

# **Atmosphere–Ocean Dynamics**

**ADRIAN E. GILL**

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## Preface

A systematic, unifying approach to the dynamics of the ocean and atmosphere is given in this book, with emphasis on the larger-scale motions (from a few kilometers to global scale). The foundations of the subject (the equations of state and dynamical equations) are covered in some detail, so that students with training in mathematics should find it a self-contained text. Knowledge of fluid mechanics is helpful but not essential. Simple mathematical models are used to demonstrate the fundamental dynamical principles with plentiful illustrations from field and laboratory. In fact, the search for suitable mathematical models during the eight years of writing stimulated several of my research papers written during that time.

Undergraduates in meteorology and oceanography should find the text a useful introduction to the dynamics of both air and sea. Having grown out of a graduate course, it is equally suitable for more advanced students, and material can be selected from many sections to give a well-structured program. For instance, my graduate course begins with a brief introduction from Chapter 1, skips to Section 5.6 and, by the third lecture, is focusing on rotation effects as covered in Chapter 7. Stratification effects are brought in later with material from Chapters 6 and 8, and then selected sections from the remaining chapters are used. Thus there is scope for considerable flexibility. Elementary courses would use material from the earlier chapters, whereas more advanced courses could be based on in-depth studies of later chapters.

Researchers should find the book attractive, not only for its systematic treatment of the dynamics, but also because of its extensive bibliography and index, the appendices, and the many useful diagrams and formulas. The treatment of many topics is novel, and considerable historical information is incorporated to make the book more readable and interesting. In fact, I became quite absorbed in pursuing historical aspects during the writing period.

## General Description of Contents

The two introductory chapters give an overall picture of how the circulations of both atmosphere and ocean are ultimately driven by the sun's energy. A somewhat novel treatment of basic thermodynamics and hydrostatics follows in Chapter 3, both ocean and atmosphere being discussed together. The fundamental equations for moving fluids are derived in Chapter 4, with particular reference to air containing moisture and water containing dissolved salts. Various types of energy are introduced, and the use of a rotating frame of reference is dealt with.

The fundamental aim is to understand the circulations of the atmosphere and ocean and the observed distributions of physical quantities such as temperature. The temperature distribution can be viewed (following Halley) as the result of a "competition" between the *sun*, which tries to warm the tropics more than the poles (and so create horizontal contrasts), and *gravity*, which tries to remove horizontal contrasts and arrange for warmer fluid to overlies colder fluid. This "competition" is complicated by such effects as the rotation of the earth, the variation of the angle between gravity and the rotation axis (the beta effect), and contrasts between the properties of air and water. Accordingly, we start with as simple a situation as possible and proceed by adding complicating effects one at a time.

The first step is taken in Chapter 5, where we consider adjustment under gravity of a homogeneous layer of fluid in the absence of rotation and external forcing effects. The results are directly applicable to phenomena such as seiches and tides in lakes, estuaries, and narrow seas. This chapter also introduces the important "hydrostatic approximation," which leads to the "shallow-water" equations. Effects of density stratification are then incorporated in Chapter 6, beginning with the two-layer system, like the oil-over-water arrangement that so intrigued Benjamin Franklin in 1762. Several aspects of wave motion are also introduced in Chapters 5 and 6. For instance, group velocity, introduced in Chapter 5 for surface waves, is applied in Chapter 6 to internal gravity waves in a continuously stratified fluid. Waves produced at a horizontal boundary, and possible refraction, reflection, or absorption in the fluid above, are also discussed.

Chapter 7, perhaps the most important in the whole book, introduces effects that are due to the earth's rotation. Although Laplace included these in his tidal equations in 1778, and Kelvin investigated wave motions in a rotating fluid a hundred years later, some of the fundamental ideas were developed relatively recently by Rossby in the 1930s. The "Rossby adjustment problem" brings out many facets of the behavior of rotating fluids, such as the tendency to attain "geostrophic equilibrium," the significance of "potential vorticity," and the importance of the length scale known as the Rossby radius of deformation.

Wave motion in a stratified rotating fluid is examined in Chapter 8 with applications to flow of air over hills and mountains. Propagation in a slowly varying medium, ray-tracing techniques, the internal wave spectrum in the ocean, and effects of waves on the mean flow are also examined. Chapter 9 introduces forcing by effects such as wind action, tide-producing forces, and solar heating. Inertial oscillations in the ocean surface layer are an example of forced motion, and these are dynamically

related to the nocturnal jet in the atmosphere. Hurricanes and the ocean's response to storms are also considered.

Phenomena associated with lateral boundaries are treated in Chapter 10. Dynamical studies that stem from Kelvin's work in 1879 can explain the main features of the very destructive North Sea Surge of 1953. Coastal upwelling, which is of great importance to fisheries, can be studied using similar analysis. Other classes of coastally trapped waves are also discussed. Equatorially trapped waves, considered in Chapter 11, have similar dynamics and are used to introduce both the beta effect and the mid-latitude beta-plane approximation for studying quasi-geostrophic flow. The tropical circulations of the atmosphere and ocean are also dealt with in this chapter.

In extratropical latitudes (Chapter 12), slow small-amplitude adjustments take place by means of planetary waves. These can be used, for instance, to describe how the ocean response to the wind has a highly asymmetric character with strong western boundary currents like the Gulf Stream and the Kuroshio. They are also useful for understanding the stationary wave patterns in the atmosphere. The omega equations, which provide a useful diagnostic tool, are also discussed.

The mid-latitude atmosphere is dominated by cyclones and anticyclones, which result from an instability of the basic wind distribution. Models illustrating how the potential energy of the zonal flow is converted into the kinetic energy of the cyclone systems are studied in Chapter 13. Fronts that develop in evolving cyclones and eddies in the ocean are also discussed. The book concludes with a global view of the atmosphere-ocean system.

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## How the Ocean–Atmosphere System Is Driven

### 1.1 Introduction

This book is about winds, currents, and the distribution of heat in the atmosphere and ocean. Since these are due to the sun, this first chapter looks at some of the essential processes that determine how the atmosphere and ocean respond to radiation from the sun. Ideally, one would like to be able to deduce this response in all its details from a knowledge of the appropriate properties of the earth and of its ocean and atmosphere, but this is not a simple matter. The nearest approach to a solution of this problem is by means of numerical models, but these still rely to some extent on observations of the real system, e.g., for determining the effects of processes (like those associated with individual clouds) that have a scale small compared with the grid used in the model.

The aim of the numerical models is to include the effects of *all* processes that play a significant part in determining the response of the ocean–atmosphere system. The aim of this chapter, on the other hand, is to consider only the most basic processes and to show how an equilibrium state **can be reached**. One such basic process is the *absorption of radiation* by certain gases (principally water vapor, carbon dioxide, and ozone), and so the “greenhouse” effect is discussed. The density field that results from radiation processes acting in isolation is not in dynamical equilibrium, because air near the ground is so warm that it is lighter than the air above. Consequently, vertical convection takes place and stirs up the lower atmosphere. Calculations of the equilibrium established when convective and radiative processes are both active is discussed in Section 1.5. These calculations, however, neglect variations in the horizontal,

which are, of course, extremely important since they are responsible for the winds and currents that are the main subject of this book. A brief discussion of the effects of horizontal variations is given in Section 1.6. Finally, since radiation is the source of energy for the atmosphere-ocean system, variations in the radiative input are discussed in Section 1.7.

## 1.2 The Amount of Energy Received by the Earth

Energy from the sun is received in the form of *radiation*, nearly all the energy being at wavelengths between 0.2 and 4  $\mu\text{m}$ . About 40% is in the visible part of the spectrum (0.4–0.67  $\mu\text{m}$ ). The average energy flux from the sun at the mean radius of the earth is called the solar constant  $S$  and has the value (Hickey *et al.*, 1980)

$$S = 1.376 \text{ kW m}^{-2}. \quad (1.2.1)$$

(A great variety of units is used for energy flux. The relation between these is given in Appendix 1.) In other words, a 1-m-diameter dish in space could collect enough energy from the sun to run a 1-kW electric heater! Since the earth's orbit is elliptical rather than circular, the actual energy received varies seasonally by  $\pm 3.5\%$  (Kondratyev, 1969, Section 1.1), the maximum amount being received at the beginning of January.

The total energy received from the sun per unit time is

$$\pi R^2 S, \quad (1.2.2)$$

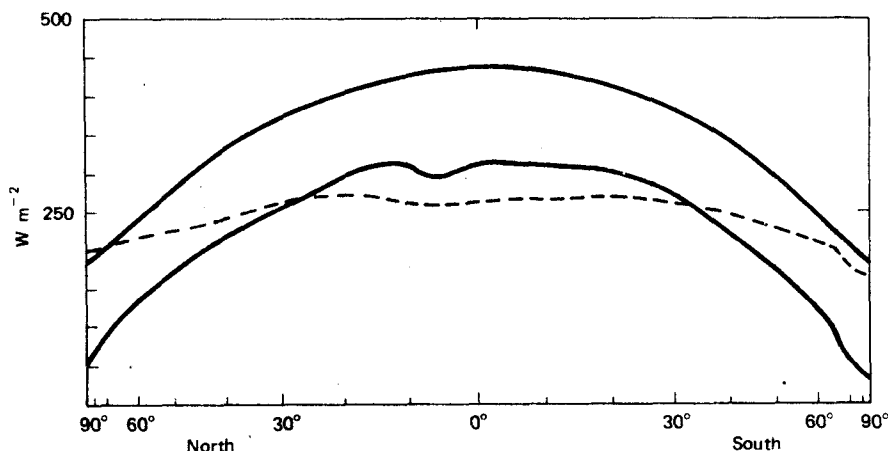
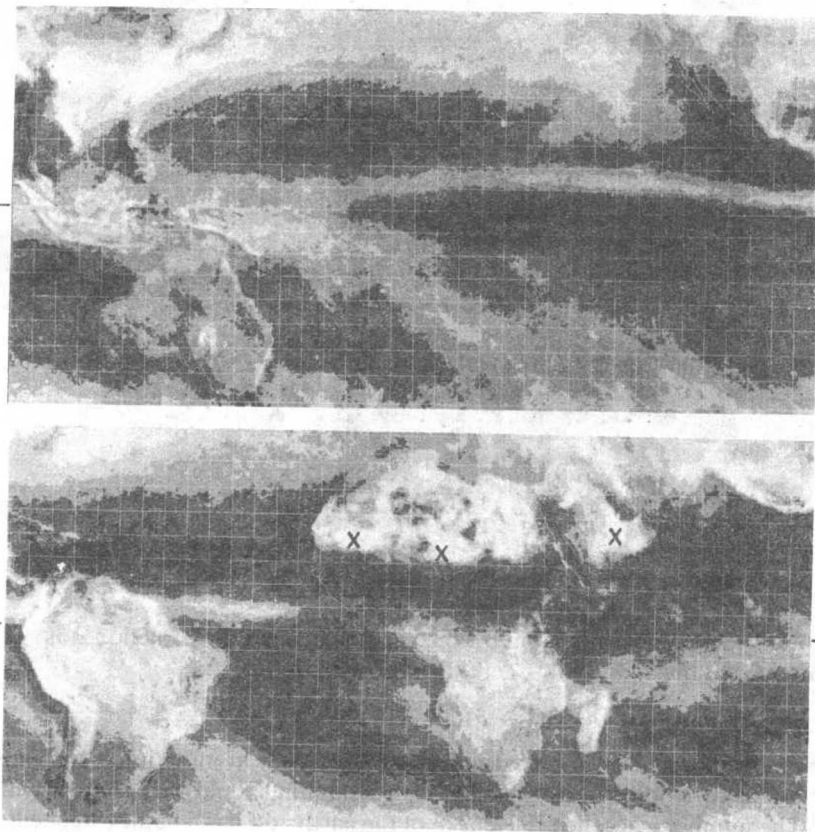


Fig. 1.1. The radiation balance of the earth. The upper solid curve shows the average flux of solar energy reaching the outer atmosphere. The lower solid curve shows the average amount of solar energy absorbed; the dashed line shows the average amount of outgoing radiation. The lower curves are average values from satellite measurements between June 1974 and February 1978, and are taken from Volume 2 of Winston *et al.* (1979). Values are in watts per square meter. The horizontal scale is such that the spacing between latitudes is proportional to the area of the earth's surface between them, i.e., is linear in the sine of the latitude.

where  $R$  is the radius of the earth. Since the area of the earth's surface is  $4\pi R^2$ , the average amount of energy received per unit area of the earth's surface per unit time is

$$\frac{1}{4}S = 344 \text{ W m}^{-2}. \quad (1.2.3)$$

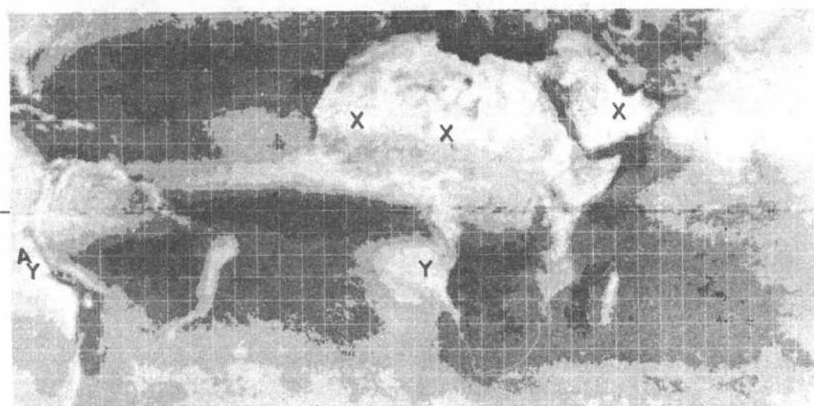
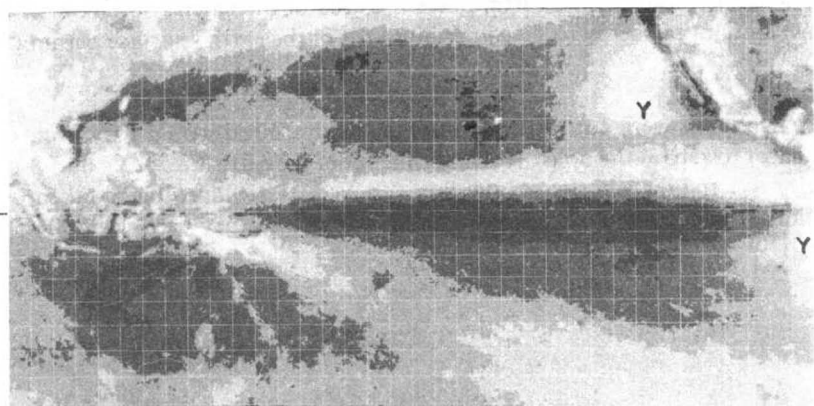
If the earth's axis were not tilted, the average flux received would vary from  $\pi^{-1}S$  at the equator to zero at the poles. However, the tilt of the earth ( $23.5^\circ$ ) results in seasonal variations in the distribution of the flux received. When account is taken of these variations, the average flux received in 1 yr is found to vary with latitude as shown in Fig. 1.1.



(a)

**Fig. 1.2.** The geographical distribution of reflectivity for (a) January 1967–1970 and (b) July 1969–1970, as determined from satellite observations. Most of the bright areas in the figure are characterized by persistent cloudiness and relatively heavy precipitation. However, the following exceptions should be noted: areas indicated by X's denote desert regions where the earth's surface is highly reflective and areas indicated by Y's denote regions of persistent low, nonprecipitating cloud decks. Tick marks along the side denote the position of the equator; the Mercator grid lines are spaced at intervals of  $5^\circ$  of latitude and longitude. [From U.S. Air Force and U.S. Department of Commerce, *Global Atlas of Relative Cloud Cover, 1967–1970*, Washington, D.C., 1971.]





(b)

Fig. 1.2 (Continued)

Not all the energy impinging on the earth is absorbed. A fraction  $\bar{\alpha}$  is reflected or scattered, so the average flux actually absorbed is

$$\frac{1}{4}(1 - \bar{\alpha})S = 240 \text{ W m}^{-2}. \quad (1.2.4)$$

The amount reflected or scattered is about  $100 \text{ W m}^{-2}$  at all latitudes, as shown in Fig. 1.1. (There is no obvious reason that this amount should vary so little with latitude.) The number  $\bar{\alpha}$  is called the *albedo* of the earth and has a value (Stephens *et al.*, 1981) of about

$$\bar{\alpha} = 0.3. \quad (1.2.5)$$

Similarly, the albedo  $\alpha$  can be defined for a particular place and particular time as the fraction of the impinging radiation that is reflected or scattered. The reflected light is the light by which the earth may be photographed from space, and such photographs (see Fig. 1.2, which is effectively the result of combining many such photographs to give the mean reflectivity) show that the albedo can vary enormously with such factors as the amount of cloud, and whether the ground is covered by ice or