CULTURE AND CIVILIZATION IN THE MIDDLE EAST

Classical Mathematics from al-Khwārizmī to Descartes

Roshdi Rashed

Translated by Michael H. Shank

مركز دراسات الوحدة العربية
CENTRE FOR ARAB UNITY STUDIES

ROUTLEDGE

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常州大学山书馆
藏书章

Routledge
Taylor & Francis Group
LONDON AND NEW YORK

مركز دراسات الوحدة العربية
CENTRE FOR ARAB UNITY STUDIES

First published 2015
by Routledge
2 Park Square, Milton Park, Abingdon, Oxon OX14 4RN

and by Routledge
711 Third Avenue, New York, NY 10017

Routledge is an imprint of the Taylor & Francis Group, an informa business

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British Library Cataloguing in Publication Data
A catalogue record for this book is available from the British Library

Library of Congress Cataloging in Publication Data
A catalog record for this book has been requested

ISBN: 978-0-415-83388-2 (hbk)
ISBN: 978-1-315-75386-7 (ebk)

Typeset in Times New Roman
by Cenveo Publisher Services

This book was prepared from press-ready files supplied by the editor

CLASSICAL MATHEMATICS FROM AL-KHWĀRIZMĪ TO DESCARTES

This book follows the development of classical mathematics and the relation between work done in the Arab and Islamic worlds and that undertaken by the likes of Descartes and Fermat.

'Early modern' mathematics is a term widely used to refer to the mathematics which developed in the West during the sixteenth and seventeenth century. For many historians and philosophers this is the watershed which marks a radical departure from 'classical mathematics,' to more modern mathematics; heralding the arrival of algebra, geometrical algebra, and the mathematics of the continuous. In this book, Roshdi Rashed demonstrates that 'early modern' mathematics is actually far more composite than previously assumed, with each branch having different traceable origins which span the millennium. Going back to the beginning of these parts, the aim of this book is to identify the concepts and practices of key figures in their development, thereby presenting a fuller reality of these mathematics.

This book will be of interest to students and scholars specialising in Islamic science and mathematics, as well as to those with an interest in the more general history of science and mathematics and the transmission of ideas and culture.

Roshdi Rashed is one of the most eminent authorities on Arabic mathematics and the exact sciences. A historian and philosopher of mathematics and science and a highly celebrated epistemologist, he is currently Emeritus Research Director (distinguished class) at the Centre National de la Recherche Scientifique (CNRS) in Paris, and is the former Director of the Centre for History of Medieval Science and Philosophy at the University of Paris (Denis Diderot, Paris VII). He also holds an Honorary Professorship at the University of Tokyo and an Emeritus Professorship at the University of Mansourah in Egypt.

Michael H. Shank is Professor of the History of Science at the University of Wisconsin-Madison, where he teaches surveys of the history of science from antiquity to Newton. His research interests focus on, and often stray beyond, the late-medieval Viennese astronomical and natural philosophical traditions.

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FOREWORD

Classical Mathematics from al-Khwārizmī to Descartes includes two new chapters – one on the transmission of Greek heritage into Arabic and the other on Descartes's mathematics – that did not appear in the original French of *D'al-Khwārizmī à Descartes*. Conversely, I have omitted here the chapter on burning mirrors ('Les miroirs ardents, anaclastique et dioptrique'), a subject to which I devoted an entire book, which is now available in English.[1]

The English translation of the present work by Professor Michael Shank has benefited greatly from both his competence in the history and philosophy of science and his refined bilingualism. I mention this to express my profound gratitude for his hard work and for apposite comments that improved the text.

I warmly thank Aline Auger (Centre National de la Recherche Scientifique), who focused her competence and flawless attention to detail on assembling the index and preparing the book for the press.

I am also grateful to Kathryn Rylance and Joe Whiting at Routledge for the care that they gave to this project at every stage of its development.

Last but not least, Dr Khair El-Din Haseeb has spared no effort in bringing this historical research to audiences beyond the original francophone one. I hereby offer him my friendly gratitude.

Roshdi RASHED

[1] *Geometry and Dioptrics in Classical Islam*, London, al-Furqān, 2005.

TRANSLATOR'S NOTE

In agreeing to undertake this translation, I hoped to learn much. The project has exceeded my expectations in many ways. Most directly, it has given me a new appreciation for the exceptional range and importance of Professor Rashed's contributions to the histories of mathematics and of the mathematical sciences, from manuscript discoveries to pointed analyses. Most strikingly, his scholarship offers a panoramic and up-to-date view of Arabic mathematics, of its historical and conceptual connections with Greek, medieval Latin, and early-modern mathematics. If this book has taught me the most about new research in the remarkable range of Arabic mathematics, its expositions have also given me a new appreciation for the significance of both Apollonius and Fermat. Not least, looking to the future, the book also makes clear that much remains to be done in the history of Arabic mathematics and mathematical sciences. May it serve to inspire young scholars when it hints that exciting discoveries are likely to reward the patient reader of Arabic manuscript holdings.

In translating, I have striven for clarity above all. When this could be achieved by retaining the gallic flavour of the original, I have done so, sometimes in the case of mathematical terminology.

Whenever they were readily available, I have used standard English translations of primary sources. Since every translation is also an interpretation, I have in some instances (usually noted) modified an existing translation to make it consistent with Professor Rashed's interpretation of the source.

Many thanks to Professor Rashed for his confidence, and to Carol Troyer-Shank for her able assistance contributions. Without her managerial acumen, and editorial and clerical skills, I doubt that I could have completed the translation.

Michael H. SHANK

PREFACE

When historians become concerned with organizing and illuminating the various stages of mathematical thought, it is not unusual for them to isolate mathematics at the dawn of the modern age by distinguishing it from ancient and medieval mathematics. The expressions 'early modern mathematics' or 'mathematics in the classical age' are the ones customarily used to designate the mathematics that developed during the 16th and 17th centuries in Western Europe – to the exclusion of every other territory. For many historians, and even more so for philosophers such as Edmund Husserl, it is in this era that mathematicians carried out a radical revolution by breaking with ancient and medieval ways of thinking in order to forge new ones that heralded modern times. According to these historians and philosophers, it is precisely in this era that one encounters the beginning of algebra, the eruption of geometrical algebra, and the emergence of the mathematics of the continuous. It is thus easy for them to take the next step: these new contributions, which were born in the 16th and 17th centuries, allegedly mark this radical revolution and seal the unity of classical mathematics. On this account, classical mathematics would therefore be characterized by the simultaneous presence of the aforementioned chapters, among others.

If one examines the components of this 'early modern mathematics', however, one soon notices that these chapters are far from being contemporaneous: each has its own history, and the inventions or discoveries are by no means simultaneous. More generally, the global landscape of mathematics in the 16th and 17th centuries appears as a composite, an edifice constituted by different elements with origins traceable to many different dates. Indeed, some chapters, such as plane geometry, the geometry of the conic sections, and the geometry of the sphere, go back a millennium. They take us back to Euclid, Apollonius, and Menelaus. Without Apollonius, for example, how can one understand anything about Mydorge, Descartes, Fermat, Desargues, and Wallis, to mention only a few? These ancient geometers are effectively fixtures in the landscape of 'early modern mathematics', a point that obviously undermines the thesis of a 'revolution'. If one now wishes to consider algebra, the books of al-Khwārizmī, Abū Kāmil, and Fibonacci are prerequisites for anyone who wishes to deal with

the authors of the 16th and 17th centuries. In algebraic geometry, finally, the surest way of seeing novelty where it does not exist and overlooking it where it does is to forget all about the works of al-Khayyām and Sharaf al-Dīn al-Ṭūsī.

One can easily multiply similar examples from the study of projections, the theory of parallels, spherical geometry, trigonometry, Diophantine analysis, number theory, combinatorial analysis, infinitesimal geometry, etc. And the same analysis holds also for such mathematical sciences as astronomy, optics, and statics. In each of these cases, one can see that a global look is necessarily deceptive, unsuited to depicting the landscape of early modern mathematics, and especially to tracing the history of its various components. Now that new results have accumulated in the history of Arabic and Latin science, one can be even less satisfied than before with the sketchy and simplistic frame that hems in early modern mathematics. It is therefore necessary to return this mathematics to a horizon that is truly its own by shattering the old tripartite periodization inherited from political history – ancient, medieval, modern – in order to conceive of a new one that will be faithful to the facts. Indeed, the very expression 'early modern mathematics' does not even have the same conceptual or temporal extension depending on the chapter of mathematics under consideration. As far as plane geometry is concerned, for example, the differences between the practice of a 3rd-century BC mathematician and that of a 17th-century mathematician are reduced to a few nuances of style that are insufficient to distinguish them from each other. And in order to understand the algebraic geometry of the early modern period, it will be necessary to integrate into it some of the work carried out in the 11th to the 12th centuries. The same goes for other chapters of mathematics and for the mathematical sciences.

To sketch a picture that conforms better to the reality of these mathematics, I have thought it indispensable to go back to the beginnings of these various chapters in order to identify the concepts and the practices that these mathematicians put to work, and to understand some of their main developments and corrections. For several decades, this effort has been the goal of my research in the history of ancient, medieval, and classical (early modern) mathematics. The studies collected here represent only a few of these results. Although carried out in a variety of circumstances, they all nevertheless grow out of the same intention and follow the same method.

With competence and rigour, Aline Auger (CNRS), has prepared the manuscript for print and compiled the index.

Roshdi RASHED, Bourg-la-Reine – 30 March 2011

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