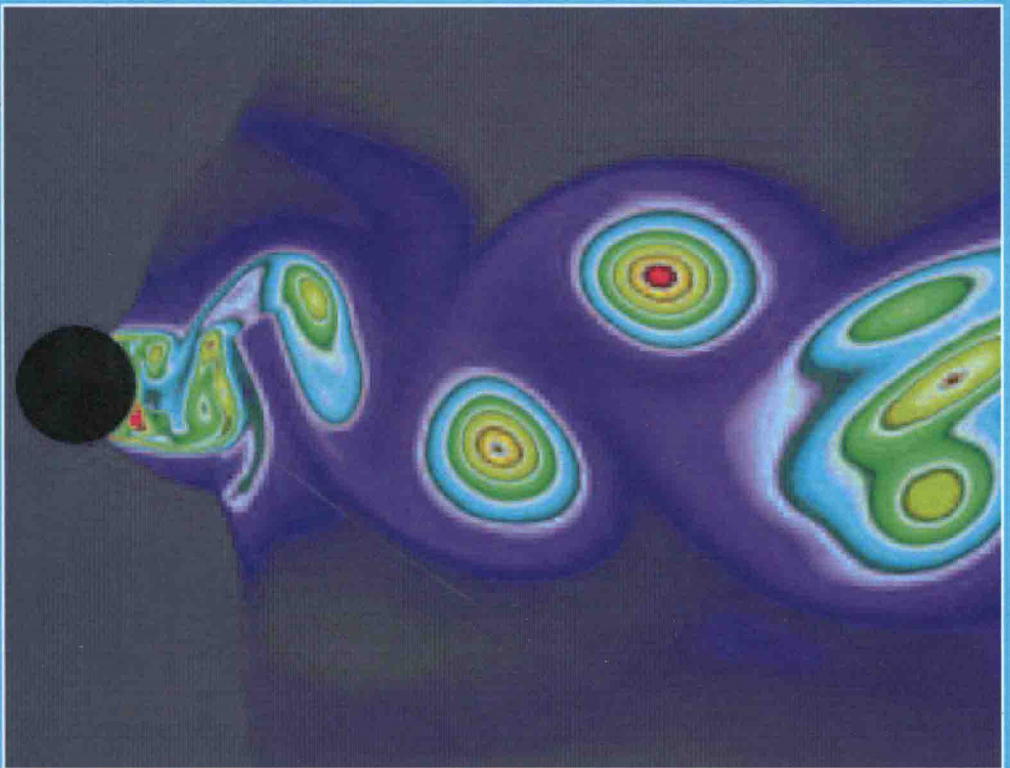


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# FLOW AROUND CIRCULAR CYLINDERS

## VOL 1: FUNDAMENTALS

M. M. ZDRAVKOVICH



This is an extremely comprehensive survey of experimental data, theoretical models, and computer simulations of flow past circular cylinders. The circular cylinder is the most widely used shape in engineering making appearances in aeronautical, chemical, civil, electrical, mechanical, nuclear, offshore, and wind engineering. The book will be useful to postgraduates and practising engineers, applied mathematicians and physicists in these fields of study and application.

Based on over 30 years of research by the author, more than 1000 papers from over 100 journals in a variety of languages are surveyed. The reference lists are an extraordinary research tool in themselves being annotated according to content and grouped by subject area.

**Dr M. M. Zdravkovich** MASME is a Reader, Telford Institute, University of Salford and a Member of Selwyn College, Cambridge.

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ZDRAVKOVICH

FLOW AROUND CIRCULAR CYLINDERS

OXFORD

# FLOW AROUND CIRCULAR CYLINDERS

*A Comprehensive Guide Through Flow Phenomena,  
Experiments, Applications, Mathematical  
Models, and Computer Simulations*

---

M. M. ZDRAVKOVICH

*Reader  
Telford Institute  
University of Salford*

VOLUME 1

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# Flow Around Circular Cylinders



A drawing from the Windsor Collection  
Leonardo da Vinci (1452–1519)

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*"A little learning is a dang'rous thing  
Drink deep, or taste not the Pierian spring  
Their shallow draughts intoxicate the brain,  
And drinking largely sobers us again."*

Alexander Pope (1688–1744) *Essay on Criticism*

## PREFACE

This book is intended to be a useful guide for: researchers in institutes, practising engineers in industry, academics and students in the fields of aerospace, civil, hydraulic, electric transmission lines, mechanical, nuclear, offshore, and wind engineering. The book may be useful reading for applied mathematicians and physicists engaged in fluid mechanics.

The unique feature of the book is that it deals exclusively with a single class of bluff bodies with circular cross-section. This class has attracted a great deal of research due to the highly complex flow structures generated. Since most of bluff bodies used in engineering applications are circular cylinders, I felt they warranted this in-depth treatment.

This volume of *Flow Around Circular Cylinders* is intended to be the first of two volumes and deals with Fundamentals in three parts, A-C. The second volume, Applications, to follow, will also be in three parts, D-F. The two volumes have been written as self-contained books. This first volume covers:

- (A) disturbance-free flow around a nominally two-dimensional cylinder across the whole Reynolds number range;
- (B) various theoretical models such as free streamline, vortex, vortex-sheet, and other miscellaneous less well-known models;
- (C) real free-stream effects such as turbulence, shear, compressibility, sound, cavitation, as well as non-Newtonian fluids.

The second volume will cover:

- (D) effects of geometry, such as: surface roughness, aspect ratio, taper, blockage, finite cylinder, yaw, rotation, wall proximity, pairs, groups, and arrays of cylinders;
- (E) unsteady free stream, such as: acceleration, deceleration, reversal, oscillatory flow, waves and combinations of waves and current, as well as streaming;
- (F) flow-induced oscillations, free and forced oscillations, synchronisation, influencing and governing parameters, means for suppressing oscillations, interfering cylinders, arrays, theoretical models, and ovaling mode of oscillation.

This book may be compared to a strange and incomplete jig-saw puzzle of a peculiar kind. It is composed like the jig-saw puzzle of hundreds and hundreds of 'pieces', which vary in size and do not fit nicely together. This reflects the complexity of Fluid Mechanics, where more research is always needed to fit together the remaining pieces. If the reader bears this in mind, it will help him to appreciate fully this book.



It has been my life's work to study and research into all aspects of flow around cylinders. This book has grown out of this long-term effort. It was not my intention to write a large book but I felt it necessary to make extensive use of descriptions since knowledge of flow past bluff bodies is still largely acquired from observations. The main emphasis is on the physical explanation of phenomena. Each description of what is happening is accompanied by a possible explanation as to why it is happening. Every effort has been made to deal with all possible aspects of flow and the book can therefore lay claim to being comprehensive.

Many papers written before World War II, which were milestones in their time are included, in order to revive their fading significance in reaching the current overall understanding. The reader is reminded that papers and information are produced nowadays at a rate greater than the capability of and time available to scientists and engineers to absorb them. Hence, the guide like this book is an attempt to offer an overview and insight in a time-saving manner.

I have introduced in each reference important descriptions to allow further insight into the contents. In order to facilitate the handling of the large number of references, they are classified into four categories:

- (i) Books, including collected works (only parts of which are relevant).
- (ii) Reviews - highly relevant and usable as initial reading.
- (iii) Papers published in learned journals (easy access).
- (iv) Papers published in conference proceedings, reports, theses, etc, (harder access).

I am sure that there will be many criticisms, comments, and questions about the book from you, the readers, which you would like to share. I would be only too happy to respond and try to answer them. I would also particularly welcome any future research reports, papers reprints, etc, on the subject which you may wish to send me.

Last but not least, I only hope that part of my enthusiasm, and devotion to the subject will pass to you.

*Salford, Lancashire*  
*25 September*  
*Anno Domini 1994*

M.M.Z.

## ACKNOWLEDGEMENTS

A large number of my colleagues and friends have shared their ideas with me and have given me invaluable suggestions on various topics. Those underlined in the list below have read one or more chapters of the book and moderated some of my views. I owe an immense debt of gratitude to all of them for their comments, frank criticisms, and for generously giving their precious time. I cannot, unfortunately, include here all the names of those who have helped or advised me in whatever ways over the years because the list would be too large. However, I am sincerely grateful to:

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The abundancy of figures in this book is due to the permission granted by the following journals:

Acoustica, Advances in Applied Mechanics, Aeronautical Quarterly (ceased), Aeronautical Research Council (British) Reports and Memoranda, AGARDograph, AIAA Journal, American Institute of Chemical Engineers, American Society of Civil Engineers Proceedings: Journal of Engineering Mechanics Division, Journal of Hydraulics Division and Journal of Structural Division, American Society of Mechanical Engineers Transactions: Journal of Basic Engineering and Journal of Fluids Engineering, Annual Review of Fluid Mechanics, Australian Journal of Scientific Research, Bulletin Japan Society of Mechanical Engineers (now JSME International Journal), Comptes Rendus Academie des Sciences, (Paris), Experiments in Fluids, Fluid Dynamics Research, Fluid Mechanics Soviet Research, Forschung aus dem Gebiete Ingenieurwesens, Ingenieur Archiv, International Chemical Engineering, International Journal of Heat and Mass Transfer, Izvestiya Akademii Nauk SSSR, Journal of Acoustic So-

ciety of America, Journal of Aeronautical Sciences (ceased), Journal of Aircraft, Journal of Applied Physics, Journal of Fluid Mechanics, Journal of Fluids and Structures, Journal de Mecanique, Journal of Mechanical Engineering Sciences, Journal of Physical Society of Japan, Journal of the Royal Aeronautical Society, Journal of Sound and Vibration, Journal of Wind Engineering and Industrial Aerodynamics, Luftfahrtforschung (ceased), NACA and NASA Technical Notes and Reports, Philosophical Magazine, Philosophical Transactions A, Physics of Fluids, Physikalische Zeitschrift, Proceedings of Cambridge Philosophical Society, Proceedings of Royal Society A, Progress in Aeronautical and (subsequently) Aerospace Sciences, Recherche Aerospatiale, Revues Francaise de Mecanique, Wärme und Stoffübertragung, Weather, Zeitschrift für angewandte Mathematik und Mechanik, Zeitschrift für angewandte Mathematik und Physik, and Zeitschrift für Flugwissenschaften und Luftfahrtforschung.

The relevant papers written in five languages are condensed into a single cover and language. The readers are reminded that science is international and tantamount to it multilingual.

Finally, there was a dozen typists who typed and re-typed the text many times over the years. The final L<sup>A</sup>T<sub>E</sub>X version was produced by Dr S. G. Sajjadi of the Salford University.

M.M.Z.

# CONTENTS

## CONTENTS TO VOLUME I: FUNDAMENTALS

xviii

<b>1</b>	<b>CONCEPTUAL OVERVIEW</b>	<b>3</b>
1.1	Regions of disturbed flow	3
1.2	Transition in disturbed regions	4
1.3	Governing and influencing parameters	6
1.4	Disturbance-free flow regimes	8
1.5	Laminar state of flow, L	9
1.6	Transition-in-wake state of flow, TrW	12
1.7	Transition-in-shear-layers, TrSL, or Subcritical state	13
1.8	Transition-in-boundary-layers, TrBL, or Critical state	13
1.9	Fully turbulent state of flow, T	16
1.10	Variation of fluid-dynamic force	16
<b>2</b>	<b>STEADY LAMINAR WAKE</b>	<b>19</b>
2.1	Creeping flow, L1	19
2.2	Pressure distribution in creeping flow	20
2.3	Resistance in creeping flow	21
2.4	Initiation and development of separation	23
2.5	Pressure distribution in the L2 regime	25
2.6	Flow structures in closed near-wake	26
2.7	Near-wake geometry	27
2.8	Velocity field outside near-wake	29
<b>3</b>	<b>PERIODIC LAMINAR REGIME</b>	<b>33</b>
3.1	Closed near-wake instability	33
3.2	Excitation and suppression of trail instability	34
3.3	Incipient roll up of trail wave	38
3.4	Kármán-Bénard eddy street	38
3.5	Spanwise structure of eddy street	41
3.6	Velocity fluctuations in periodic wake	46



3.7	Strength of eddies	52
3.8	Arrangement of eddies in wake	55
3.9	Frequency of eddy shedding; Strouhal number	59
3.10	Controversy over the $St-Re$ relationship	61
3.11	Universal $St-Re$ relationship	63
3.12	Interaction of shedding modes	67
3.13	Pressure, friction and drag	69
3.14	Secondary eddy street in the far-wake	73
3.15	Spanwise structure of secondary eddy street	76
<b>4</b>	<b>TRANSITION-IN-WAKE STATE</b>	<b>79</b>
4.1	Lower and upper transition-in-wake regimes	79
4.2	Distortions of laminar eddy filaments-'fingers'	79
4.3	Low-frequency irregularities	81
4.4	Variation of eddy shedding frequency	82
4.5	Two modes of eddy shedding	83
4.6	Overlapping of two modes of eddy shedding	84
4.7	Transition of eddies to turbulence	86
4.8	Pressure, skin friction, separation and drag	89
4.9	Suppression of transition	92
<b>5</b>	<b>TRANSITION-IN-SHEAR-LAYERS STATE</b>	<b>94</b>
5.1	TrSL1 or lower subcritical flow regime	94
5.1.1	Structure of turbulent eddies	94
5.1.2	Wake spanwise structure	95
5.1.3	Gerrard-Bloor transition waves	97
5.1.4	Convection speed of eddies	99
5.1.5	Pressure distribution and separation	100
5.2	TrSL2 or intermediate subcritical flow regime	100
5.2.1	Transition eddies	100
5.2.2	Frequency of transition eddies	102
5.2.3	Fluctuating and mean pressure, lift and drag	105
5.2.4	Strength of eddies	107
5.2.5	Strouhal number	111
5.3	TrSL3 or upper subcritical regime	111
5.3.1	Quasi-invariable flow	111

5.3.2	Characteristic regions in the near-wake	112
5.3.3	Modulation of velocity fluctuations	114
5.3.4	Three-dimensional near-wake	115
5.3.5	Spanwise correlation	116
5.3.6	Strouhal number	119
5.3.7	Mechanism of eddy shedding	120
5.3.8	Topological models of eddy streets	124
5.3.9	Strength and path of eddies	129
5.3.10	Mean and fluctuating pressure	130
5.3.11	Fluctuating stagnation and separation points	133
5.3.12	Instantaneous pressure distribution	134
5.4	Far-wake	139
5.4.1	Dynamic similarity	139
5.4.2	Energy balance across far-wake	141
5.4.3	Intermittency and entrainment	142
5.4.4	Hypothetical double-roller eddies	146
5.4.5	Development of secondary flow structure	147
5.4.6	Spanwise breakdown of eddies	153
5.4.7	Other large-scale structures	157
5.4.8	Spectra of turbulence in far-wakes	157
<b>6</b>	<b>TRANSITION-IN-BOUNDARY-LAYERS STATE</b>	<b>163</b>
6.1	Interaction of transition and separation	163
6.2	Variation in mean pressure distribution	163
6.3	Development of boundary layers	166
6.4	Precritical or TrBL0 regime	169
6.5	Single bubble or TrBL1 regime	173
6.6	Two-bubble or TrBL2 regime	176
6.7	Supercritical or TrBL3 regime	178
6.8	Mean pressure and drag coefficients	180
6.9	Postcritical or TrBL4 regime	183
6.10	Mean and fluctuating pressures	187
6.11	Mean and fluctuating forces	190
6.12	Skin friction	193
6.13	Separation and reattachment	196
6.14	Adverse pressure recovery	198

<b>7</b>	<b>FULLY TURBULENT STATE</b>	<b>201</b>
7.1	Preliminary background and the T1 regime	201
7.2	Three-dimensional cylinders in disturbed flow	201
7.3	Ultimate or T2 regime ( $Re \rightarrow \infty$ )	203
<b>A</b>	<b>A. REFERENCES</b>	<b>207</b>
<b>8</b>	<b>SOLUTIONS OF THE N-S EQUATIONS</b>	<b>245</b>
8.1	The Navier-Stokes equations	245
8.2	Hele-Shaw's flow	247
8.3	Oseen's approximate equations	249
8.4	Asymptotic behaviour of Lamb's solution	252
8.5	Periodic solution in Oseen approximation	255
8.6	Numerical solution of the Navier-Stokes equations	257
8.7	Computer solutions of the Navier-Stokes equations	263
8.8	Steady flow computations	264
8.9	'Simulated' steady flow	266
8.10	Unsteady flow computations	268
8.11	Oseen's approximation applied to far wake	272
<b>9</b>	<b>BOUNDARY LAYER APPROXIMATION</b>	<b>276</b>
9.1	Prandtl's concept of thin boundary layer	276
9.2	Von Mises' transformation	278
9.3	Hiemenz's solution	282
9.4	Thom's approximate method	284
9.5	Bairstow's method of calculation	286
9.6	Further computations	287
9.7	Schlichting's theory for far wake	287
<b>10</b>	<b>FREE STREAMLINE MODELS</b>	<b>291</b>
10.1	'Dead-water' bounded by a discontinuity surface	291
10.2	Free streamline theory	291
10.3	Brodetsky's approximate transformation	293
10.4	Displacement of detachment point	294

10.5	Modification of base pressure	295
10.6	Free streamline velocity variation	298
10.7	Mimura's approximation of measured pressure	300
10.8	Cavity flows	302
10.9	Parkinson and Jandali's wake-source model	304
10.10	Kiya and Arie wake-source-sink model	307
10.11	Combination of free streamlines and boundary layers	309
10.12	Asymptotic flow with vanishing viscosity	311
10.13	Simulation of turbulent diffusion	313
<b>11</b>	<b>VORTEX MODELS AND STABILITY</b>	<b>317</b>
11.1	Inviscid vortex models	317
11.2	Viscous diffusion of a point-vortex	319
11.3	Arbitrary initial velocity profile	322
11.4	General solution of vorticity diffusion	325
11.5	Pair of point-vortices	328
11.6	Row of point-vortices	331
11.7	Two rows of point-vortices	332
11.8	Semi-finite point vortex street	335
11.9	Kármán's stability criteria	335
11.10	Instability of the point-vortex street	337
11.11	Instability of viscous vortex streets	341
11.12	Absolute and convective instability	344
11.13	Hooker's partially viscous vortex street	345
11.14	Effect of vortex diffusion	348
<b>12</b>	<b>VORTEX SHEET MODELS</b>	<b>351</b>
12.1	Structure of vortex sheets	351
12.2	Rolling up of vortex sheet	351
12.3	Discrete vortex simulation of a single vortex sheet	355
12.4	Discrete vortex simulation of two vortex sheets	357
12.5	Apparent loss of vorticity	361
12.6	Gerrard's modelling of discrete vortex sheet	362



12.7	Further computations of discrete vortices	365
12.8	Sarpkaya's discrete vortex model	369
<b>13</b>	<b>MISCELLANEOUS MODELS</b>	<b>373</b>
13.1	Introduction	373
13.2	Universal Strouhal number	373
13.3	Shaw's acoustic model	381
13.4	Birkhoff's model of vortex street	383
13.5	Ackeret's temperature recovery theory	384
13.6	Momentum balance in a vortex street	386
13.7	Kronauer's minimum drag model	390
<b>B</b>	<b>B. REFERENCES</b>	<b>396</b>
<b>14</b>	<b>FREE STREAM TURBULENCE</b>	<b>431</b>
14.1	Introduction	431
14.2	Interaction of free stream turbulence	432
14.3	TrW or transition-in-wake state of flow	433
14.4	TrSL or transition in free shear layers state of flow	434
14.5	TrSL2 or disturbance-sensitive regime	435
14.6	TrSL3 or upper subcritical regime	439
14.7	Taylor's number for 'level' of turbulence	442
14.8	TrBL or transition in boundary layers state	443
14.9	Turbulent state of flow	452
14.10	Distortion of free stream turbulence	453
<b>14</b>	<b>NON-UNIFORM FREE STREAM</b>	<b>458</b>
15.1	Introduction	458
15.2	Linear shear flow	458
15.3	Two types of shear and cylinder orientation	459
15.4	Linear shear across cylinder	459
15.4.1	Taylor's theory	462
15.4.2	Creeping linear shear flow without separation	463
15.4.3	Laminar linear shear flow with separation	464
15.4.4	Suppression of eddy shedding by linear shear	465
15.4.5	Drag, lift, and torque coefficients	467