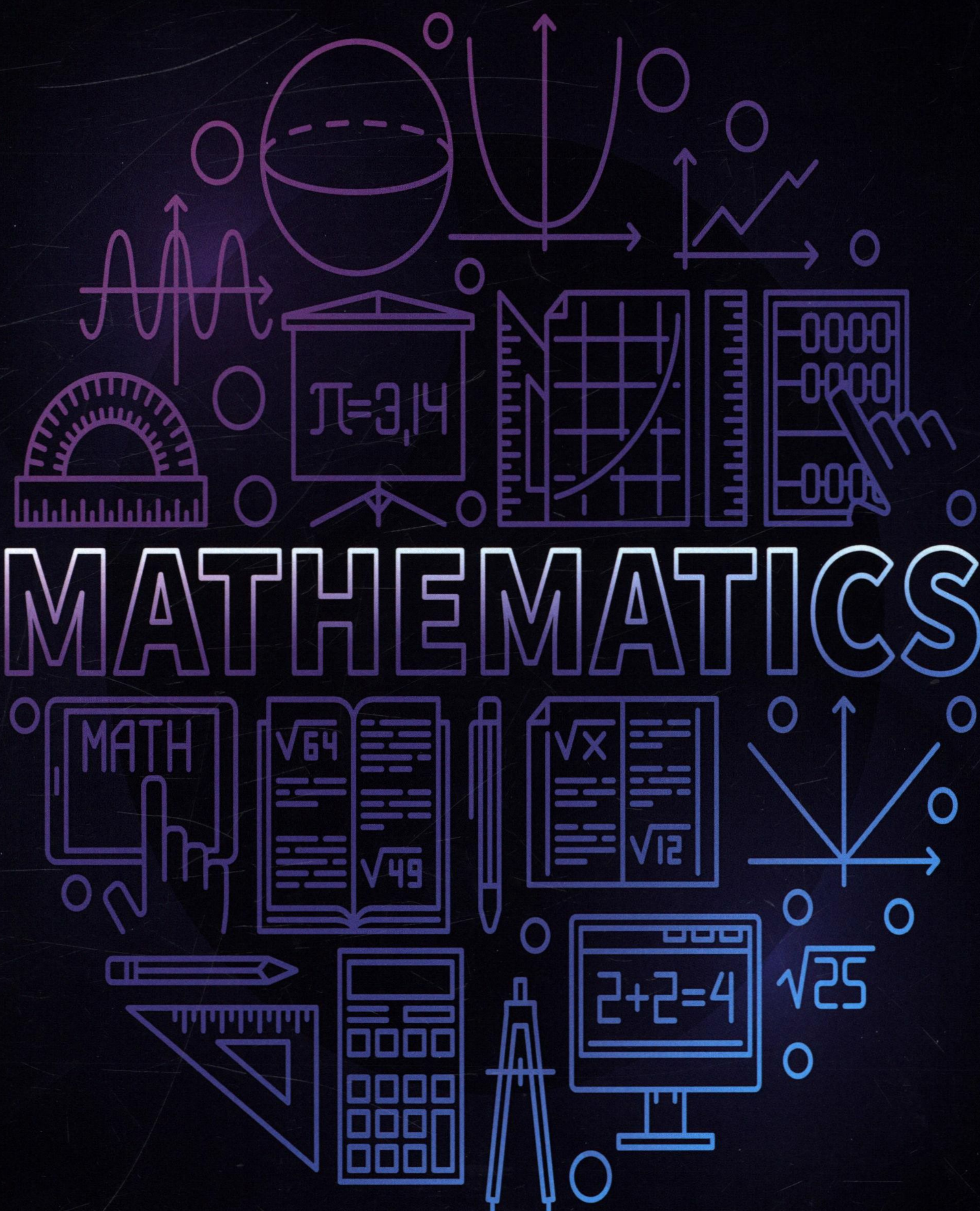
Linear Algebra: Theorems and Applications

Contributors: Francesco Aldo Costabile, Elisabetta Longo et al.



About the Book

Linear algebra is essential in analysis, applied math, and even in theoretical mathematics. Linear algebra is central to both pure and applied mathematics. For instance, abstract algebra arises by relaxing the axioms of a vector space, leading to a number of generalizations. Functional analysis studies the infinite-dimensional version of the theory of vector spaces. Combined with calculus, linear algebra facilitates the solution of linear systems of differential equations. Techniques from linear algebra are also used in analytic geometry, engineering, physics, natural sciences, computer science, computer animation, advanced facial recognition algorithms and the social sciences. Because linear algebra is such a well-developed theory, nonlinear mathematical models are sometimes approximated by linear models. Finally, linear algebra is important partly because it is the science of the only types of equations that we know how to solve easily. Fortunately, many non-linear situations can be studied using linear methods. The best-known example of this is calculus, which is entirely about understanding curvy functions and objects using straight, linear approximations to them. Like any good tool, linear algebra can be put to an incredible variety of uses.

Linear Algebra- Theorems and Applications contains selected topics in linear algebra, which represent the recent contributions in the most famous and widely problems. It includes a wide range of theorems and applications in different branches of linear algebra, such as linear systems, matrices, operators, inequalities, etc. However, all major topics are also presented in an alternative manner with an emphasis on nonstandard and neat proofs of known theorems. It contributes new insights to matrix theory and finite dimensional linear algebra in their algebraic, arithmetic, combinatorial, geometric, or numerical aspects. It also give significant applications of matrix theory or linear algebra to other branches of mathematics and to other sciences and provides perspectives on the historical development of matrix theory and linear algebra. It continues to be a definitive resource for researchers, scientists and graduate students.

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ISBN 978-1-78802-021-3
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9 781788 020213

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Published by Auris Reference Limited

www.aurisreference.com

United Kingdom

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ISBN: 978-1-78802-021-3

British Library Cataloguing in Publication Data
A CIP record for this book is available from the British Library

Printed in the United Kingdom

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Preface

Linear algebra is essential in analysis, applied math, and even in theoretical mathematics. Linear algebra is central to both pure and applied mathematics. For instance, abstract algebra arises by relaxing the axioms of a vector space, leading to a number of generalizations. Functional analysis studies the infinite-dimensional version of the theory of vector spaces. Combined with calculus, linear algebra facilitates the solution of linear systems of differential equations. Techniques from linear algebra are also used in analytic geometry, engineering, physics, natural sciences, computer science, computer animation, advanced facial recognition algorithms and the social sciences. Because linear algebra is such a well-developed theory, nonlinear mathematical models are sometimes approximated by linear models. Finally, linear algebra is important partly because it is the science of the only types of equations that we know how to solve easily. Fortunately, many non-linear situations can be studied using linear methods. The best-known example of this is calculus, which is entirely about understanding curvy functions and objects using straight, linear approximations to them. Like any good tool, linear algebra can be put to an incredible variety of uses.

Linear Algebra- Theorems and Applications contains selected topics in linear algebra, which represent the recent contributions in the most famous and widely problems. It includes a wide range of theorems and applications in different branches of linear algebra, such as linear systems, matrices, operators, inequalities, etc. However, all major topics are also presented in an alternative manner with an emphasis on nonstandard and neat proofs of known theorems. It contributes new insights to matrix theory and finite dimensional linear algebra in their algebraic, arithmetic, combinatorial, geometric, or numerical aspects. It also give significant applications of matrix theory or linear algebra to other branches of mathematics and to other sciences and provides perspectives on the historical development of matrix theory and linear algebra. It continues to be a definitive resource for researchers, scientists and graduate students.

Contents

1

ALGEBRAIC THEORY OF APPELL POLYNOMIALS WITH APPLICATION TO GENERAL LINEAR INTERPOLATION PROBLEM

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INTRODUCTION

In 1880 P. E. Appell ([1]) introduced and widely studied sequences of nn-degree polynomials

$$A_n(x), \quad n = 0, 1, \ldots \tag{1}$$

satisfying the differential relation

$$DA_n(x) = nA_{n-1}(x), \quad n = 1, 2, \ldots \tag{2}$$

Sequences of polynomials, verifying the (□), nowadays called Appell polynomials, have been well studied because of their remarkable applications not only in different branches of mathematics ([2], [3]) but also in theoretical physics and chemistry ([4], [5]). In 1936 an initial bibliography was provided by Davis (p. 25[6]). In 1939 Sheffer ([7]) introduced a new class of polynomials which extends the class of Appell polynomials; he called these polynomials of type zero, but nowadays they are called Sheffer polynomials. Sheffer also noticed the similarities between Appell polynomials and the umbral calculus, introduced in the second half of the 19th century with the work of such mathematicians as Sylvester, Cayley and Blissard (for examples, see [8]). The Sheffer theory is mainly based on formal power series. In 1941 Steffensen ([9]) published a theory on Sheffer polynomials based on formal power series too. However, these theories were not suitable as they did not provide sufficient computational tools. Afterwards Mullin, Roman and Rota ([10], [11], [12]), using operators method, gave a beautiful theory of umbral calculus, including Sheffer polynomials. Recently, Di Bucchianico and Loeb ([13]) summarized and documented more than five hundred old and new findings related to Appell polynomial sequences. In last years attention has centered on finding a novel representation of Appell polynomials. For instance, Lehemer ([14]) illustrated six different approaches to representing the sequence of Bernoulli polynomials, which is a special case of Appell polynomial sequences. Costabile ([15], [16]) also gave a new form of Bernoulli polynomials, called determinantal form, and later these ideas have been extended to Appell polynomial sequences. In fact, in 2010, Costabile and Longo ([17]) proposed an algebraic and elementary approach to Appell polynomial sequences. At the same time, Yang and Youn ([18]) also gave an algebraic approach, but with different methods. The approach to Appell polynomial sequences via linear algebra is an easily comprehensible mathematical tool, specially for non-specialists; that is very good because many polynomials arise in physics, chemistry and engineering. The present work concerns with these topics and it is organized as follows: in Section □ we mention the Appell method ([1]); in Section □ we provide the determinantal approach ([17]) and prove the equivalence with other definitions; in Section □ classical and non-classical examples are given; in Section □, by using elementary tools of linear algebra, general properties of Appell polynomials are provided; in Section □ we mention Appell polynomials of second kind ([19], [20]) and, in Section □ two classical examples are given; in Section □ we provide an application to general linear interpolation problem([21]), giving, in Section □, some examples; in Section □ the Yang and Youn approach ([18]) is sketched; finally, in Section □ conclusions close the work.

THE APPELL APPROACH

Let $\{A_n(x)\}_n$ be a sequence of n-degree polynomials satisfying the differential relation (□). Then we have

Remark 1 There is a one-to-one correspondence of the set of such sequences $\{A_n(x)\}_n$ and the set of numerical sequences $\{\alpha_n\}_n, \ \alpha_0 \neq 0$ given by the explicit representation

$$A_n(x) = \alpha_n + \binom{n}{1}\alpha_{n-1}x + \binom{n}{2}\alpha_{n-2}x^2 + \cdots + \alpha_0 x^n, \ n = 0, 1, \ldots \quad (3)$$

Equation (□), in particular, shows explicitly that for each $n \geq 1$ the polynomial $A_n(x)$ is completely determined by $A_{n-1}(x)$ and by the choice of the constant of integration αnαn.

Remark 2 Given the formal power series

$$a(h) = \alpha_0 + \frac{h}{1!}\alpha_1 + \frac{h^2}{2!}\alpha_2 + \cdots + \frac{h^n}{n!}\alpha_n + \cdots, \quad \alpha_0 \neq 0, \quad (4)$$

with αii=0,1,...αii=0,1,... real coefficients, the sequence of polynomials, $A_n(x)$, determined by the power series expansion of the product $a(h)e^{hx}$, i.e.

$$a(h)e^{hx} = A_0(x) + \frac{h}{1!}A_1(x) + \frac{h^2}{2!}A_2(x) + \cdots + \frac{h^n}{n!}A_n(x) + \cdots \quad (5)$$

satisfies (□).

The function a(h)ah is said, by Appell, ‹generating function› of the sequence $\{A_n(x)\}_n$.

Appell also noticed various examples of sequences of polynomials verifying (□).

He also considered ([1]) an application of these polynomial sequences to linear differential equations, which is out of this context.

THE DETERMINANTAL APPROACH

Let be $\beta_i \in \mathbb{R}, \ i = 0, 1, \ldots,$ with $\beta_0 \neq 0.$

We give the following

Definition 1 The polynomial sequence defined by

$$\begin{cases} A_0(x) = \frac{1}{\beta_0}, \\ A_n(x) = \frac{(-1)^n}{(\beta_0)^{n+1}} \begin{vmatrix} 1 & x & x^2 & \cdots & \cdots & x^{n-1} & x^n \\ \beta_0 & \beta_1 & \beta_2 & \cdots & \cdots & \beta_{n-1} & \beta_n \\ 0 & \beta_0 & \binom{2}{1}\beta_1 & \cdots & \cdots & \binom{n-1}{1}\beta_{n-2} & \binom{n}{1}\beta_{n-1} \\ 0 & 0 & \beta_0 & \cdots & \cdots & \binom{n-1}{2}\beta_{n-3} & \binom{n}{2}\beta_{n-2} \\ \vdots & & & \ddots & & \vdots & \vdots \\ \vdots & & & & \ddots & \vdots & \vdots \\ 0 & \cdots & \cdots & \cdots & 0 & \beta_0 & \binom{n}{n-1}\beta_1 \end{vmatrix}, \ n = 1, 2, \ldots \end{cases} \quad (6)$$

is called Appell polynomial sequence for .βi.

Then we have

Theorem 1 If $A_n(x)$ is the Appell polynomial sequence for βiβi the differential relation (□) holds.

Using the properties of linearity we can differentiate the determinant (□), expand the resulting determinant with respect to the first column and recognize the factor $A_{n-1}(x)$ after multiplication of the i-th row by $i-1, \ i = 2, \ldots, n$ and j-th column by $\frac{1}{j}, \ j = 1, \ldots, n.$

Theorem 2 If $A_n(x)$ is the Appell polynomial sequence for βi we have the equality (□) with

$$\alpha_0 = \frac{1}{\beta_0},$$

$$\alpha_i = \frac{(-1)^i}{(\beta_0)^{i+1}} \begin{vmatrix} \beta_1 & \beta_2 & \cdots & \cdots & \beta_{i-1} & \beta_i \\ \beta_0 & \binom{2}{1}\beta_1 & \cdots & \cdots & \binom{i-1}{1}\beta_{i-2} & \binom{i}{1}\beta_{i-1} \\ 0 & \beta_0 & \cdots & \cdots & \binom{i-1}{2}\beta_{i-3} & \binom{i}{2}\beta_{i-2} \\ \vdots & & \ddots & & \vdots & \vdots \\ \vdots & & & \ddots & \vdots & \vdots \\ 0 & \cdots & \cdots & 0 & \beta_0 & \binom{i}{i-1}\beta_1 \end{vmatrix} =$$

$$= -\frac{1}{\beta_0}\sum_{k=0}^{i-1}\binom{i}{k}\beta_{i-k}\alpha_k, \qquad i = 1, 2, \ldots, n.$$

(7)

From (□), by expanding the determinant $A_n(x)$ with respect to the first row, we obtain the (□) with αi given by (□) and the determinantal form in (□); this is a determinant of an upper Hessenberg matrix of order i ([16]), then setting $\overline{\alpha}_i = (-1)^i(\beta_0)^{i+1}\alpha_i$ for $i = 1, 2, \ldots, n$, we have

$$\overline{\alpha}_i = \sum_{k=0}^{i-1}(-1)^{i-k-1}h_{k+1,i}q_k(i)\overline{\alpha}_k,$$

(8)

where:

$$h_{l,m} = \begin{cases} \beta_m & \text{for } l = 1, \\ \binom{m}{l-1}\beta_{m-l+1} & \text{for } 1 < l \leq m+1, \quad l, m = 1, 2, \ldots, i, \\ 0 & \text{for } l > m+1, \end{cases}$$

(9)

$$q_k(i) = \prod_{j=k+2}^{i} h_{j,j-1} = (\beta_0)^{i-k-1}, \quad k = 0, 1, \ldots, i-2,$$

$$q_{i-1}(i) = 1.$$

(10)

By virtue of the previous setting, (□) implies

$$\overline{\alpha}_i = \sum_{k=0}^{i-2}(-1)^{i-k-1}\binom{i}{k}\beta_{i-k}(\beta_0)^{i-k-1}\overline{\alpha}_k + \binom{i}{i-1}\beta_1\overline{\alpha}_{i-1} =$$

$$= (-1)^i(\beta_0)^{i+1}\left(-\frac{1}{\beta_0}\sum_{k=0}^{i-1}\binom{i}{k}\beta_{i-k}\alpha_k\right),$$

(11)

and the proof is concluded.

Remark 3 We note that (□) and (□) are equivalent to

$$\sum_{k=0}^{i}\binom{i}{k}\beta_{i-k}\alpha_k = \begin{cases} 1 & i = 0 \\ 0 & i > 0 \end{cases}$$

(12)

and that for each sequence of Appell polynomials there exist two sequences of numbers αiαi and βi related by (□).

Corollary 1 If $A_n x$ is the Appell polynomial sequence for βi we have

$$A_n(x) = \sum_{j=0}^{n}\binom{n}{j}A_{n-j}(0)x^j, \quad n = 0, 1, \ldots$$

(13)

Follows from Theorem □ being

$$A_i(0) = \alpha_i, \quad i = 0, 1, \ldots, n. \tag{14}$$

Remark 4 For computation we can observe that αnαn is a_n-order determinant of a particular upper Hessenberg form and it's known that the algorithm of Gaussian elimination without pivoting for computing the determinant of an upper Hessenberg matrix is stable (p. 27[22]).

Theorem 3 If a(h) is the function defined in (□) and $A_n x$ is the polynomial sequence defined by (□), setting

$$\begin{cases} \beta_0 = \dfrac{1}{\alpha_0}, \\ \beta_n = -\dfrac{1}{\alpha_0}\left(\displaystyle\sum_{k=1}^{n}\binom{n}{k}\alpha_k\beta_{n-k}\right), \quad n = 1, 2, \ldots, \end{cases} \tag{15}$$

we have that $A_n(x)$ satisfies the (□), i.e. $A_n(x)$ is the Appell polynomial sequence for βi.

Let be

$$b(h) = \beta_0 + \frac{h}{1!}\beta_1 + \frac{h^2}{2!}\beta_2 + \cdots + \frac{h^n}{n!}\beta_n + \cdots \tag{16}$$

with βn as in (□). Then we have $a(h)b(h) = 1$, where the product is intended in the Cauchy sense, i.e.:

$$a(h)b(h) = \sum_{n=0}^{\infty}\sum_{k=0}^{n}\binom{n}{k}\alpha_k\beta_{n-k}\frac{h^n}{n!}. \tag{17}$$

Let us multiply both hand sides of equation

$$a(h)e^{hx} = \sum_{n=0}^{\infty}A_n(x)\frac{h^n}{n!} \tag{19}$$

for $\frac{1}{a(h)}$ and, in the same equation, replace functions e^{hx} and $\frac{1}{a(h)}$ by their Taylor series expansion at the origin; then (□) becomes

$$\sum_{n=0}^{\infty}\frac{x^n h^n}{n!} = \sum_{n=0}^{\infty}A_n(x)\frac{h^n}{n!}\sum_{n=0}^{\infty}\frac{h^n}{n!}\beta_n. \tag{20}$$

By multiplying the series on the left hand side of (□) according to the Cauchy-product rules, previous equality leads to the following system of infinite equations in the unknown $A_n(x), \ n = 0, 1, \ldots$

$$\begin{cases} A_0(x)\beta_0 = 1, \\ A_0(x)\beta_1 + A_1(x)\beta_0 = x, \\ A_0(x)\beta_2 + \binom{2}{1}A_1(x)\beta_1 + A_2(x)\beta_0 = x^2, \\ \vdots \\ A_0(x)\beta_n + \binom{n}{1}A_1(x)\beta_{n-1} + \ldots + A_n(x)\beta_0 = x^n, \\ \vdots \end{cases} \tag{21}$$

From the first one of (□) we obtain the first one of (□). Moreover, the special form of the previous system (lower triangular) allows us to work out the unknown $A_n x$ operating with the first n+1 equations, only by applying the Cramer rule:

$$A_n(x) = \frac{1}{(\beta_0)^{n+1}}\begin{vmatrix} \beta_0 & 0 & 0 & \cdots & 0 & 1 \\ \beta_1 & \beta_0 & 0 & \cdots & 0 & x \\ \beta_2 & \binom{2}{1}\beta_1 & \beta_0 & \cdots & 0 & x^2 \\ \vdots & & & \ddots & & \vdots \\ \beta_{n-1} & \binom{n-1}{1}\beta_{n-2} & \cdots & \cdots & \beta_0 & x^{n-1} \\ \beta_n & \binom{n}{1}\beta_{n-1} & \cdots & \cdots & \binom{n}{n-1}\beta_1 & x^n \end{vmatrix}. \tag{22}$$

By transposition of the previous, we have