATIVE METHODS

A Managerial Emphasis

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A native of Michigan, Dr. Gross attained his BA and MBA degrees from Michigan State University. His DBA, with concentration in Management Science, Marketing, and Business Policy was earned from the University of Colorado. He has written several articles which have appeared in professional journals, was coauthor of a text titled *Business Forecasting*, and has been active in the American Institute for Decision Sciences, the American Marketing Association, and the Southwestern Marketing Association, and holds memberships in Beta Gamma Sigma and Beta Alpha Psi.

Many of the concepts and examples for this text were abstracted from Dr. Gross's practical experiences obtained from three years in full-time consulting and two years on the corporate marketing staff of a leading automobile manufacturer. Still active in consulting, his involvement extends to many industries, including heavy and light manufacturing, distributions, resort and tourism, retailing and government. His work has included many types of projects including marketing research, new product concept testing, plant layout, site selection, product mix, production, forecasts, jury selection models, and competitive strategy.

Preface

This book is an introduction to quantitative methods, sometimes called management science or operations research, written for the upper-level business MBA student who does not have an extensive quantitative background. We assume that the reader has had introductory courses in statistics and algebra; a course in computer programming would be useful, but not essential. Written for the future manager who desires an overview of the techniques rather than in-depth knowledge, we have emphasized a managerial perspective rather than proofs and theorems. However, unlike many introductory texts, the materials are sufficiently rigorous to allow development of a sound knowledge base.

Every effort has been made to make this a readable book. Where possible, realistic cases are used as a framework for expanding on technique. Further, where possible, we have attempted to organize techniques around problems, rather than the reverse. Classroom experience indicates that student comprehension and interest increase as a result.

Far more topics are included than can be covered in a single course, especially those on the quarter system. All materials, however, are not essential—

an instructor can pick those topics most important for students within his or her school. Further, some institutions are likely to find this text can also be used in a two-sequence course, especially those on a quarter system. Sufficient topic breadth exists to fit the needs of most such programs. An especially interesting feature is the final chapter which presents several problems stated as mini-cases. Because the appropriate solution technique is not identified, classroom experience shows that the material helps to firm-up student understanding.

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H. L. Verma
C. W. Gross

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Introduction to Quantitative Methods

Pure mathematics consists entirely of such asseverations as that, if such and such a proposition is true of anything, then such and such another proposition is true of that thing. It is essential not to discuss whether the first proposition is really true, and not to mention what the anything is of which it is supposed to be true. . . . If our hypothesis is about anything and not about some one or more particular things, then our deductions constitute mathematics. Thus mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true.

BERTRAND RUSSELL

INTRODUCTION

Managers are constantly facing situations where they have to make decisions. To illustrate, the president of an automobile company has to decide whether or not a new compact car should be added to the product line. Top management of a utility must decide whether or not a nuclear plant should be built. The branch manager of a bank must decide how many teller windows to keep open at any given time. All of these represent decisions. Some may be more complex and farther reaching than those faced by most managers. Nevertheless, all managers must make decisions, and their decisions must be effective.

The need to make decisions is as age-old as business itself. Today, however, managers face an environment with more intense competition than ever before. Further, the magnitude of capital outlay as a result of a decision in many businesses is almost bewildering. One more stamping press in the auto industry, for example, can cost as much as several million dollars. A new steel plant could cost as much as a hundred million dollars or more. A new airplane for an airline's fleet can cost nearly as much. Consequently, the need for effective decision making in business has never been more apparent.

WHY USE QUANTITATIVE METHODS?

Where do quantitative methods fit into decision making? Most, if not all, business decisions are evaluated in quantifiable measures. Profitability, distances, time, and sales are but a few of the ways decisions are evaluated. A machine might be very attractive, with shiny knobs, gleaming gauges, and fancy colors, but if it does not work—in terms of the appropriate quantifiable measure such as production per hour—the machine is not effective. For business success, managers must make decisions that minimize poor performances. Here is where quantitative methods enter the picture. They enable the manager to better understand the potential consequences of each alternative through more effective means of analysis.

What About Judgment and “Gut Feel”?

Perhaps you have heard some manager state “Experience is the best teacher,” or “Executive experience is the best way to solve problems in the real world.” This is a common feeling among many naive managers.

To assess this, Buffa conducted an experiment with fifty-two highly successful executives attending the UCLA Executive Program. All were top executives of the level of president, vice president, head of a major division, and so forth. Realistic problems were constructed, where plain business judgment could be used to select the appropriate alternative. Each problem was structured so that a formal quantitative analysis could be made and the “right answer” calculated.

Results were surprising. “Only 10 percent of the executives made the best possible decision and, significantly, all of these men worked out the solution