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A New Fracture Criterion for Composite Materials

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复合材料的新断裂准则

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Introduction

In this book, a newly developed Z-fracture criterion is introduced. Experimental results show that Z-fracture criterion can be successfully applied in composite structure design. The beauty of the newly developed criterion is that it can be used precisely predicting the crack propagation direction in composite materials.

The author also introduced the composite fracture mechanics, the fracture criteria and their inadequacy, especially the S-criterion.

The introduced Z-criterion is an academic achievement in author's research work on the area of composite fracture mechanics. It makes valuable contributions in the theory of composite fracture.

These achievements make the book a valuable reference book for university students, professors, engineers and researchers in the related engineering areas.

内容简介

本书详细介绍了复合材料断裂理论领域里的最新研究成果：复合材料的 Z-断裂准则。大量的试验证明了该准则不仅可以解决复杂的复合材料抗断裂设计问题，而且可以对复合材料中裂纹的扩展方向进行正确预测。

书中还详细介绍了复合材料断裂力学基础、工程和研究中常使用的断裂准则以及这些准则的缺陷，特别是 S 准则的缺陷。书中介绍的 Z-断裂准则是作者长期在国外从事断裂理论研究中的成果,对复合材料结构的安全设计有着重要意义。

这些成果使得该书成为一本很好的专业参考书，可供相关领域里的大学生、研究生、大学教师、工程师和研究人员参考使用。

PREFACE

Fiber composites are an emerging class of structural materials widely used in many critical applications such as national defense and space technology. The features of the fiber composites that make them so promising as engineering materials are their low density, high specific strength, high specific stiffness, and the opportunities to tailor the material properties through the control of fiber and matrix combination and fabrication processing.

Cracks in composite are generally subjected to mixed mode deformations due to the highly complex nature of the material, such as the variable manufacturing procedure, different notches, and the varied material mechanical properties. They may also be created between layers and through layers at the same time and generally behave as fully developed cracks.

To study the crack problem from the micro-view, quantum mechanics knowledge will be needed. Between macroscopic and microscopic knowledge of dislocation movements, formation of subgrain precipitates slipbands and grain inclusions will be needed. Due to the highly complex nature of crack propagation and the lack of a full physical understanding, as well as the lack or sufficiently powerful mathematical tools, there is no single theory to cope with the crack problem from the above mentioned points of view. In this book the macro approach will be applied.

In chapter 1 the basic knowledge of mechanics in composite materials is introduced. It will be used for introductions of composite fracture mechanics in chapter 2 and chapter 3.

Regarding fracture criteria, there are many crack initiation criteria currently available, such as the stress intensity factor criterion, the maximum circumference stress criterion, COD criterion, J integral criterion and the S criterion. The S criterion has been demonstrated to be a good criterion and brought the prospect of studying the crack problem using a single parameter. However some numerical and experimental results have indicted that the S criterion is not adequate when applied to some fiber composites. A successful and more versatile criterion is essential for the analysis of fiber composites.

Generally speaking, a successful fracture criterion is supported by many experiments, but under certain special conditions almost any fracture criterion may be found to be unsatisfactory. In this book a newly developed Z-criterion for the composite materials has been fully introduced. Detailed information for the stress fields as well as the total, dilatational and distortional strain energy density factors for mixed mode cracks has been fully covered.

In addition, an application of J-integral in Z-criterion for composite materials is also introduced. In the final chapter we introduced how to use the Z-criterion to analyze cracked composite plate under bending conditions. The book can be used as a reference book for engineers and graduate students who wish to get more information in advanced fracture mechanics for composite materials.

I would like to take this opportunity to thank Professors Bor Z Jang, Bruce Valaire and Jeffrey Suhling for their valuable guidance during my Ph.D study in Auburn University in U.S.A.. I also thank Shanxi Province Nature Science Foundation and Taiyuan Heavy Machinery Institute to support me to publish this book, and my graduate student Mr. Chen Yonggang for his meticulous care in typing the manuscript.

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May 15, 2000

CONTENTS

1 Mechanics of Composite Materials

1.1 Stress-Strain Relations

From the macro-point of view fiber reinforced composites can be treated as uniform materials with anisotropic mechanical properties. In this book we only study such materials with linear elastic property. For anisotropic materials the stress and strain is related as follows [1]:

$$\sigma_i = C_{ij}\,\varepsilon_j \qquad (i, j = 1,2,\ldots,6) \tag{1.1}$$

where σ_i is stress component,

ε_j is strain component,

C_{ij} is stiffness matrix component.

It can be proved that stiffness matrix $\boldsymbol{C}$ is a symmetric matrix

$$C_{ij} = C_{ji} \tag{1.2}$$

So $\boldsymbol{C}$ has 21 independent components for anisotropic materials. Each component is a material constant.

For orthotropic materials

There are three mutually perpendicular elastic symmetric planes in the material. It can also be proved that $\boldsymbol{C}$ only has 9 independent components. In this case we have

$$C_{14} = C_{15} = C_{16} = C_{24} = C_{25} = C_{26} = C_{34} = C_{35} = C_{36} = C_{45} = C_{46} = C_{56} = 0 \tag{1.3}$$

Then equation (1.1) can be written as

$$\begin{pmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \tau_{23}\\ \tau_{31}\\ \tau_{12}\end{pmatrix}=\begin{bmatrix}C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0\\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & C_{66}\end{bmatrix}\begin{pmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \gamma_{23}\\ \gamma_{31}\\ \gamma_{12}\end{pmatrix} \tag{1.4}$$

For isotropic materials

C_{ij} has only two independent components. Then equation (1.1) can be written as

$$\begin{pmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \tau_{23}\\ \tau_{31}\\ \tau_{12}\end{pmatrix}=\begin{bmatrix}C_{11} & C_{12} & C_{12} & 0 & 0 & 0\\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0\\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0\\ 0 & 0 & 0 & \frac{C_{11}-C_{12}}{2} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{C_{11}-C_{12}}{2} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{C_{11}-C_{12}}{2}\end{bmatrix}\begin{pmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \gamma_{23}\\ \gamma_{31}\\ \gamma_{12}\end{pmatrix} \tag{1.5}$$

Compliance matrix

Equation (1.1) can be written as

$$\varepsilon_i = B_{ij}\sigma_j \quad (i, j = 1,2,\ldots,6) \tag{1.6}$$

where B_{ij} is called compliance matrix components, and

$$\boldsymbol{B} = \boldsymbol{C}^{-1} \tag{1.7}$$

$\boldsymbol{B}$ is called compliance matrix,

$\boldsymbol{C}$ is called stiffness matrix .

For anisotropic materials

Similarly matrix $\boldsymbol{B}$ is a symmetric matrix, and has 21 independent components.

For orthotropic materials

Compliance matrix $\boldsymbol{B}$ has 9 independent components. Strain-stress relations are as follows

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{pmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{13} & 0 & 0 & 0 \\ B_{12} & B_{22} & B_{23} & 0 & 0 & 0 \\ B_{13} & B_{23} & B_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & B_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & B_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & B_{66} \end{bmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{pmatrix} \tag{1.8}$$

For isotropic materials

Compliance matrix $\boldsymbol{B}$ has 2 independent components. Strain-stress relations are as follows

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{pmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{12} & 0 & 0 & 0 \\ B_{12} & B_{11} & B_{12} & 0 & 0 & 0 \\ B_{12} & B_{12} & B_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{B_{11}-B_{12}}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \dfrac{B_{11}-B_{12}}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{B_{11}-B_{12}}{2} \end{bmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{pmatrix} \tag{1.9}$$

1.2 Elastic Constants for Orthotropic Composite Materials

It is convenient using elastic constants (Young's modulus, Poisson's ratio and shear modulus) to express the compliance matrix $\boldsymbol{B}$ as follows:

$$
\boldsymbol{B} = \begin{bmatrix}
\frac{1}{E_1} & \frac{-\nu_{21}}{E_2} & \frac{-\nu_{31}}{E_3} & 0 & 0 & 0 \\
\frac{-\nu_{12}}{E_1} & \frac{1}{E_2} & \frac{-\nu_{32}}{E_3} & 0 & 0 & 0 \\
\frac{-\nu_{13}}{E_1} & \frac{-\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G_{23}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}}
\end{bmatrix} \tag{1.10}
$$

where E_i (i = 1,2,3) is Young's modulus in the i th elastic principal direction in the orthotropic materials (under the condition of $\sigma_i \neq 0, \sigma_j = 0$, $i \neq j$). ν_{ij} is Poisson's ratio and it is defined as $\nu_{ij} = -\frac{\varepsilon_j}{\varepsilon_i}$ (under condition of $\sigma_i \neq 0$, other stresses are zero). G_{ij} (i , j=1,2,3) is the shear modulus in the i - j plane.

Considering the symmetric property of $\boldsymbol{B}$ from equation (1.10) we have the reciprocal law for orthotropic materials:

$$
\frac{\nu_{ij}}{E_i} = \frac{\nu_{ji}}{E_j} \qquad (i, j = 1,2,3) \tag{1.11}
$$

Noting

$$
\boldsymbol{C}\,\boldsymbol{B} = \boldsymbol{I} \tag{1.12}
$$

The stiffness matrix $\boldsymbol{C}$ is easily derived from (1.10) as

$$
C_{11} = \frac{1-\nu_{23}\nu_{32}}{E_2 E_3 \Delta}, \quad C_{22} = \frac{1-\nu_{13}\nu_{31}}{E_1 E_3 \Delta}, \quad C_{33} = \frac{1-\nu_{12}\nu_{21}}{E_1 E_2 \Delta}
$$

$$
C_{12} = \frac{\nu_{12}+\nu_{13}\nu_{34}}{E_1 E_3 \Delta} = \frac{\nu_{21}+\nu_{23}\nu_{31}}{E_2 E_3 \Delta} = C_{21}
$$

$$
C_{13} = \frac{\nu_{13}+\nu_{12}\nu_{23}}{E_1 E_2 \Delta} = \frac{\nu_{31}+\nu_{32}\nu_{21}}{E_2 E_3 \Delta} = C_{31}
$$

$$C_{23} = \frac{\nu_{23} + \nu_{21}\nu_{13}}{E_1 E_2 \Delta} = \frac{\nu_{32} + \nu_{31}\nu_{12}}{E_1 E_3 \Delta} = C_{32}$$

$$C_{44} \doteq G_{23}\,,\quad C_{55} = G_{31}\,, C_{66} = G_{12}\,,$$

$$\Delta = \begin{vmatrix} \frac{1}{E_1} & \frac{-\nu_{21}}{E_2} & \frac{-\nu_{31}}{E_3} \\ \frac{-\nu_{12}}{E_1} & \frac{1}{E_2} & \frac{-\nu_{32}}{E_3} \\ \frac{-\nu_{13}}{E_1} & \frac{-\nu_{23}}{E_2} & \frac{1}{E_3} \end{vmatrix} \tag{1.13}$$

1.3 Limitation of Elastic Constants

For orthotropic materials

We know that it is always positive definite for the strain energy density function. Then the stiffness matrix $\boldsymbol{C}$ and the compliance matrix $\boldsymbol{B}$ are both positives definite. So we have [2]

$$\Delta = \frac{1 - \nu_{12}\nu_{21} - \nu_{23}\nu_{32} - \nu_{31}\nu_{13} - 2\nu_{21}\nu_{32}\nu_{13}}{E_1 E_2 E_3} > 0 \tag{1.14}$$

$$E_1 > 0\,, E_2 > 0\,, E_3 > 0 \tag{1.15}$$

$$1 - \nu_{23}\nu_{32} > 0\,,\quad 1 - \nu_{13}\nu_{31} > 0\,,\quad 1 - \nu_{12}\nu_{21} > 0 \tag{1.16}$$

$$|\nu_{21}| < \left(\frac{E_2}{E_1}\right)^{\frac{1}{2}},\quad |\nu_{12}| < \left(\frac{E_1}{E_2}\right)^{\frac{1}{2}}$$

$$|\nu_{32}| < \left(\frac{E_3}{E_2}\right)^{\frac{1}{2}},\quad |\nu_{23}| < \left(\frac{E_2}{E_3}\right)^{\frac{1}{2}}$$

$$|\nu_{13}| < \left(\frac{E_1}{E_3}\right)^{\frac{1}{2}},\quad |\nu_{31}| < \left(\frac{E_3}{E_1}\right)^{\frac{1}{2}} \tag{1.17}$$

By using equation (1.11) and (1.17), the following equation can be derived:

$$\nu_{21}\nu_{32}\nu_{13} < \frac{1}{2}\left(1 - \nu_{21}^2\frac{E_1}{E_2} - \nu_{32}^2\frac{E_2}{E_3} - \nu_{13}^2\frac{E_3}{E_1}\right) < \frac{1}{2} \tag{1.18}$$

It tells us that the product of three Poisson's ratio is less than 0.5.

For isotropic materials

Similarly it can be proved that the Poisson's Ratio ν may take the value within the following range

$$-1 < \nu < \frac{1}{2} \tag{1.19}$$

1.4 Stress and Strain Relations of Orthotropic Composite Plate

Coordinate axis parallels to elastic principal direction

As shown in figure 1.1 the coordinate axis is parallel to material elastic principal direction.

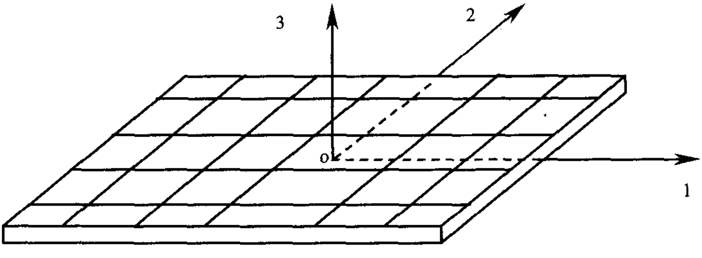

Figure 1.1 Coordinate system in orthotropic plate

The stress and strain relations for plane-stress state are as follows

$$\sigma_3 = \tau_{23} = \tau_{31} = 0 \tag{1.20}$$

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} = \begin{pmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{pmatrix} = \begin{pmatrix} \dfrac{1}{E_1} & \dfrac{-\nu_{21}}{E_2} & 0 \\ \dfrac{-\nu_{12}}{E_1} & \dfrac{1}{E_2} & 0 \\ 0 & 0 & \dfrac{1}{G_{12}} \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{pmatrix} \tag{1.21}$$

$$\varepsilon_3 = -\frac{\nu_{13}}{E_1}\sigma_1 - \frac{\nu_{23}}{E_2}\sigma_2 \tag{1.22}$$

$$\gamma_{23} = \gamma_{31} = 0 \tag{1.23}$$

From the equation (1.21) the in-plane stress strain relations are derived as:

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{pmatrix} = \begin{pmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{pmatrix} \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} = \boldsymbol{Q} \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} \tag{1.24}$$

where $\boldsymbol{Q}$ is in-plane stiffness matrix for orthotropic composite plate.

Actually $\boldsymbol{Q}$ is part of material stiffness matrix $\boldsymbol{C}$. The components of $\boldsymbol{Q}$ are as follows:

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}$$

$$Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} = \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}}$$

$$Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}$$

$$Q_{66} = G_{12} \tag{1.25}$$

Coordinate axis is inclined to elastic principal direction

As shown in figure 1.2 the coordinate axis is inclined to material elastic principal direction.