



国际著名力学图书

——影印版系列

Engineering Mechanics

STATICS Second Edition

工程力学

静力学 第2版

Andrew Pytel · Jaan Kiusalaas



清华大学出版社

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Engineering Mechanics

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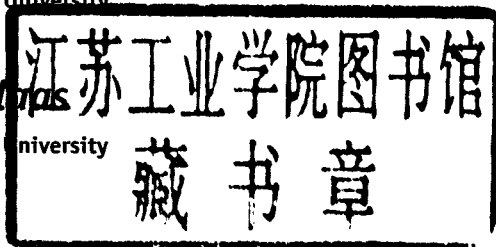
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STATICS(Second Edition)

Andrew Pytel • Jaan Kiusalaas

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Engineering Mechanics

Statics(Second Edition)

影 印 版 序

工程力学(包括理论力学和材料力学)是许多工程学科的基础,被列为我国高等院校理工科大学学生所必修的基础课。学习工程力学需要有清晰的物理概念和形象的几何直观,准确地理解基本原理和基本方法,熟练地掌握数学推理,因此工程力学是大学阶段较难学好的课程之一。另一方面,正因为它具有科学严密性和应用灵活性紧密结合的魅力,是培养学生综合研究素质的训练园地,工程力学又为许多大学生所青睐。

本书由美国宾夕法尼亚州立大学的两位教授所著,包括静力学和动力学两部分,本册为静力学部分。书中对静力学基本概念和基本原理的叙述简明、清晰,通过例题的讨论(共118个)帮助学生掌握应用原理的方法和解题技巧,附有大量习题(共983个)供学生训练,每章后面还配有供学生自我检查的测试题,是一本典型的美式教材,出版后被美国多所大学采用。作者并不追求理论上的系统、完整,而更重视静力学基本原理的熟练应用。作者强调在开始进行具体计算以前首先要从整体上分析和把握问题的求解思路,判断问题中的未知量数和独立方程数,这是学好工程力学的经验之谈。习题经过精心挑选,有应用基本原理可以直接求解的,也有来自工程实际、较有趣而更复杂的。

本书共10章,内容包括:静力学引论,力系的基本运算,力系的合成,平面内的平衡分析,平面外(三维空间)的平衡分析,梁和索,干摩擦,分布载荷的中心,面积的惯性矩和惯性积,虚功和势能。其中带星号(*)的章节是要求较高的补充材料,由教师决定取舍。配有数值积分、方程的求根和常用材料的密度三个附录。最后附有双号习题的答案和名词检索。汇总第一版出版以来新的教学经验和读者的建议,第二版对多数章节进行了修改,统一了符号,更新了近三分之一的习题。

我国目前的工程力学课程一般按研究对象(刚体和变形体)划分为理论力学和材料力学两部分。美国则按运动状态将其划分为静力学和动力学两部分,总体上说相当于我国的理论力学,但在静力学中除了包含平面桁架外又增加了材料力学中梁的内力分析内容。

本书是美国现有工程力学教材中值得一读的优秀教材之一,内容丰富、语言流畅,可作为我国高等工业院校工程力学和理论力学课程的主要英文参考书。

陆明万

清华大学工程力学系

To Jean, Leslie, Lori, John, Nicholas

and

To Judy, Nicholas, Jennifer, Timothy

Preface

Statics and dynamics form the foundation of many engineering disciplines and are, therefore, essential to the training of an engineer. Mastery of these subjects requires a clear understanding of the principles, and experience in applying the principles to a wide range of situations. Because the applications require reasoning rather than memorization, statics and dynamics are perceived by students as difficult courses, making their teaching particularly challenging.

This textbook on statics and its companion volume on dynamics were developed over many years of teaching. The salient features of *Statics* are as follows:

- The selection of homework problems strives for a balanced presentation. It includes many “textbook” problems that illustrate the principles in a straightforward manner. In addition, there are numerous problems of direct engineering relevance, which are more interesting and challenging.
- The problems are evenly divided between SI and U.S. Customary units.
- The analysis of equilibrium is presented in three articles. The first teaches how to draw a free-body diagram, and the second shows how to write the equilibrium equations from a given free-body diagram. The third article combines the methods just learned to arrive at a logical scheme for the complete analysis of a problem.
- The Sample Problems separate the method of analysis from the mathematical details of the solution. This approach to equilibrium analysis teaches the student to “think before calculating.”
- The text continually emphasizes the importance of comparing the number of independent equations with the number of unknowns.

The book contains several optional topics, which are marked by an asterisk (*). This material can be omitted without jeopardizing the presentation of other parts of *Statics*. An asterisk is also used to mark problems that require advanced reasoning.

New to the Second Edition The considerable feedback we received from the users of the first edition was very helpful in the preparation of the second edition. As the result of their comments and of suggestions made by the reviewers, we made the following changes:

- The number of homework problems has been increased.
- Approximately one-third of the homework problems are new or have changed numerical values.
- A new problem set involving units and dimensions has been added to the first chapter.

- Several chapter introductions have been rewritten to place the topics in a better perspective.
- A number of improvements in notation have been made.

Ancillary *Study Guide to Accompany Pytel and Kiusalaas, Engineering Mechanics, Statics, Second Edition*, J. L. Pytel, 1999. The goal of this study guide is to help students master the problem-solving skills required in their study of *Statics*. Students are prompted to interact with the material as they work through “guided” problems. The study guide contains additional problems that are accompanied by complete solutions.

Acknowledgments Statics and dynamics is a mature field of study that has been built over many generations. Thus any new textbook pays tacit homage to the books that preceded it and to their authors. We are also grateful to the following reviewers for their valuable suggestions: Duane Castaneda, University of Alabama–Birmingham; Scott G. Danielson, North Dakota State University; Richard N. Downer, University of Vermont; Howard Epstein, University of Connecticut; Ralph E. Flori, University of Missouri–Rolla; Li-Sheng Fu, Ohio State University; Susan L. Gerth, Kansas State University; Edward E. Hornsey, University of Missouri–Rolla; Cecil O. Huey, Clemson University; Thomas J. Kosc, Texas A&M University; Dahsin Liu, Michigan State University; Mark Mear, University of Texas–Austin; Satish Nair, University of Missouri–Columbia; Hamid Nayeb-Hashemi, Northeastern University, Boston; Robert Price, Louisiana Tech University; Robert Schmidt, University of Detroit–Mercy; Robert Seabloom, University of Washington–Seattle; Kassim M. Tarhini, Valparaiso University; Dennis Vandenbrink, Western Michigan University; Carl Vilmann, Michigan Tech University.

We especially recognize our indebtedness to Dr. Christine Masters, who checked the solutions to the homework problems.

Andrew Pytel
Jaan Kiusalaas

Contents

CHAPTER 1

Introduction to Statics

1

- 1.1 Introduction 1
- 1.2 Newtonian Mechanics 3
- 1.3 Fundamental Properties of Vectors 10
- 1.4 Representation of Vectors Using Rectangular Components 16
- 1.5 Vector Multiplication 25

CHAPTER 2

Basic Operations with Force Systems

33

- 2.1 Introduction 33
- 2.2 Equivalence of Vectors 33
- 2.3 Force 34
- 2.4 Reduction of Concurrent Force Systems 35
- 2.5 Moment of a Force about a Point 44
- 2.6 Moment of a Force about an Axis 54
- 2.7 Couples 66
- 2.8 Changing the Line of Action of a Force 78

CHAPTER 3

Resultants of Force Systems

87

- 3.1 Introduction 87
- 3.2 Reduction of a Force System to a Force and a Couple 87
- 3.3 Definition of Resultant 95
- 3.4 Resultants of Coplanar Force Systems 96

- 3.5 Resultants of Noncoplanar Force Systems 104

- 3.6 Introduction to Distributed Normal Loads 115

CHAPTER 4

Coplanar Equilibrium Analysis

129

- 4.1 Introduction 129
- 4.2 Definition of Equilibrium 129
- PART A: Analysis of Single Bodies 130
 - 4.3 Free-Body Diagram of a Body 130
 - 4.4 Coplanar Equilibrium Equations 138
 - 4.5 Writing and Solving Equilibrium Equations 140
 - 4.6 Equilibrium Analysis for Single-Body Problems 148
- PART B: Analysis of Composite Bodies 159
 - 4.7 Free-Body Diagrams Involving Internal Reactions 159
 - 4.8 Equilibrium Analysis of Composite Bodies 170
 - 4.9 Special Cases: Two-Force and Three-Force Bodies 178
- PART C: Analysis of Plane Trusses 189
 - 4.10 Description of a Truss 189
 - 4.11 Method of Joints 190
 - 4.12 Method of Sections 197

CHAPTER 5

Noncoplanar Equilibrium

209

- 5.1 Introduction 209
- 5.2 Definition of Equilibrium 209
- 5.3 Free-Body Diagrams 209
- 5.4 Independent Equilibrium Equations 220

5.5	Improper Constraints	223	
5.6	Writing and Solving Equilibrium Equations	224	
5.7	Equilibrium Analysis	232	
CHAPTER 6			
Beams and Cables		247	
6.1	Introduction	247	
PART A: Beams		247	
6.2	Internal Force Systems	247	
6.3	Analysis of Internal Forces	255	
6.4	Area Method for Drawing V- and M-Diagrams	268	
PART B: Cables		282	
6.5	Cables under Distributed Loads	282	
6.6	Cables under Concentrated Loads	293	
CHAPTER 7			
Dry Friction		303	
7.1	Introduction	303	
7.2	Coulomb's Theory of Dry Friction	303	
7.3	Problem Classification and Analysis	306	
7.4	Impending Tipping	322	
7.5	Angle of Friction; Wedges and Screws	329	
*7.6	Ropes and Flat Belts	337	
*7.7	Disk Friction	344	
CHAPTER 8			
Centroids and Distributed Loads		353	
8.1	Introduction	353	
8.2	Centroids of Plane Areas and Curves	353	
8.3	Centroids of Curved Surfaces, Volumes, and Space Curves	370	
8.4	Theorems of Pappus-Guldinus	388	
8.5	Center of Gravity and Center of Mass	393	
8.6	Distributed Normal Loads	401	
CHAPTER 9			
Moments and Products of Inertia of Areas		419	
9.1	Introduction	419	
9.2	Moments of Inertia of Areas and Polar Moments of Inertia	419	
9.3	Products of Inertia of Areas	436	
9.4	Transformation Equations and Principal Moments of Inertia of Areas	442	
*9.5	Mohr's Circle for Moments and Products of Inertia	450	
CHAPTER 10			
Virtual Work and Potential Energy		461	
*10.1	Introduction	461	
*10.2	Planar Kinematics of a Rigid Body	461	
*10.3	Virtual Work	464	
*10.4	Method of Virtual Work	466	
*10.5	Instant Center of Rotation	480	
*10.6	Equilibrium and Stability of Conservative Systems	487	
APPENDIX A			
Numerical Integration		499	
A.1	Introduction	499	
A.2	Trapezoidal Rule	499	
A.3	Simpson's Rule	500	
APPENDIX B			
Finding Roots of Functions		503	
B.1	Introduction	503	
B.2	Newton's Method	503	
B.3	Secant Method	504	
APPENDIX C			
Densities of Common Materials		507	
Answers to Even-Numbered Problems		509	
Index		515	

*Indicates optional articles.

Introduction to Statics

1.1 Introduction

a. What is engineering mechanics?

Statics and dynamics are introductory engineering mechanics courses, and they are among the first engineering courses encountered by most students. Therefore, it is appropriate that we begin with a brief exposition on the meaning of the term *engineering mechanics* and on the role that these courses play in engineering education. Before defining engineering mechanics, we must first consider the similarities and differences between physics and engineering.

In general terms, *physics* is the science that relates the properties of matter and energy, excluding biological and chemical effects. Physics includes the study of mechanics,* thermodynamics, electricity and magnetism, and nuclear physics. On the other hand, *engineering* is the application of the mathematical and physical sciences (physics, chemistry, and biology) to the design and manufacture of items that benefit humanity. *Design* is the key concept that distinguishes engineers from scientists. According to the Accreditation Board for Engineering and Technology (ABET), engineering design is the process of devising a system, component, or process to meet desired needs.

Mechanics is the branch of physics that considers the action of forces on bodies or fluids that are *at rest* or *in motion*. Correspondingly, the primary topics of mechanics are statics and dynamics. The first topic that you studied in your initial physics course, in either high school or college, was undoubtedly mechanics. Thus, *engineering mechanics* is the branch of engineering that applies the principles of mechanics to mechanical design (i.e., any design that must take into account the effect of forces). The primary goal of engineering mechanics courses is to introduce the student to the engineering applications of mechanics. Statics and Dynamics are generally followed by one or more courses that introduce material properties and deformation, usually called Strength of Materials or Mechanics of Materials. This sequence of courses is then followed by formal training in mechanical design.

Of course, engineering mechanics is an integral component of the education of engineers whose disciplines are related to the mechanical sciences,

*When discussing the topics included in physics, the term *mechanics* is used without a modifier. Quite naturally, this often leads to confusion between "mechanics" and "engineering mechanics."

such as aerospace engineering, architectural engineering, civil engineering, and mechanical engineering. However, a knowledge of engineering mechanics is also useful in most other engineering disciplines, because there, too, the mechanical behavior of a body or fluid must often be considered. Because mechanics was the first physical science to be applied to everyday life, it follows that engineering mechanics is the oldest branch of engineering. Given the interdisciplinary character of many engineering applications (e.g., robotics and manufacturing), a sound training in engineering mechanics continues to be one of the most important aspects of engineering education.

b. Problem formulation and the accuracy of solutions

As you will soon discover, your mastery of the principles of engineering mechanics will be reflected in your ability to formulate and solve problems. Unfortunately, there is no simple method for teaching problem-solving skills. Nearly all individuals require a considerable amount of practice in solving problems before they begin to develop the analytical skills that are so necessary for success in engineering. For this reason, a relatively large number of sample problems and homework problems are placed at strategic points throughout the text.

To help you develop an “engineering approach” to problem analysis, we think that you will find it instructive to divide your solution for each homework problem into the following parts:

1. **GIVEN:** After carefully reading the problem statement, list all the data provided. If a figure is required, sketch it neatly and approximately to scale.
2. **FIND:** State precisely the information that is to be determined.
3. **SOLUTION:** Solve the problem, showing all the steps that you used in the analysis. Work neatly so that your work can be easily followed by others.
4. **VALIDATE:** Many times, an invalid solution can be uncovered by simply asking yourself, “Does the answer make sense?”

When reporting your answers, use only as many digits as the least accurate value in the given data. For example, suppose that we are required to convert 12 500 ft (assumed to be accurate to three significant digits) to miles. Using a calculator, we would divide 12 500 ft by 5280 ft/mi and report the answer as 2.37 mi (three significant digits), although the quotient displayed on the calculator would be 2.367 424 2. Reporting the answer as 2.367 424 2 implies that all eight digits are significant, which is, of course, untrue. It is your responsibility to round off the answer to the correct number of digits. *In this text*, you should assume that given data are accurate to three significant digits unless stated otherwise. For example, a length that is given as 3 ft should be interpreted as 3.00 ft.

When performing intermediate calculations, a good rule of thumb is to carry one more digit than will be reported in the final answer; for example, use four-digit intermediate values if the answer is to be significant to three digits. Furthermore, it is common practice to report an additional digit if the first digit in an answer is 1; for example, use 1.392 rather than 1.39.

1.2 Newtonian Mechanics

a. Scope of Newtonian mechanics

In 1687 Sir Isaac Newton (1642–1727) published his celebrated laws of motion in *Principia (Mathematical Principles of Natural Philosophy)*. Without a doubt, this work ranks among the most influential scientific books ever published. We should not think, however, that its publication immediately established classical mechanics. Newton's work on mechanics dealt primarily with celestial mechanics and was thus limited to particle motion. Another two hundred or so years elapsed before rigid-body dynamics, fluid mechanics, and the mechanics of deformable bodies were developed. Each of these areas required new axioms before it could assume a usable form.

Nevertheless, Newton's work is the foundation of classical, or Newtonian, mechanics. His efforts have even influenced two other branches of mechanics, born at the beginning of the twentieth century: relativistic and quantum mechanics. *Relativistic mechanics* addresses phenomena that occur on a cosmic scale (velocities approaching the speed of light, strong gravitational fields, etc.). It removes two of the most objectionable postulates of Newtonian mechanics: the existence of a fixed or inertial reference frame and the assumption that time is an absolute variable, "running" at the same rate in all parts of the universe. (There is evidence that Newton himself was bothered by these two postulates.) *Quantum mechanics* is concerned with particles on the atomic or subatomic scale. It also removes two cherished concepts of classical mechanics: determinism and continuity. Quantum mechanics is essentially a probabilistic theory; instead of predicting an event, it determines the likelihood that an event will occur. Moreover, according to this theory, the events occur in discrete steps (called *quanta*) rather than in a continuous manner.

Relativistic and quantum mechanics, however, have by no means invalidated the principles of Newtonian mechanics. In the analysis of the motion of bodies encountered in our everyday experience, both theories converge on the equations of Newtonian mechanics. Thus the more esoteric theories actually reinforce the validity of Newton's laws of motion.

b. Newton's laws for particle motion

Using modern terminology, Newton's laws of particle motion may be stated as follows:

1. If a particle is at rest (or moving with constant velocity in a straight line), it will remain at rest (or continue to move with constant velocity in a straight line) unless acted on by a force.
2. A particle acted on by a force will accelerate in the direction of the force. The magnitude of the acceleration is proportional to the magnitude of the force and inversely proportional to the mass of the particle.
3. For every action, there is an equal and opposite reaction; that is, the forces of interaction between two particles are equal in magnitude and opposite in direction.

Although the first law is simply a special case of the second law, it is customary to state the first law separately because of its importance to the subject of statics.

c. Inertial reference frames

When applying Newton's second law, attention must be paid to the coordinate system in which the accelerations are measured. An *inertial reference frame* (also known as a Newtonian or Galilean reference frame) is defined to be any rigid coordinate system in which Newton's laws of particle motion relative to that frame are valid with an acceptable degree of accuracy. In most design applications used on the surface of the earth, an inertial frame can be approximated with sufficient accuracy by attaching the coordinate system to the earth. In the study of earth satellites, a coordinate system attached to the sun usually suffices. For interplanetary travel, it is necessary to use coordinate systems attached to the so-called fixed stars.

It can be shown that any frame that is translating with constant velocity relative to an inertial frame is itself an inertial frame. It is a common practice to omit the word *inertial* when referring to frames for which Newton's laws obviously apply.

d. Units and dimensions

The standards of measurement are called *units*. The term *dimension* refers to the type of measurement, regardless of the units used. For example, kilogram and foot/second are units, whereas mass and length/time are dimensions. Throughout this text we use two standards of measurement: U.S. Customary system and SI system (from *Système internationale d'unités*). In the U.S. Customary system the base (fundamental) dimensions are force $[F]$, length $[L]$, and time $[T]$. The corresponding base units are pound (lb), foot (ft), and second (s). The base dimensions in the SI system are mass $[M]$, length $[L]$, and time $[T]$, and the base units are kilogram (kg), meter (m), and second (s). All other dimensions or units are combinations of the base quantities. For example, the dimension of velocity is $[L/T]$, the units being ft/s, m/s, and so on.

A system with the base dimensions $[FLT]$ (such as the U.S. Customary system) is called a *gravitational system*. If the base dimensions are $[MLT]$ (as in the SI system), the system is known as an *absolute system*. In each system of measurement, the base units are defined by physically reproducible phenomena or physical objects. For example, the second is defined by the duration of a specified number of radiation cycles in a certain isotope, the kilogram is defined as the mass of a certain block of metal kept near Paris, France, and so on.

All equations representing physical phenomena must be *dimensionally homogeneous*; that is, each term of an equation must have the same dimension. Otherwise, the equation will not make physical sense (it would be meaningless, for example, to add a force to a length). Checking equations for dimensional homogeneity is a good habit to learn, as it can reveal mistakes made during algebraic manipulations.

e. Mass, force, and weight

If a force \mathbf{F} acts on a particle of mass m , Newton's second law states that

$$\mathbf{F} = m\mathbf{a} \quad (1.1)$$

where \mathbf{a} is the acceleration vector of the particle. For a gravitational $[FLT]$ system, dimensional homogeneity of Eq. (1.1) requires the dimension of mass to be

$$[M] = \left[\frac{FT^2}{L} \right] \quad (1.2a)$$

In the U.S. Customary system, the derived unit of mass is called a *slug*. A slug is defined as the mass that is accelerated at the rate of 1.0 ft/s^2 by a force of 1.0 lb . Substituting units for dimensions in Eq. (1.2a), we get for the unit of a slug

$$1.0 \text{ slug} = 1.0 \text{ lb} \cdot \text{s}^2/\text{ft}$$

For an absolute $[MLT]$ system of units, dimensional homogeneity of Eq. (1.1) yields for the dimension of force

$$[F] = \left[\frac{ML}{T^2} \right] \quad (1.2b)$$

The derived unit of force in the SI system is a *newton* (N), defined as the force that accelerates a 1.0-kg mass at the rate of 1.0 m/s^2 . From Eq. (1.2b), we obtain

$$1.0 \text{ N} = 1.0 \text{ kg} \cdot \text{m/s}^2$$

Weight is the force of gravitation acting on a body. Denoting gravitational acceleration (free-fall acceleration of the body) by g , the weight W of a body of mass m is given by Newton's second law as

$$W = mg \quad (1.3)$$

Note that mass is a constant property of a body, whereas weight is a variable that depends on the local value of g . The gravitational acceleration on the surface of the earth is approximately 32.2 ft/s^2 , or 9.81 m/s^2 . Thus the mass of a body that weighs 1.0 lb on earth is $(1.0 \text{ lb})/(32.2 \text{ ft/s}^2) = 1/32.2 \text{ slug}$. Similarly, if the mass of a body is 1.0 kg , its weight on earth is $(9.81 \text{ m/s}^2)(1.0 \text{ kg}) = 9.81 \text{ N}$.

At one time, the pound was also used as a unit of mass. The *pound mass* (lbm) was defined as the mass of a body that weighs 1.0 lb on the surface of the earth. Although pound mass is an obsolete unit, it is still used occasionally, giving rise to confusion between mass and weight. In this text, we use the pound exclusively as a unit of force.

f. Conversion of units

A convenient method for converting a measurement from one set of units to another is to multiply the measurement by appropriate conversion factors. For example, to convert 240 mi/h into ft/s , we proceed as follows:

$$240 \text{ mi/h} = 240 \frac{\text{mi}}{\text{h}} \times \frac{1.0 \text{ h}}{3600 \text{ s}} \times \frac{5280 \text{ ft}}{1.0 \text{ mi}} = 352 \text{ ft/s}$$

where the multipliers $1.0 \text{ h}/3600 \text{ s}$ and $5280 \text{ ft}/1.0 \text{ mi}$ are conversion factors. Because $1.0 \text{ h} = 3600 \text{ s}$ and $5280 \text{ ft} = 1.0 \text{ mi}$, we see that each conversion factor is dimensionless and of magnitude 1. Therefore, a measurement is unchanged when it is multiplied by conversion factors—only its units are altered. Note that it is permissible to cancel units during the conversion as if they were algebraic quantities.

Conversion factors applicable to mechanics are listed inside the front cover of the book.

g. Law of gravitation

In addition to his many other accomplishments, Newton also proposed the law of universal gravitation. Consider two particles of mass m_A and m_B that are separated by a distance R , as shown in Fig. 1.1. The law of gravitation states that the two particles are attracted to each other by forces of magnitude F that act along the line connecting the particles, where

$$F = G \frac{m_A m_B}{R^2} \quad (1.4)$$

The universal gravitational constant G is equal to $3.44 \times 10^{-8} \text{ ft}^4/(\text{lb} \cdot \text{s}^4)$, or $6.67 \times 10^{-11} \text{ m}^3/(\text{kg} \cdot \text{s}^2)$. Although this law is valid for particles, Newton showed that it is also applicable to spherical bodies, provided that their masses are distributed uniformly. (When attempting to derive this result, Newton was forced to develop calculus.)

If we let $m_A = M_e$ (the mass of the earth), $m_B = m$ (the mass of a body), and $R = R_e$ (the mean radius of the earth), then F in Eq. (1.4) will be the weight W of the body. Comparing $W = GM_e m/R_e^2$ with $W = mg$, we find that $g = GM_e/R_e^2$. Of course, adjustments may be necessary in the value of g for some applications in order to account for local variation of the gravitational attraction.

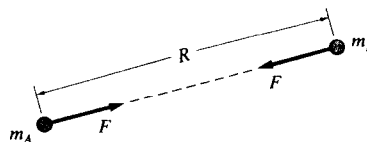


Fig. 1.1

Sample Problem 1.1

Convert 5000 lb/in.² to Pa (1 Pa = 1 N/m²).

Solution

Using the conversion factors listed inside the front cover, we obtain

$$\begin{aligned} 5000 \text{ lb/in.}^2 &= 5000 \frac{\text{lb}}{\text{in.}^2} \times \frac{4.448 \text{ N}}{1.0 \text{ lb}} \times \left(\frac{39.37 \text{ in.}}{1.0 \text{ m}} \right)^2 \\ &= 34.5 \times 10^6 \text{ N/m}^2 = 34.5 \text{ MPa} \end{aligned} \quad \text{Answer}$$

Sample Problem 1.2

The acceleration a of a particle is related to its velocity v , its position coordinate x , and time t by the equation

$$a = Ax^3t + Bvt^2 \quad (\text{a})$$

where A and B are constants. The dimension of the acceleration is length per unit time squared; that is, $[a] = [L/T^2]$. The dimensions of the other variables are $[v] = [L/T]$, $[x] = [L]$, and $[t] = [T]$. Derive the dimensions of A and B if Eq. (a) is to be dimensionally homogeneous.

Solution

For Eq. (a) to be dimensionally homogeneous, the dimension of each term on the right-hand side of the equation must be $[L/T^2]$, the same as the dimension for a . Therefore, the dimension of the first term on the right-hand side of Eq. (a) becomes

$$[Ax^3t] = [A][x^3][t] = [A][L^3][T] = \left[\frac{L}{T^2} \right] \quad (\text{b})$$

Solving Eq.(b) for the dimension of A , we find

$$[A] = \frac{1}{[L^3][T]} \left[\frac{L}{T^2} \right] = \frac{1}{[L^2T^3]} \quad \text{Answer}$$

Performing a similar dimensional analysis on the second term on the right-hand side of Eq. (a) gives

$$[Bvt^2] = [B][v][t^2] = [B] \left[\frac{L}{T} \right] [T^2] = \left[\frac{L}{T^2} \right] \quad (\text{c})$$

Solving Eq. (c) for the dimension of B , we find

$$[B] = \left[\frac{L}{T^2} \right] \left[\frac{T}{L} \right] \left[\frac{1}{T^2} \right] = \left[\frac{1}{T^3} \right] \quad \text{Answer}$$