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GAUGE INVARIANCE, PATH-DEPENDENCE AND WILSON LINES

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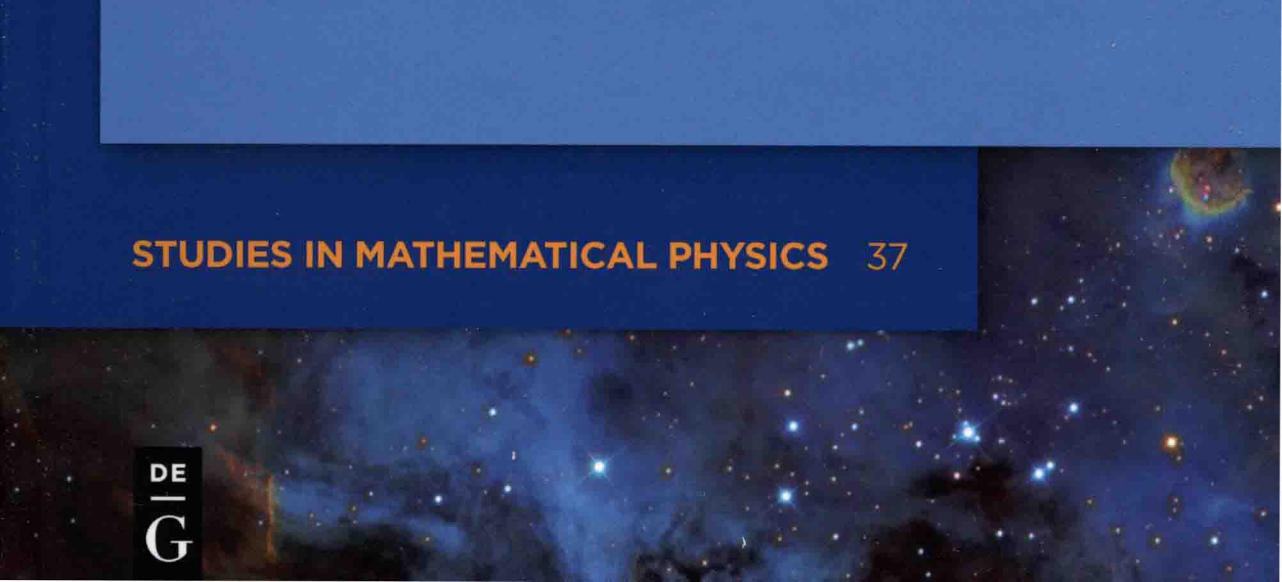

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Preface

The application of quantum chromodynamics (QCD) in practice implies computation of various hadronic and vacuum gauge-invariant path-dependent matrix elements that often contain complicated systems of Wilson lines and loops. The latter may include light-like segments, (semi-)infinite parts, simple obstructions such as cusps, that is the points where the path itself is continuous, while the derivative is not, and self-intersections, which make a path nonplanar. The purpose of this book is to give a systematic pedagogical introduction into the quantum field theory approach to quantitative analysis of Wilson path-ordered exponentials in QCD and its applications of this formalism to the study of gauge-invariant quark and gluon correlation functions, which can be associated with the three-dimensional transverse momentum-dependent parton density functions, commonly known nowadays as *TMD pdfs* or simply *TMDs*.

The strong interest in TMD pdfs is due, in the first place, to the rapid theoretical and experimental development and impressive recent results achieved in the study of the three–dimensional structure of the nucleon, which suggests that not only the longitudinal fraction of the struck parton momenta, normally associated with the Bjorken-x variable, but also the two transversal components $\boldsymbol{k}_\perp = (k_x, k_y)$ are taken into account. A new era in the investigation of the quark and gluon contents of nucleons has been launched in the research programmes dealing with high-energy semi-inclusive reactions with polarized and unpolarized hadrons, where the transverse motion and the spin–orbit correlations of the partons are directly accessible. Understanding the partonic structure of nucleons beyond the collinear approximation calls for an appropriate development of the theory. In classical *inclusive* processes, such as deep-inelastic ep-scattering (DIS) or electron–positron annihilation to hadrons, where no more than one hadron is identified in the initial state, the so-called *collinear QCD factorization* approach is applicable. The latter suggests that the longitudinal (parallel to a large light-like momentum in a suitable system) momenta of the patrons are intrinsic (non-perturbative), while their transverse momenta can be created by perturbative radiation effects (parton showers). In *less inclusive* processes, such as the Drell–Yan lepton pair production, semi-inclusive DIS, hadron–hadron annihilation to jets, Higgs and heavy-flavor production, where two or more hadrons in the initial or final state are detected, one is tempted to go beyond the collinear approximation. The reason is that now the momenta of the particles participating and detected in the process entail a nonplanar kinematical setting, which makes it natural to keep not only the collinear but also the transverse components of the parton momenta unintegrated. The so-called *transverse momentum dependent* factorization framework is believed to be a promising tool in these situations. It is expected to provide a unifying QCD-based framework with both mechanisms of the transverse momentum creation taken into account, that is intrinsic (essentially non-perturbative) as well as the perturbative radiation in parton showers.

From the phenomenological prospective, observation of large *single-spin asymmetries* in experiments with polarized hadrons certainly demands the development of relevant theoretical tools that must include the non-perturbative intrinsic transverse momenta of the partons. The use of TMD parton densities as non-perturbative input provides such a framework because they contain explicit correlations between partonic transverse momenta and the orbital momenta and spins of the nucleons. Moreover, the TMD factorization approach is considered to be a promising tool for the QCD study of some *unpolarized high-energy processes* in the specific regimes. Namely, the Drell–Yan vector boson production for low-q_T and the high-energy hadronic collisions with fixed momentum transfer at small Bjorken-x, where the gluon longitudinal momentum fractions become small, while the transverse momentum components dominate, give us examples of the TMD-related regimes accessible at the Large Hadron Collider (LHC). The TMD approach is also applicable to unpolarized processes with sensitivity to polarized gluon distributions as well as the Higgs, jet and heavy flavor production processes at the LHC.

Among other currently operating and planned facilities with the most promising TMD-related experimental programmes we name the following ones:
- Relativistic Heavy Ion Collider (RHIC in Brookhaven National Laboratory) hosts various reactions with polarized protons and nuclei.
- One-third of the already approved experiments for the 12 GeV Upgrade of the Thomas Jefferson National Accelerator Facility (JLab) are devoted to the investigation of three-dimensional structure of the nucleon with strong TMD-related programme.
- Planned Electron-Ion Collider (EIC) is designed as a high luminosity machine with particularly interesting TMD experiments with polarized hadrons.

Technically, our text is meant to be a continuation of our previous monograph:
- I.O. Cherednikov, T. Mertens and F.F. Van der Veken: *Wilson lines in quantum field theory"*, De Gruyter, Berlin (2014)

where we are mostly concerned about the mathematical foundations of Wilson loops and geometrical properties of the generalized loop space. In the present book, we start with identifying and explaining the most important concepts and ideas that we shall use in the main body of the text and then develop ab initio calculation techniques applicable to generic piecewise-linear Wilson lines. We present also the practical tools for its implementation. Emphasis is put on the issues of gauge-invariance of nonlocal path-dependent QCD correlation functions with different Wilson lines keeping in mind their connection with the geometrical properties of generalized loop space. The present volume can be used as a primer and an introductory text to the advanced

expositions presented in the following fundamental books, which deal partially with similar topics:
- R. Gambini and J. Pullin: *"Loops, knots, gauge theories and quantum gravity"*, Cambridge (1996)
- Y. Makeenko: *"Methods of contemporary gauge theory"*, Cambridge (2002)
- J. Collins: *"Foundations of perturbative QCD"*, Cambridge (2011)

The following key topics will be in the center of our exposition:
- Integrated and unintegrated (transverse momentum-dependent) parton density functions
- Normal-, time- and path-ordering and -ordered exponentials in quantum mechanics and quantum field theory
- Abelian and non-Abelian Wilson lines and loops
- QCD factorization in inclusive and semi-inclusive hadronic processes
- Path dependence and its consequences in the practical calculations of gauge-invariant correlation functions with Wilson lines

Antwerp, December 2016
I.O. Cherednikov
F.F. Van der Veken

Contents

1 Introduction

The purpose of the first two chapters of our book is to identify and explain the most important concepts, which we shall need to study gauge-invariant path-dependent quantum correlation functions with Wilson lines. They provide, therefore, an extended technical introduction to the forthcoming material.

1.1 Main Properties of QCD

Quantum chromodynamics (QCD) is the quantum field theory of strong interaction. The QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{quark}} + \mathcal{L}_{\text{gluon}} + \mathcal{L}_{\text{quark-gluon}} \tag{1.1}$$

contains the terms describing kinematics of quark and gluon fields, separately, and their interaction and self-interaction (for gluons). The most efficient approach to deal with a quantum field theory is to consider interactions as "weak" (in certain sense) perturbations and to use the Feynman diagram techniques to evaluate matrix elements, which can be associated with measurable quantities.

For the theory of electrons and photons, quantum electrodynamics (QED), this approach works more or less straightforwardly. In contrast to QED, QCD possesses some remarkable properties, which make the direct application of this methodology impossible.

- *Confinement:* The QCD Lagrangian is formulated in terms of the quark (fermion) and gluon (boson) fields, which are considered then as the fundamental degrees of freedom of the strong interaction. However, these objects do not exist as physical particles. At the same time, there are no QCD fields for the physical strongly interacting particles – hadrons. Therefore, the QCD matrix elements should be calculated for the physical (hadronic) states, while the operators correspond to the quark and gluon fields.
- *Running coupling:* After renormalization, the QCD coupling α_s starts running, that is it changes with characteristic energy scale. The same happens to the electromagnetic charge in QED, but the behaviour is different: in QED, the coupling decreases together with the energy scale, while the strong coupling grows up if the energy goes down. This behaviour implies the property of *asymptotic freedom*: at high energy or, equivalently, at small distance, the quarks and gluons can be considered as free particles and one can expect that the perturbative approach is applicable.

Hence, we see that one can work with QCD as with a "normal" quantum field theory by making use of the perturbative expansion only in the asymptotic freedom regime. However, any hadronic process (even at very high energy) contains not only large energy scale, but also a remarkably small one, the hadronization scale, at which quarks and gluons recombine to make up real hadrons and any perturbative methods are not applicable.

1.2 Principal Tools to Work with QCD in the High-Energy Regime

Any scattering process in which strongly interacting particles – hadrons – participate, even those which occur at high centre-of-mass energy

$$s = (E_1 + E_2)^2,$$

include also dependence on a low-energy scale, which can be associated with typical hadron mass M_h. The large s, however, does not say us anything about the momentum scale which characterizes the partonic subprocess and which sets up the scale for the running coupling. The energy of a probe which allows us to penetrate a hadron to reveal its partonic substructure is called *hard*. If in a given process, there is no other energy scale which can be treated as "hard" in the above sense, then the process is considered as *soft*. An important example of such process is given by, e.g. the elastic hadron–hadron scattering. The methodology of analysis of such reactions goes beyond pure perturbative techniques and will not be considered here.

We shall focus on the so-called hard hadronic processes in the high-energy regime. For an inclusive scattering reaction in the t-channel,

$$P_1 + P_2 \to P_1' + \text{anything} \tag{1.2}$$

it means that besides the large centre-of-mass energy

$$s = (P_1 + P_2)^2 \gg M_\mathrm{h}^2, \tag{1.3}$$

where M_h is a hadronic mass scale, such as the proton mass, there is also the large momentum transfer

$$t = \Delta P_1^2 = (P_1' - P_1)^2 \gg M_\mathrm{h}^2. \tag{1.4}$$

Given the Heisenberg uncertainty relation, the large momentum transfer suggests that we can "measure" the intrinsic structure of the hadron with the spatial resolution

$$\Delta r^2 \sim \Delta P_1^{-2} = t^{-1}. \tag{1.5}$$

If t is large enough, we can access to the partonic subprocesses, where the strong coupling is small and the perturbation theory can be reasonably justified.

Still, we have to specify how the low-energy part of the total reaction (1.2) should be taken into account. The *QCD factorization approach* provides an appropriate method to merge the high- and low-energy regimes and perform efficient calculations.

- In a few words, the idea of QCD factorization consists in the consistent separation of large-distance (essentially nonperturbative, hadronization level) **S** and small-distance **H** (perturbatively calculable matrix elements, partonic level) contributions to a given process. The latter is being said to be factorizable if the differential cross section can be presented as the convolution of the hard and soft parts

$$d\sigma = \mathbf{H}_{\text{small distance}} \otimes \mathbf{S}_{\text{large distance}}. \tag{1.6}$$

  It is important to note that **S** is expected to be universal, while **H** depends on a particular process.
- Contribution of the soft part of the factorization formula (1.6) can be presented in terms of *parton density functions*[1] (PDFs), which accumulate information about the intrinsic structure of hadrons. More precise, the PDFs determine the probability distributions of quarks and gluons confined in the hadron. This is essentially the basic assumption of the *parton model*. The parton model is, however, not equivalent to QCD. The consistent construction of QCD-improved parton picture and the appropriate factorization scheme call for a suitable field-theoretical operator definitions for the PDFs.

Parton density functions in the momentum space can formally be obtained from the correlation functions of the appropriate quantum field operators of the following generic form[2]:

$$\boldsymbol{\Phi}(k) = \int \frac{d^4 z}{(2\pi)^4}\, e^{-ikz} \langle P|\overline{\psi}_H(z)\mathcal{U}^{\gamma}_{z,0}\psi_H(0)|P\rangle_H, \tag{1.7}$$

where the field operators $\overline{\psi}, \psi$ and the hadronic vectors of state $|P\rangle$ are taken in the Heisenberg representation and $\mathcal{U}^{\gamma}_{(z;0)}$ is a Wilson line or a system of lines which connects the points z and 0 with an arbitrary trajectory γ and make the correlation function (1.7) gauge-invariant

$$\mathcal{U}^{\gamma}_{(z;0)} = \mathcal{P}\exp\left(ig\int_0^z d\zeta^{\mu} t^a A^a_{\mu}(\zeta)\right). \tag{1.8}$$

The matrix elements (1.7) are associated with the hadronic expectation values of the parton number operators

1 Also parton distribution or fragmentation functions.

2 Here and in what follows we discuss mostly quark correlation functions.

$$\Phi(Q) \sim \langle a_Q^\dagger a_Q \rangle = \langle N_Q \rangle, \tag{1.9}$$

where N_Q stands for the operator which returns the number of particles possessing quantum numbers $\{Q\}$ (momenta, spins, colours, etc.) in a given state. The latter property connects the field-theoretical definition (1.7) with the intuitive ideas of the original parton model.

Nevertheless, equation (1.7) is too symbolic to make real calculations and useful predictions possible. The rest of the book is devoted to the identification of the most important issues and development of the methods to deal with them.