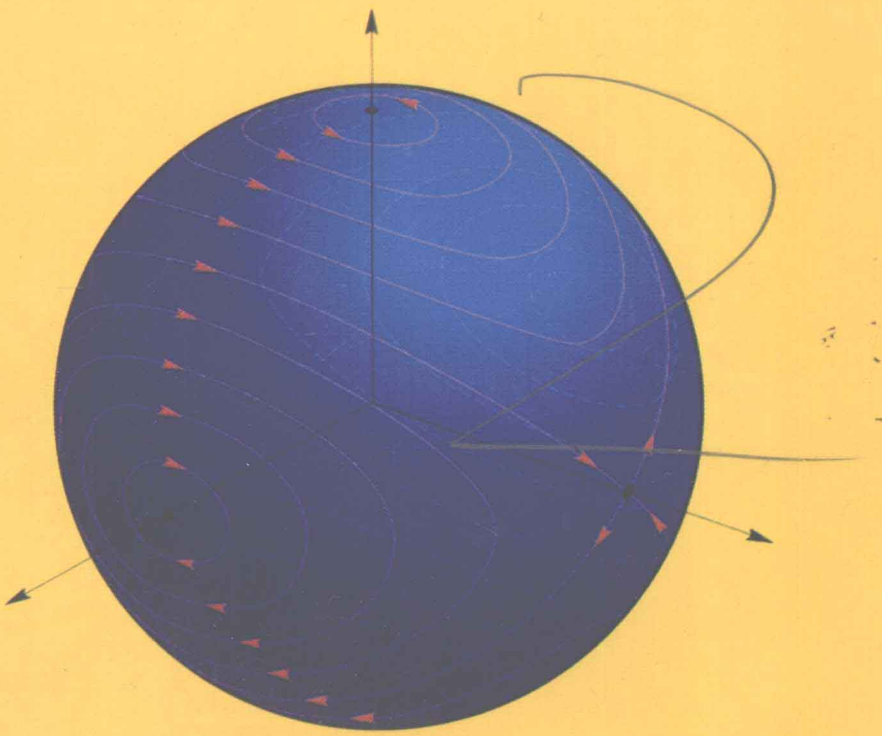
Jerrold E. Marsden ▪ Tudor S. Ratiu

INTRODUCTION to MECHANICS and SYMMETRY

Second Edition

力学和对称性导论 第2版



Springer

世界图书出版公司

www.wpcbj.com.cn

Texts in Applied Mathematics 17

图书在版编目（CIP）数据

力学和对称性导论=Introduction to Mechanics and Symmetry：英文／(美)马斯登（Marsden,J.E.）著.
—北京：世界图书出版公司北京公司，2008.4
ISBN 978-7-5062-9182-8

Ⅰ.力… Ⅱ.马… Ⅲ.经典力学—研究—英文 Ⅳ.031

中国版本图书馆CIP数据核字（2008）第021778号

书　　名:	Introduction to Mechanics and Symmetry 2nd ed.
作　　者:	Jerrold E. Marsden, Tudor S. Ratiu
中 译 名:	力学和对称性导论 第2版
责任编辑:	高春 刘慧
出 版 者:	世界图书出版公司北京公司
印 刷 者:	三河国英印务有限公司
发　　行:	世界图书出版公司北京公司（北京朝内大街137号 100010）
联系电话:	010-64015659, 64038348
电子信箱:	kjsk@vip.sina.com
开　　本:	24开
印　　张:	25.5
版　　次:	2008年3月第1次印刷
版权登记:	图字:01-2007-4445
书　　号:	978-7-5062-9182-8 / O・573　　定　价: 69.00元

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Mathematics Subject Classification (1991): 58-01, 70-01, 70Hxx

Library of Congress Cataloging-in-Publication Data
Marsden, Jerrold E.
Introduction to mechanics and symmetry / Jerrold E. Marsden.
Tudor S. Ratiu – 2nd Ed.
p. cm. — (Texts in applied mathematics; v. 17)
Includes bibliographical references and index.
ISBN 0-387-98643-X (alk. paper)
1. Mechanics, Analytic. 2. Symmetry (Physics). I. Ratiu, Tudor S.
II. Title. III. Series.
QA808.M33 1999
531—dc21 98-44773

ISBN 0-387-98643-X

To Barbara and Lilian for their love and support

Series Preface

Mathematics is playing an ever more important role in the physical and biological sciences, provoking a blurring of boundaries between scientific disciplines and a resurgence of interest in the modern as well as the classical techniques of applied mathematics. This renewal of interest, both in research and teaching, has led to the establishment of the series Texts in Applied Mathematics (TAM).

The development of new courses is a natural consequence of a high level of excitement on the research frontier as newer techniques, such as numerical and symbolic computer systems, dynamical systems, and chaos, mix with and reinforce the traditional methods of applied mathematics. Thus, the purpose of this textbook series is to meet the current and future needs of these advances and to encourage the teaching of new courses.

TAM will publish textbooks suitable for use in advanced undergraduate and beginning graduate courses, and will complement the Applied Mathematical Sciences (AMS) series, which will focus on advanced textbooks and research-level monographs.

Pasadena, California	J.E. Marsden
Providence, Rhode Island	L. Sirovich
Houston, Texas	M. Golubitsky
College Park, Maryland	S.S. Antman

Preface

Symmetry and mechanics have been close partners since the time of the founding masters: Newton, Euler, Lagrange, Laplace, Poisson, Jacobi, Hamilton, Kelvin, Routh, Riemann, Noether, Poincaré, Einstein, Schrödinger, Cartan, Dirac, and to this day, symmetry has continued to play a strong role, especially with the modern work of Kolmogorov, Arnold, Moser, Kirillov, Kostant, Smale, Souriau, Guillemin, Sternberg, and many others. This book is about these developments, with an emphasis on concrete applications that we hope will make it accessible to a wide variety of readers, especially senior undergraduate and graduate students in science and engineering.

The geometric point of view in mechanics combined with solid analysis has been a phenomenal success in linking various diverse areas, both within and across standard disciplinary lines. It has provided both insight into fundamental issues in mechanics (such as variational and Hamiltonian structures in continuum mechanics, fluid mechanics, and plasma physics) and provided useful tools in specific models such as new stability and bifurcation criteria using the energy–Casimir and energy–momentum methods, new numerical codes based on geometrically exact update procedures and variational integrators, and new reorientation techniques in control theory and robotics.

Symmetry was already widely used in mechanics by the founders of the subject, and has been developed considerably in recent times in such diverse phenomena as reduction, stability, bifurcation and solution symmetry breaking relative to a given system symmetry group, methods of finding explicit solutions for integrable systems, and a deeper understanding of special systems, such as the Kowalewski top. We hope this book will provide a reasonable avenue to, and foundation for, these exciting developments.

Because of the extensive and complex set of possible directions in which one can develop the theory, we have provided a fairly lengthy introduction. *It is intended to be read lightly at the beginning and then consulted from time to time as the text itself is read.*

This volume contains much of the basic theory of mechanics and should prove to be a useful foundation for further, as well as more specialized, topics. Due to space limitations we warn the reader that many important topics in mechanics are not treated in this volume. We are preparing a second volume on general reduction theory and its applications. With luck, a little support, and yet more hard work, it will be available in the near future.

Solutions Manual. A solution manual is available for instructors. It contains complete solutions to many of the exercises, as well as other supplementary comments. For further information, see

`http://www.cds.caltech.edu/~marsden/books/`.

Internet Supplements. To keep the size of the book within reason, we are making some material available (free) on the Internet. These are a collection of sections whose omission does not interfere with the main flow of the text. See `http://www.cds.caltech.edu/~marsden/books/`. Updates and information about the book can also be found at this website.

What Is New in the Second Edition? In this second edition, the main structural changes are the creation of a solutions manual (along with many more exercises in the text) and the Internet supplements. The Internet supplements contain, for example, the material on the Maslov index that was not needed for the main flow of the book. As for the substance of the text, much of the book was rewritten throughout to improve the flow of material and to correct inaccuracies. Some examples: The material on the Hamilton–Jacobi theory was completely rewritten, a new section on Routh reduction (§8.9) was added, Chapter 9 on Lie groups was substantially improved and expanded. The presentation of examples of coadjoint orbits (Chapter 14) was improved by stressing matrix methods throughout.

Acknowledgments. We thank Rudolf Schmid, Rich Spencer, and Alan Weinstein for helping with an early set of notes that helped us on our way. Our many colleagues, students, and readers, especially Henry Abarbanel, Vladimir Arnold, Larry Bates, Michael Berry, Tony Bloch, Dong-Eui Chang, Hans Duistermaat, Marty Golubitsky, Mark Gotay, George Haller, Aaron Hershman, Darryl Holm, Phil Holmes, Sameer Jalnapurkar, Edgar Knobloch, P.S. Krishnaprasad, Naomi Leonard, Debra Lewis, Robert Littlejohn, Richard Montgomery, Phil Morrison, Richard Murray, Peter Olver, Oliver O'Reilly, Juan-Pablo Ortega, George Patrick, Octavian Popp, Mason Porter, Matthias Reinsch, Shankar Sastry, Tanya Schmah, Juan Simo, Hans Troger, Loc Vu-Quoc, Alan Weinstein, and Steve Wiggins, have our deepest gratitude for their encouragement and suggestions. We also collectively thank all our students and colleagues who have used these notes and have provided valuable advice.

We are also indebted to Carol Cook, Anne Kao, Nawoyuki Gregory Kubota, Sue Knapp, Barbara Marsden, Marnie McElhiney, June Meyermann, Teresa Wild, and Ester Zack for their dedicated and patient work on the typesetting and artwork for this book. We want to single out with special thanks Hendra Adiwidjaja, Nawoyuki Gregory Kubota, and Wendy McKay for their special effort with the typesetting, the scripts for automatic conversion of references, the macros for indexing, and the figures (including the cover illustration). We also thank the staff at Springer-Verlag, especially Achi Dosanjh, Laura Carlson, MaryAnn Cottone, David Kramer, Ken Dreyhaupt, and Rüdiger Gebauer for their skillful editorial work and production of the book.

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Jerrold E. Marsden is professor of control and dynamical systems at Caltech. He received his B.Sc. at Toronto in 1965 and his Ph.D. from Princeton University in 1968, both in applied mathematics. He has done extensive research in mechanics, with applications to rigid-body systems, fluid mechanics, elasticity theory, plasma physics, as well as to general field theory. His current interests include dynamical systems and control theory and how it relates to mechanical systems with symmetry. He was one of the founders in the early 1970s of reduction theory for mechanical systems with symmetry, which remains an active research area today. He was the recipient of the prestigious Norbert Wiener prize of the American Mathematical Society and the Society for Industrial and Applied Mathematics in 1990, was elected a fellow of the American Academy of Arts and Sciences in 1997, and recieved a Max Planck research award in 2000. He has been a Carnegie Fellow at Heriot–Watt University (1977), a Killam Fellow at the University of Calgary (1979), recipient of the Jeffrey–Williams prize of the Canadian Mathematical Society in 1981, a Miller Professor at the University of California, Berkeley (1981–1982), a recipient of the Humboldt Prize in Germany (1991), and a Fairchild Fellow at Caltech (1992). He has served in several administrative capacities, such as director of the Research Group in Nonlinear Systems and Dynamics at Berkeley, 1984–86, the Advisory Panel for Mathematics at NSF, the Advisory committee of the Mathematical Sciences Institute at Cornell, and as director of the Fields Institute, 1990–1994. He is an editor for Springer-Verlag's *Applied Mathematical Sciences* series since 1982 and serves on the editorial boards of several journals in mechanics, dynamics, and control.

Tudor S. Ratiu is professor of mathematics at the University of California, Santa Cruz and the Swiss Federal Institute of Technology, in Lausanne. He received his B.Sc. in mathematics and M.Sc. in applied mathematics, both at the University of Timişoara, Romania, and his Ph.D. in mathematics at Berkeley in 1980. He has previously taught at the University of Michigan, Ann Arbor, as a T. H. Hildebrandt Research Assistant Professor (1980–1983) and at the University of Arizona, Tucson (1983–1987). His research interests center on geometric mechanics, symplectic geometry, global analysis, and infinite-dimensional Lie theory, with applications to integrable systems, nonlinear dynamics, continuum mechanics, plasma physics, and bifurcation theory. He was a National Science Foundation Postdoctoral Fellow (1983–86), a Sloan Foundation Fellow (1984–87), a Miller Research Professor at Berkeley (1994), and a recipient of of the Humboldt Prize in Germany (1997). Since his arrival at UC Santa Cruz in 1987, he has been on the executive committee of the Nonlinear Sciences Organized Research Unit. He is managing editor of the AMS *Surveys and Monographs* series and on the editorial board of the *Annals of Global Analysis* and the *Annals of the University of Timişoara*. He was also a member of various research institutes such as MSRI in Berkeley, the Center for Nonlinear Studies at Los Alamos, the Max Planck Institute in Bonn, MSI at Cornell, IHES in Bures-sur-Yvette, The Fields Institute in Toronto (Waterloo), the Erwin Schrödinger Institute for Mathematical Physics in Vienna, the Isaac Newton Institute in Cambridge, and RIMS in Kyoto.

Contents

1
Introduction and Overview

1.1 Lagrangian and Hamiltonian Formalisms

Mechanics deals with the dynamics of particles, rigid bodies, continuous media (fluid, plasma, and elastic materials), and field theories such as electromagnetism and gravity. This theory plays a crucial role in quantum mechanics, control theory, and other areas of physics, engineering, and even chemistry and biology. Clearly, mechanics is a large subject that plays a fundamental role in science. Mechanics also played a key part in the development of mathematics. Starting with the creation of calculus stimulated by Newton's mechanics, it continues today with exciting developments in group representations, geometry, and topology; these mathematical developments in turn are being applied to interesting problems in physics and engineering.

Symmetry plays an important role in mechanics, from fundamental formulations of basic principles to concrete applications, such as stability criteria for rotating structures. The theme of this book is to emphasize the role of symmetry in various aspects of mechanics.

This introduction treats a collection of topics fairly rapidly. The student should not expect to understand everything perfectly at this stage. *We will return to many of the topics in subsequent chapters.*

Lagrangian and Hamiltonian Mechanics. Mechanics has two main points of view, ***Lagrangian mechanics*** and ***Hamiltonian mechanics***. In one sense, Lagrangian mechanics is more fundamental, since it is based on variational principles and it is what generalizes most directly to the general relativistic context. In another sense, Hamiltonian mechanics is more fundamental, since it is based directly on the energy concept and it is what is more closely tied to quantum mechanics. Fortunately, in many cases these branches are equivalent, as we shall see in detail in Chapter 7. Needless to say, the merger of quantum mechanics and general relativity remains one of the main outstanding problems of mechanics. In fact, the methods of mechanics and symmetry are important ingredients in the developments of string theory, which has attempted this merger.

Lagrangian Mechanics. The Lagrangian formulation of mechanics is based on the observation that there are variational principles behind the fundamental laws of force balance as given by Newton's law $\mathbf{F} = m\mathbf{a}$. One chooses a configuration space Q with coordinates q^i, $i = 1, \ldots, n$, that describe the ***configuration*** of the system under study. Then one introduces the ***Lagrangian*** $L(q^i, \dot{q}^i, t)$, which is shorthand notation for $L(q^1, \ldots, q^n, \dot{q}^1, \ldots, \dot{q}^n, t)$. Usually, L is the kinetic *minus* the potential energy of the system, and one takes $\dot{q}^i = dq^i/dt$ to be the system velocity. The ***variational principle of Hamilton*** states

$$\delta \int_a^b L(q^i, \dot{q}^i, t)\, dt = 0. \tag{1.1.1}$$

In this principle, we choose curves $q^i(t)$ joining two fixed points in Q over a fixed time interval $[a, b]$ and calculate the integral regarded as a function of this curve. Hamilton's principle states that this function has a critical point at a solution within the space of curves. If we let δq^i be a variation, that is, the derivative of a family of curves with respect to a parameter, then by the chain rule, (1.1.1) is equivalent to

$$\sum_{i=1}^{n} \int_a^b \left(\frac{\partial L}{\partial q^i} \delta q^i + \frac{\partial L}{\partial \dot{q}^i} \delta \dot{q}^i \right) dt = 0 \tag{1.1.2}$$

for all variations δq^i.

Using equality of mixed partials, one finds that

$$\delta \dot{q}^i = \frac{d}{dt} \delta q^i.$$

Using this, integrating the second term of (1.1.2) by parts, and employing the boundary conditions $\delta q^i = 0$ at $t = a$ and b, (1.1.2) becomes

$$\sum_{i=1}^{n} \int_a^b \left[\frac{\partial L}{\partial q^i} - \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}^i} \right) \right] \delta q^i \, dt = 0. \tag{1.1.3}$$

Since δq^i is arbitrary (apart from being zero at the endpoints), (1.1.2) is equivalent to the ***Euler–Lagrange equations***

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}^i} - \frac{\partial L}{\partial q^i} = 0, \quad i = 1, \ldots, n. \tag{1.1.4}$$