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CEHUI KEJI ZHUANZHU CHUBAN JIJIN ZIZHU

**THEORY ON FUZZY SPATIAL OBJECT  
MODELING WITH ITS APPLICATIONS**

唐新明 著

**模糊空间对象模型  
理论及其应用**

(中文摘要·英文版)



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# 模糊空间对象模型理论及其应用

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· 北京 ·

## 内 容 简 介

本书是模糊空间要素建模在理论和方法上的专著。作者探讨了模糊空间要素的定义、模糊空间要素的拓扑关系、模糊空间要素建模、模糊空间要素的生成方法、模糊空间要素的查询方法以及基于模糊空间要素的推理等。在理论和方法的基础上,采用最大似然法对三亚市的 TM 影像进行了模糊分类,生成了模糊的空间土地覆盖要素,提出了模糊要素查询的多种方法,设计了模糊空间要素的查询界面,并在模糊空间要素的基础上,对三亚市的土地覆盖动态变化进行了分析和对比。

本书可供有关高校和科研单位从事地理信息系统理论和技术研究人员的参考。

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## Preface

Fuzzy logic has found many applications since its introduction by L. Zadeh about thirty years ago. One of the most interesting applications is in the field of geographic information science. Many-if not most-spatial phenomena expose some degree of vagueness or uncertainty with regard to location or extension in space and time. Fuzzy sets have been applied to better understand and represent these phenomena by many geo-information scientists since nearly twenty years. The focus of these applications was mainly on fuzzy boundaries, fuzzy extensions of objects in space, or fuzzy classification.

Another fundamental issue in geographic information science is the structurally correct representation of features and relationships among them. Since the landmark paper on topological principles in cartography by James Corbett in the late 1970ies topology has been the focus of attention in GIS research and development. The derivation of topological spatial relations by Egenhofer and Franzosa in 1991 marks another milestone in geo-information science. These topological spatial relations were derived by a consequent application of topological invariants of boundary and interior of point sets.

Whenever we deal with topologies on underlying spaces we tacitly assume that the open sets are formed by regular point sets where every element has a crisp binary yes or no membership. Tang Xinming addresses three important aspects of the use of fuzzy logic in geographic information science: topological relations based on fuzzy topology, creation of fuzzy objects, and reasoning about change of fuzzy objects.

Here, for the first time topological relationships between fuzzy spatial objects based on fuzzy open sets are investigated. The complex subject of what is the interior and boundary of a fuzzy set based on a fuzzy topology is treated in great detail. The major findings in this respect are the fuzzy topological invariants of core, interior, fringe and boundary of a fuzzy set, and the topological relations that can be derived for simple fuzzy regions. Because of the number of invariants the intersection approach produces a large number of possible relations. To create an environment that makes queries according to the derived relations feasible they are classified into logical groups. A prototype querying tool is also introduced to demonstrate the principle.

To illustrate the theoretical approach an application in land cover change has been selected. Tang Xinming presents a method to create fuzzy land cover objects from satellite images that are subsequently submitted to a fuzzy reasoning process in order to infer changes in land cover over time. The use of fuzzy land cover objects and the application of a fuzzy reasoning process allow not only a more appropriate

representation of vaguely delineated natural phenomena but also reveal more insight into the changes that took place over time.

This book stands out in two respects. It is the first comprehensive treatment of fuzzy topology in a spatial context. The reader will find a wealth of mathematical theory related to fuzzy topological spaces and their invariants. In particular, the treatment of the concept of a fuzzy boundary of a fuzzy set from a thorough mathematical perspective, and the derivation of other invariants makes it worth reading. In analogy to the classical 4-intersection approach fuzzy topological spatial relations can be derived. This is done in great detail and the classification and grouping of relations makes the theoretical findings worth implementing. Finally, the analysis of fuzzy land cover objects and the fuzzy reasoning about their changes open an avenue for further interesting applications and implementations of this approach.

This book offers to both the mathematically interested and the application oriented readers a multitude of useful and practical insights into the complex domains of fuzzy topology and fuzzy logical reasoning. Moreover, it stimulates the reader to probe further into the intriguing subject of fuzzy logic in GIS.

Tang Xinming has written an important book that helps to understand the underlying theoretical principles of spatial information with uncertainties and vagueness involved as well as the application of these principles to practical problems of a changing world.

**Wolfgang Kainz**

## 序(译文)

自从 30 多年前扎德引入模糊逻辑理论以来,出现了许多关于该理论的应用,其中在地理信息科学领域的应用令人注目。因为空间信息中时间和空间的可扩展性,绝大多数空间对象都表现出一定程度的模糊性和不确定性。为了更好地认识和表达这些对象,最近 20 年来,许多地理信息科学研究者应用模糊集理论来达到目的。这些应用主要集中在模糊边界、空间对象的模糊扩张以及模糊识别等方面。

地理信息科学最根本的目的之一就是正确地、结构化地表达空间对象和表达对象之间的关系。20 世纪 70 年代后期 James Corbett 关于制图学中拓扑原理的标志性论文成为 GIS 研究和开发的焦点,1991 年 Egenhofer 和 Franzosa 提出的拓扑空间关系的提取又成为地理信息科学发展的一个里程碑,其空间关系是通过应用边界和点集内部的拓扑不变量而获得的。

当我们研究空间信息中隐含的拓扑关系时,通常情况下默认开集是由规则点集构成,这样其中每一个元素都有刚性的二元真值或者说不考虑其隶属度。本书作者唐新明博士引入了模糊逻辑在地理信息科学中应用的三个方面:基于模糊拓扑的空间拓扑关系、空间模糊对象的创建和模糊对象变化的推理。

本书首次在模糊开集的基础上研究了模糊空间对象之间的拓扑关系。在模糊拓扑理论的基础上,详细探讨了包括模糊集的内部和边界在内的模糊对象等复杂命题。主要发现有两点:一是模糊集的核心、内部、边缘和边界等拓扑性质,二是提出了简单模糊区域提取拓扑关系的可能性。采用相交矩阵的方法来得到的可能的拓扑不变量,其数量很大,本书根据可见关系把空间对象分为不同逻辑群,从而创建了一个空间拓扑关系查询环境,并为实践这种方法引入了查询模型。

为了验证书中提出的理论的正确性,作者选择了土地覆盖变化应用来进行实践。作者提出从卫星影像创建模糊土地覆盖对象的方法,并把得到的模糊空间对象用于穿越时间的土地覆盖变化查询的模糊推理过程。模糊覆盖对象的利用和模糊推理过程不仅可以更适当地表达自然现象,并且能够更加深入地研究伴随时间发生的土地利用变化。

本书有两个突出方面:其一,首次全面地在空间环境中使用模糊拓扑理论。在书中读者将饱览关于模糊拓扑空间及其拓扑不变量的丰富的数学理论。本书从数学的角度来研究模糊集的模糊边界,以及提取除常用拓扑之外其他拓扑不变量的方法,将使读者大开眼界。特别是书中使用类似 4-交矩阵的模型来提取模糊拓扑空间关系,充满了独创性。作者详细阐述了以上内容,并从关系的分类和群集分析出发,提出了理论的可行性。其二,书中提出的模糊土地覆盖对象的分析和关于其变化的模糊推理,为更深入地在地理信息领域应用模糊集理论,并使之服务于实际应用开辟了一条新路。

本书不仅从数学角度而且从实践应用的角度把读者引入到模糊拓扑和模糊逻辑推理的深入研究中,并把这个高深的领域深入浅出地介绍给读者。更重要的是,它将激发有兴趣的读者更加深入探索 GIS 中微妙的模糊逻辑课题的热情。

作者写了一本很重要的书,这本书将帮助人们认识和探索空间信息中隐含的不确定理论和模糊理论,并且把这些理论原则应用到改造世界的实际问题中。

沃尔夫冈·凯恩兹

## Abstract

Currently most GISs represent natural phenomena by crisp spatial objects. In fact many natural phenomena have fuzzy characteristics. The representation of these objects in the crisp form greatly simplifies the processing of spatial data. However, this simplification cannot describe these natural phenomena precisely, and it will lead to loss of information in these objects. In order to describe natural phenomena more precisely, the fuzziness in these natural phenomena should be considered and represented in a GIS. This will allow the derivation of better results and a better understanding of the real world to be achieved.

The central topic of this thesis focuses on the accommodation of fuzzy spatial objects in a GIS. Several issues are discussed theoretically and practically, including the definition of fuzzy spatial objects, the topological relations between them, the modeling of fuzzy spatial objects, the generation of fuzzy spatial objects and the utilization of fuzzy spatial objects for land cover changes.

A formal definition of crisp spatial objects has been derived based on the highly abstract mathematics such as set theory and topology. Fuzzy set theory and fuzzy topology are the ideal tools for defining fuzzy spatial objects theoretically, since fuzzy set theory is a natural extension of classical set theory and fuzzy topology is built based on fuzzy sets. However, owing to the extension, several properties holding between crisp sets do not hold for fuzzy sets.

The key issue of a fuzzy spatial object is its boundary. Three definitions of fuzzy boundary are revisited and one is selected for the definition of fuzzy spatial objects. Besides the fuzzy boundary, several notions such as the core, the interior, the fringe, the frontier, the internal fringe and the outer of a fuzzy set are defined in fuzzy topological space. The relationships between these notions and the interior, the boundary and the exterior of a fuzzy set are revealed. In general, the core is the crisp subset of the interior, and the fringe is a kind of boundary but shows a finer structure than the boundary of a fuzzy set in fuzzy topological space. These notions are all proven to be topological properties of a fuzzy topological space.

The definition of a simple fuzzy region is derived based on the above topological properties. It is discussed twice in the thesis. Firstly, the definition of a simple fuzzy region is given in a special fuzzy topological space called crisp fuzzy topological space, since most topological properties of a fuzzy set in the fuzzy topological space are the same as those in crisp topological space. A formal definition of a simple fuzzy region is proposed based on the discussion of the topological properties, besides the interior, the boundary and the exterior, of a fuzzy set in the general fuzzy topological space. A crisp simple region is a special

form of a simple fuzzy region.

One of the fundamental properties between fuzzy spatial objects is the topological relations. This topic is intensively discussed in the thesis. The problem of the 9-intersection approach for identifying topological relations between fuzzy spatial objects is revealed. In order to derive the topological relations between fuzzy spatial objects, the 9-intersection approach is updated into the  $3 \times 3$ -intersection approach in the crisp fuzzy topological space. Furthermore, the  $4 \times 4$ -intersection matrix is built up by using the topological properties of fuzzy sets, and the  $5 \times 5$ -intersection matrix can be built up based on a certain condition in crisp fuzzy topological space. These matrices are then updated in the general fuzzy topological space, based on topological properties, other than the interior, the boundary and the exterior, of two fuzzy sets. Two  $3 \times 3$ -intersection and one  $4 \times 4$ -intersection matrices are introduced in the general fuzzy topological space. The topological relations between simple fuzzy regions can be identified based on the topological invariants in the intersections of the matrices. Using the empty/non-empty topological invariants in the intersections, 44 and 152 relations are derived between two simple fuzzy regions.

The modeling of fuzzy spatial objects should be done not only for simple fuzzy regions, but also for fuzzy lines and fuzzy points. In order to model fuzzy lines and fuzzy points and the topological relations between fuzzy spatial objects, a fuzzy cell is proposed and a fuzzy cell complex can be constructed from fuzzy cells. A fuzzy region, a fuzzy line and a fuzzy point are then defined according to this structure. The relations between these fuzzy spatial objects are identified. The fuzzy cell complex structure constitutes a theoretic framework, since it can easily model the fuzzy spatial objects.

After proposing the theoretic framework for fuzzy spatial object modeling, the thesis addresses several practical issues on applying fuzzy spatial objects. The first issue is how to generate fuzzy spatial objects. A composite method is proposed for the generation of fuzzy land cover objects. It involves several steps, from designing membership functions to classification and refining the membership values of fuzzy land cover objects.

Another practical issue is how to retrieve fuzzy spatial objects, particularly on the basis of topological relations. In traditional GIS, the query operators are defined based on the relatively small number of topological relations. However, there are many topological relations between fuzzy spatial objects. In order to query fuzzy spatial objects, the query operators are proposed and formalized based on the common-sense operators in traditional GIS. The 44 or 152 topological relations are grouped into these operators by four different methods. These methods constitute a relatively complete covering for querying fuzzy spatial objects so as to meet the different application requirements.

The third practical issue is how to use fuzzy spatial objects in real applications. Since the dynamics of land covers is a very important topic in China, the focus lies on calculating changes of land covers. Sanya city, located in South China, is selected as the test area. A fuzzy reasoning method is proposed for calculating land cover changes. It shows that, with fuzzy representation, not only can a better result be achieved for the land cover changes, but also the details of changes can be revealed.

## 摘 要

目前所有的地理信息系统的数学模型都是建立在刚性(非模糊)要素基础上的。其主要特征是这些要素的定义是明确的,其边界是确定的,即要素的内涵和外延是一致的。例如地籍系统中,宗地的归属是确定的,宗地的权属界线是非模糊的。在常规的地理信息系统中,空间物体被抽象化为类别定义明确及边界表达确定的非模糊空间要素。有关学者已经在经典拓扑学的基础上对空间要素以及空间要素的拓扑关系进行了全面地阐述,基于代数拓扑学的空间要素模型已经广泛地应用于几乎所有的地理信息系统数据模型中。这种模型可以表达具有明确边界的空间物体。

然而,在自然界中,更多现象是模糊的,在空间分布上是连续的,没有确切的边界来区分它们。例如,山脉的范围和位置、土地利用和土地覆盖等要素的边界位置都是模糊的。这类具有不确定空间范围和边界的空间物体就叫做模糊空间要素。

可以说自然界中绝大部分地理要素的边界是模糊的,如何定义、产生、建立模糊空间要素模型是地理信息系统发展的重要方向之一。能够处理模糊空间要素的地理信息系统是新一代地理信息系统的重要任务。它的建立和开发,将大大提高地理信息系统处理空间数据的能力,提高空间分析的精度,拓展地理信息系统的应用能力和决策的准确性。

模糊的本质是为了精确,对模糊空间要素的研究离不开模糊数学,运用模糊数学方法进行分析和推理已经得到了广泛的应用,并在诸多领域取得了成功的经验。目前的方法一般是在非模糊要素的基础上采用模糊分析和推理工具在各自的应用领域进行分析和应用,这种分析方法忽视了自然界要素本身的不确定性和模糊性,使得分析结果仍然不尽合理。模糊地理信息系统的任务就是要在模糊数学、拓扑学和其他数学工具的基础上,发展并定义模糊空间要素,研究模糊空间要素之间、模糊和非模糊空间要素之间的几何关系、拓扑关系等,在此基础上,建立可以容纳模糊和非模糊空间要素的空间数据结构和模型,开发模糊空间要素查询语言,进行空间信息的显示和分析,促进地理信息系统的更广泛、更精确的应用。

本书是模糊空间要素建模在理论和方法上的专著。主要包括模糊空间要素的定义、模糊空间要素的拓扑关系、模糊空间要素模型,模糊空间要素的生成方法,模糊空间要素的查询方法,以及基于模糊空间要素的推理等。在理论和方法的基础上,采用最大似然法对三亚市的 TM 影像进行了模糊分类,产生了模糊空间土地覆盖要素,设计了模糊空间要素的查询界面,并对三亚市的土地覆盖动态变化进行了分析和对比。

模糊空间要素的核心问题是模糊边界问题。刚性物体的边界在拓扑学中已有明确的定义,关于模糊边界,其概念在模糊拓扑学中有若干种扩充。本书研究了三

种模糊边界概念的区别和联系,并选择了其中一种作为模糊空间要素的模糊边界的定义。除了上述三种概念之外,本书还拓展了和模糊边界有关的模糊拓扑概念,并对这些概念之间的关系进行了描述和证明。

模糊空间要素的定义是本书重点内容之一。本书在第三章中在一种特殊的模糊拓扑空间中建立有关的简单模糊空间区域的定义。然后在第四章中提出了在一般模糊拓扑空间中的简单模糊空间区域的定义,并阐述了两种定义的关系以及在地理信息系统中的适用性。

模糊空间要素拓扑关系是建立地理信息系统模型和空间分析最重要的基础之一。非模糊空间要素之间的拓扑关系可以采用 9-intersection 模型进行分析和描述。本书在模糊拓扑空间中分析了采用 9-intersection 模型分析模糊空间要素拓扑关系的适用性。并在模糊拓扑空间中,将 9-intersection 模型进行了拓展,提出了两个  $3 \times 3$ -intersection,一个  $4 \times 4$ -intersection,一个  $5 \times 5$ -intersection 模型,并采用  $3 \times 3$ -intersection 和  $4 \times 4$ -intersection 模型对简单模糊区域之间的拓扑关系进行了分析和表达。

地理信息系统模型中的要素主要是点、线、面,这些要素的定义源于代数拓扑。本书在第四章中采用代数拓扑学原理,在胞腔复形的基础上发展了模糊胞腔复形的概念,并将其用于模糊点、线、面的定义以及模糊点、线、面之间的拓扑关系的分析和表达,并指出模糊胞腔复形的结构可以用于模糊空间要素模型的建立。

地理信息系统中最重要任务的之一是描述和表达自然界中的地理要素。在完成模糊空间要素模型建立的理论问题之后,首先要解决如何产生模糊空间要素的问题。模糊空间要素的产生有许多方法,本书第六章提出了模糊空间要素的产生的基本步骤,并采用最常用的最大似然法对 TM 影像对土地覆盖的进行了自动分类,获得了模糊的土地覆盖要素。本书对模糊的土地覆盖的产生方法进行了详细的阐述。

地理信息系统的另一个重要任务是要素的检索和查询,如何检索和查询模糊空间要素是第七章讨论的一个重点。由于模糊空间要素的模糊特性,对模糊空间要素的检索和查询较对非模糊要素的检索和查询要复杂的多。本书着重讨论了基于模糊拓扑关系和非模糊拓扑关系的四种查询方法,指出了这些方法的区别和联系,分析了四种方法可适用的范围。

第八章探讨了基于模糊空间要素的推理。以三亚市的土地覆盖为例,根据两个时相的 TM 影像产生了模糊的土地覆盖要素,在此基础上对土地覆盖的变化进行了变化程度的推理,并与传统的基于非模糊的土地覆盖要素的推理进行了比较。分析结果证明,像土地覆盖这种自然要素的表示,采用模糊方法较采用非模糊方法可以更准确地反映自然物体以及它们的变化。

本书讨论了模糊空间要素建模方面特别是模糊拓扑建模方面的有关问题。由于本人的认识水平有限,有些问题的研究还比较肤浅甚至错误,请学者批评指正;

由于研究时间有限,有些问题例如定量的模糊拓扑关系等还有待进行进一步的研究。作者希望通过本书抛砖引玉,开展对模糊空间要素的本质问题的深入研究,为模糊空间要素的数据库建模服务,为模糊地理信息系统的开发奠定基础。

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