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国外物理名著系列 23

（影印版）

# New Foundations for Classical Mechanics

(2nd Edition)

# 经典力学新基础

（第二版）

D.Hestenes

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# 国外物理名著系列序言

对于国内的物理学工作者和青年学生来讲，研读国外优秀的物理学著作是系统掌握物理学知识的一个重要手段。但是，在国内并不能及时、方便地买到国外的图书，且国外图书不菲的价格往往令国内的读者却步，因此，把国外的优秀物理原著引进到国内，让国内的读者能够方便地以较低的价格购买是一项意义深远的工作，将有助于国内物理学工作者和青年学生掌握国际物理学的前沿知识，进而推动我国物理学科研和教学的发展。

为了满足国内读者对国外优秀物理学著作的需求，科学出版社启动了引进国外优秀著作的工作，出版社的这一举措得到了国内物理学界的积极响应和支持，很快成立了专家委员会，开展了选题的推荐和筛选工作，在出版社初选的书单基础上确定了第一批引进的项目，这些图书几乎涉及了近代物理学的所有领域，既有阐述学科基本理论的经典名著，也有反映某一学科专题前沿的专著。在选择图书时，专家委员会遵循了以下原则：基础理论方面的图书强调“经典”，选择了那些经得起时间检验、对物理学的发展产生重要影响、现在还不“过时”的著作（如狄拉克的《量子力学原理》）。反映物理学某一领域进展的著作强调“前沿”和“热点”，根据国内物理学研究发展的实际情况，选择了能够体现相关学科最新进展，对有关方向的科研人员和研究生有重要参考价值的图书。这些图书都是最新版的，多数图书都是 2000 年以后出版的，还有相当一部分是当年出版的新书。因此，这套丛书具有权威性、前瞻性和应用性强的特点。由于国外出版社的要求，科学出版社对部分图书进行了少量的翻译和注释（主要是目录标题和练习题），但这并不会影响图书“原汁原味”的感觉，可能还会方便国内读者的阅读和理解。

“他山之石，可以攻玉”，希望这套丛书的出版能够为国内物理学工作者和青年学生的工作和学习提供参考，也希望国内更多专家参与到这一工作中来，推荐更多的好书。

杨国桢

中国科学院院士

中国物理学会理事长

# Preface to the First Edition (revised)

This is a textbook on classical mechanics at the intermediate level, but its main purpose is to serve as an introduction to a new mathematical language for physics called *geometric algebra*. Mechanics is most commonly formulated today in terms of the vector algebra developed by the American physicist J. Willard Gibbs, but for some applications of mechanics the algebra of complex numbers is more efficient than vector algebra, while in other applications matrix algebra works better. Geometric algebra integrates all these algebraic systems into a coherent mathematical language which not only retains the advantages of each special algebra but possesses powerful new capabilities.

This book covers the fairly standard material for a course on the mechanics of particles and rigid bodies. However, it will be seen that geometric algebra brings new insights into the treatment of nearly every topic and produces simplifications that move the subject quickly to advanced levels. That has made it possible in this book to carry the treatment of two major topics in mechanics well beyond the level of other textbooks. A few words are in order about the unique treatment of these two topics, namely, rotational dynamics and celestial mechanics.

The spinor theory of rotations and rotational dynamics developed in this book cannot be formulated without geometric algebra, so a comparable treatment is not to be found in any other book at this time. The relation of the spinor theory to the matrix theory of rotations developed in conventional textbooks is completely worked out, so one can readily translate from one to the other. However, the spinor theory is so superior that the matrix theory is hardly needed except to translate from books that use it. In the first place, calculations with spinors are demonstrably more efficient than calculations with matrices. This has practical as well as theoretical importance. For example, the control of artificial satellites requires continual rotational computations that soon number in the millions. In the second place, spinors are essential in advanced quantum mechanics. So the utilization of spinors in the classical theory narrows the gap between the mathematical formulations of classical and quantum mechanics, making it possible for students to proceed more rapidly to advanced topics.

Celestial mechanics, along with its modern relative astromechanics, is essential for understanding space flight and the dynamics of the solar system. Thus, it is essential knowledge for the informed physicist of the space age. Yet celestial mechanics is scarcely mentioned in the typical undergraduate physics curriculum. One reason for this neglect is the belief that the subject is too advanced, requiring a complex formulation in terms of Hamilton-Jacobi theory. However, this book uses geometric algebra to develop a new formulation of perturbation theory in celestial mechanics which is well within the reach of undergraduates. The major gravitational perturbations in the solar system are discussed to bring students up to date in space age mechanics. The new mathematical techniques developed in this book should be of interest to anyone concerned with the mechanics of space flight.

To provide an introduction to geometric algebra suitable for the entire physics curriculum, the mathematics developed in this book exceeds what is strictly necessary for a mechanics course, including a substantial treatment of linear algebra and transformation groups with the techniques of geometric algebra. Since linear algebra and group theory are standard tools in modern physics, it is important for students to become familiar with them as soon as possible. There are good reasons for integrating instruction in mathematics and physics. It assures that the mathematical background will be sufficient for the needs of physics, and the physics provides nontrivial applications of the mathematics as it develops. But most important, it affords an opportunity to teach students that the design and development of an efficient mathematical language for representing physical facts and concepts is the business of theoretical physics. That is one of the objectives of this book.

There are plans to extend the geometric algebra developed here to a series of books on electrodynamics, relativity and quantum theory, in short, to provide a unified language for the whole of physics. This is a long term project, likely to be drawn out, in part because there is so much current activity in the research literature on geometric algebra.

I am happy to report that others have joined me in this enterprise. A current list of books on geometric algebra and its applications is given in the References section at the back of this book. That list can be expected to grow substantially in coming years.

The making of this book turned out to be much more difficult than I had anticipated, and could not have been completed without help from many sources. I am indebted to my NASA colleagues for educating me on the vicissitudes of celestial mechanics; in particular, Phil Roberts on orbital mechanics, Neal Hulkower on the three body problem, and, especially, Leon Blitzer for permission to draw freely on his lectures. I am indebted to Patrick Reany, Anthony Delugt and John Bergman for improving the accuracy of the text, and to Carmen Mendez and Denise Jackson for their skill and patience in typing a difficult manuscript. Most of all I am indebted to my wife Nancy for her unflagging support and meticulous care in preparing every one of the diagrams. Numerous corrections have been made in the several reprintings.

DAVID HESTENES

# Preface to the Second Edition

The second edition has been expanded by nearly a hundred pages on relativistic mechanics. The treatment is unique in its exclusive use of geometric algebra and its detailed treatment of spacetime maps, collisions, motion in uniform fields and relativistic spin precession. It conforms with Einstein's view that Special Relativity is the culmination of developments in classical mechanics.

The accuracy of the text has been improved by the accumulation of many corrections over the last decade. I am grateful to the many students and colleagues who have helped root out errors, as well as the invaluable assistance of Patrick Reany in preparing the manuscript. The second edition, in particular, has benefited from careful scrutiny by J. L. Jones and Prof. J. Vrbik. The most significant corrections are to the perturbation calculations in Chapter 8. Prof. Vrbik located the error in my calculation of the precession of the moon's orbit due to perturbation by the sun (p. 550), a calculation which vexed Newton and many others since. I am indebted to David Drewer for calling my attention to D.T. Whiteside's fascinating account of Newton's failure to master the lunar perigee calculation (see Section 8-3). Vrbik has kindly contributed a more accurate computation to this edition. He has also extended the spinor perturbation theory of Section 8-4 in a series of published applications to celestial mechanics (see References). Unfortunately, to make room for the long relativity chapter, the chapter on *Foundations of Mechanics* had to be dropped from the Second Edition. It will be worth expanding at another time. Indeed, it has already been incorporated in a new appraoch to physics instruction centered on making and using conceptual models. [For an update on Modeling Theory, see D. Hestenes, "Modeling Games in the Newtonian World," *Am. J. Phys.* **60**, 732–748 (1992).]

When using this book as a mechanics textbook, it is important to move quickly through Chapters 1 and 2 to the applications in Chapter 3. A thorough study of the topics and problems in Chapter 2 could easily take the better part of a semester, so that chapter should be used mainly for reference in a mechanics course. To facilitate identification of those elements of geometric algebra which are most essential to applications, a *Synopsis of Geometric Algebra* has been included in the beginning of this edition.

# Synopsis of Geometric Algebra

Generally useful relations and formulas for the geometric algebra $\mathcal{G}_3$ of Euclidean 3-space are listed here. Detailed explanations and further results are given in Chapter 2.

For vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, ..., and scalars $\alpha$, $\beta$, ..., the Euclidean geometric algebra for any dimension has the following properties

| | | |
|---|---|---|
| **associativity:** | $\mathbf{a}(\mathbf{bc}) = (\mathbf{ab})\mathbf{c}$ | $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$ |
| **commutivity:** | $\alpha\mathbf{b} = \mathbf{b}\alpha$ | $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$ |
| **distributivity:** | $\mathbf{a}(\mathbf{b} + \mathbf{c}) = \mathbf{ab} + \mathbf{ac}$ | $(\mathbf{b} + \mathbf{c})\mathbf{a} = \mathbf{ba} + \mathbf{ca}$ |
| **linearity:** | $\alpha(\mathbf{b} + \mathbf{c}) = \alpha\mathbf{b} + \alpha\mathbf{c} = (\mathbf{b} + \mathbf{c})\alpha$ | |
| **contraction:** | $\mathbf{a}^2 = \mathbf{aa} = \lvert\mathbf{a}\rvert^2$ | |

The *geometric product* $\mathbf{ab}$ is related to the *inner product* $\mathbf{a}\cdot\mathbf{b}$ and the *outer product* $\mathbf{a}\wedge\mathbf{b}$ by

$$\mathbf{ab} = \mathbf{a}\cdot\mathbf{b} + \mathbf{a}\wedge\mathbf{b} = \mathbf{b}\cdot\mathbf{a} - \mathbf{b}\wedge\mathbf{a} = 2\mathbf{a}\cdot\mathbf{b} - \mathbf{ba}.$$

For any *multivectors* $A$, $B$, $C$, ..., the scalar part of their geometric product satisfies

$$\langle AB\rangle_0 = \langle BA\rangle_0 .$$

Selectors without a grade subscript select for the scalar part, so that

$$\langle\ldots\rangle \equiv \langle\ldots\rangle_0 .$$

*Reversion* satisfies

$$(AB)^\dagger = B^\dagger A^\dagger , \qquad \mathbf{a}^\dagger = \mathbf{a} , \qquad \langle A^\dagger\rangle_0 = \langle A\rangle_0^\dagger = \langle A\rangle_0 .$$

The unit *righthanded pseudoscalar* $i$ satisfies

$$i^2 = -1 , \qquad \mathbf{a}i = i\mathbf{a} = \mathbf{a}\cdot i .$$

The vector *cross product* $\mathbf{a}\times\mathbf{b}$ is implicitly defined by

$$\mathbf{a}\wedge\mathbf{b} = i(\mathbf{a}\times\mathbf{b}) = i\mathbf{a}\times\mathbf{b} .$$

Inner and outer products are related by the *duality relations*

$$\mathbf{a}\wedge(i\mathbf{b}) = (\mathbf{a}\cdot\mathbf{b})i , \qquad \mathbf{a}\cdot(i\mathbf{b}) = (\mathbf{a}\wedge\mathbf{b})i = \mathbf{b}\times\mathbf{a} .$$

Every multivector $A$ can be expressed uniquely in the *expanded form*

$$A = \alpha + \mathbf{a} + i\mathbf{b} + i\beta = \textstyle\sum_{k=0}^{3} \langle A \rangle_k ,$$

where the $k$-vector parts are

$$\langle A \rangle_0 = \alpha , \qquad \langle A \rangle_1 = \mathbf{a} , \qquad \langle A \rangle_2 = i\mathbf{b} , \qquad \langle A \rangle_3 = i\beta .$$

The even part is a *quaternion* of the form

$$\langle A \rangle_+ = \alpha + i\mathbf{b} .$$

The *conjugate* $\widetilde{A}$ of $A$ is defined by

$$\widetilde{A} = \langle A^\dagger \rangle_+ - \langle A^\dagger \rangle_- = \alpha - \mathbf{a} - i\mathbf{b} + i\beta .$$

**Algebraic Identities:**

$$\mathbf{a} \cdot (\mathbf{b} \wedge \mathbf{c}) = (\mathbf{a} \cdot \mathbf{b})\mathbf{c} - (\mathbf{a} \cdot \mathbf{c})\mathbf{b} = \mathbf{a} \cdot \mathbf{b}\mathbf{c} - \mathbf{a} \cdot \mathbf{c}\mathbf{b} = (\mathbf{b} \times \mathbf{c}) \times \mathbf{a} ,$$

$$\mathbf{a} \wedge \mathbf{b} \wedge \mathbf{c} = i[\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})] ,$$

$$(\mathbf{a} \wedge \mathbf{b} \wedge \mathbf{c}) \cdot \mathbf{d} = (\mathbf{a} \wedge \mathbf{b})\mathbf{c} \cdot \mathbf{d} - (\mathbf{a} \wedge \mathbf{c})\mathbf{b} \cdot \mathbf{d} + (\mathbf{b} \wedge \mathbf{c})\mathbf{a} \cdot \mathbf{d} ,$$

$$\mathbf{a} \wedge \mathbf{b} \wedge \mathbf{c} \wedge \mathbf{d} = 0 .$$

For further identites, see Exercise (4.8) on page 71.

**Exponential and Trigonometric Functions:**

$$e^{i\mathbf{a}} = \cos \mathbf{a} + i \sin \mathbf{a} = \cos |\mathbf{a}| + i\hat{\mathbf{a}} \sin |\mathbf{a}| .$$

See pages 73, 282 and 661 for more.

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Chapter 1

# Origins of Geometric Algebra

There is a tendency among physicists to take mathematics for granted, to regard the development of mathematics as the business of mathematicians. However, history shows that most mathematics of use in physics has origins in successful attacks on physical problems. The advance of physics has gone hand in hand with the development of a mathematical language to express and exploit the theory. Mathematics today is an immense and imposing subject, but there is no reason to suppose that the evolution of a mathematical language for physics is complete. The task of improving the language of physics requires intimate knowledge of how the language is to be used and how it refers to the physical world, so it involves more than mathematics. It is one of the fundamental tasks of theoretical physics.

This chapter sketches some historical high points in the evolution of geometric algebra, the mathematical language developed and applied in this book. It is not supposed to be a balanced historical account. Rather, the aim is to identify explicit principles for constructing symbolic representations of geometrical relations. Then we can see how to *design* a compact and efficient geometrical language tailored to meet the needs of theoretical physics.

## 1-1. Geometry as Physics

Euclid's systematic formulation of Greek geometry (in 300 BC) was the first comprehensive theory of the physical world. Earlier attempts to describe the physical world were hardly more than a jumble of facts and speculations. But Euclid showed that from a mere handful of simple assumptions about the nature of physical objects a great variety of remarkable relations can be deduced. So incisive were the insights of Greek geometry that it provided a foundation for all subsequent advances in physics. Over the years it has been extended and reformulated but not changed in any fundamental way.

The next comparable advance in theoretical physics was not consummated until the publication of Isaac Newton's *Principia* in 1687. Newton was fully aware that geometry is an indispensible component of physics; asserting,

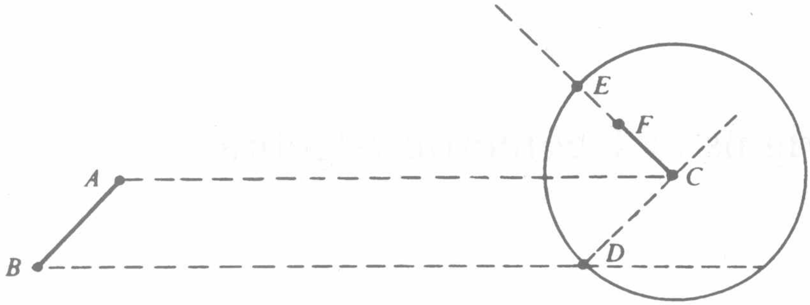


Fig. 1.1. Congruence of Line Segments.

Euclid's axioms provide rules which enable one to compare any pair of line segments. Segments $AB$ and $CF$ can be compared as follows.

First, a line parallel to $AB$ can be drawn through $C$. And a line parallel to $AC$ can be drawn through $B$. The two lines intersect at a point $D$. The line segment $CD$ is *congruent* to $AB$.

Second, a circle with center $C$ can be drawn through $D$. It intersects the line CF at a point $E$. The segment $CE$ is congruent to $CD$ and, by the assumed transitivity of the relation, congruent to $AB$.

Third, the point $F$ is either inside, on, or outside the circle, in which cases we say that the *magnitude* of $CF$ is respectively, less than, equal to, or greater than the magnitude of $AB$.

The procedure just outlined can, of course, be more precisely characterized by a formal deductive argument. But the point to be made here is that this procedure can be regarded as a theoretical formulation of basic physical operations involved in measurement.

If $AB$ is regarded as the idealization of a standard stick called a "ruler", the first step above may be regarded as a description of the *translation* of the stick to the place $CD$ without changing its magnitude. Then the second step idealizes the reorientation of the ruler to place it contiguous to an idealized body $CF$ so that a comparison (third step) can be made. Further assumptions are needed to supply the ruler with a "graduated scale" and so assign a unique magnitude to $CF$.

". . . the description of right lines and circles, upon which geometry is founded, belongs to mechanics. Geometry does not teach us to draw these lines, but requires them to be drawn . . . To describe right lines and circles are problems, but not geometrical problems. The solution of these problems is required from mechanics and by geometry the use of them, when so solved, is shown; and it is the glory of geometry that from those few principles, brought from without, it is able to produce so many things. *Therefore geometry is founded in mechanical practice, and is nothing but that part of universal mechanics which accurately proposes and demonstrates the art of measuring* . . ." (italics added)

As Newton avers, geometry is the theory on which the practice of measurement is based. Geometrical figures can be regarded as idealizations of physical bodies. The theory of congruent figures is the central theme of geometry, and it provides a theoretical basis for measurement when it is regarded as an idealized description of the physical operations involved in classifying physical bodies according to size and shape (Figure 1.1). To put it

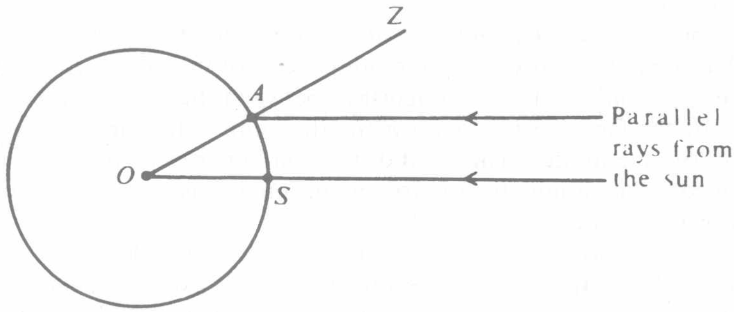


Fig. 1.2. Measurement of the Earth.

The most accurate of the early measurements of the earth's circumference was made by Eratosthenes (~200 BC). He observed that at noon on the day of the summer solstice the sun shone directly down a deep well at Syene. At the same time at Alexandria, taken to be due north and 5000 stadia (≈500 miles) away, the sun cast a shadow indicating it was 1/50 of a circle from zenith. By the equality of corresponding angles in the diagram this gives $50 \times 500 = 25\,000$ miles for the circumference of the earth.

another way, the theory of congruence specifies a set of rules to be used for classifying bodies. Apart from such rules the notions of size and shape have no meaning.

Greek geometry was certainly not developed with the problem of measurement in mind. Indeed, even the idea of measurement could not be conceived until geometry had been created. But already in Euclid's day the Greeks had carried out an impressive series of applications of geometry, especially to optics and astronomy (Figure 1.2), and this established a pattern to be followed in the subsequent development of trigonometry and the practical art of measurement. With these efforts the notion of an experimental science began to take shape.

Today, "to measure" means to assign a number. But it was not always so. Euclid sharply distinguished "number" from "magnitude". He associated the notion of number strictly with the operation of counting, so he recognized only integers as numbers; even the notion of fractions as numbers had not yet been invented. For Euclid a magnitude was a line segment. He frequently represented a whole number $n$ by a line segment which is $n$ times as long as some other line segment chosen to represent the number 1. But he knew that the opposite procedure is impossible, namely, that it is impossible to distinguish all line segments of different length by labeling them with numerals representing the counting numbers. He was able to prove this by showing the