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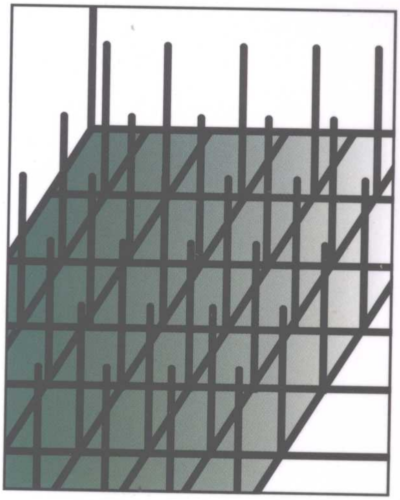

Probability: Theory and Examples

Third Edition

概率论 第3版

Rick Durrett

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Theory and Examples

Third Edition

Richard Durrett
Cornell University

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影印版前言

概率论是研究随机现象的一门数学，它的研究范围包括随机事件、随机度量和随机过程。概率论既有深刻的理论也有广泛的应用，其重要性越来越受到人们的关注。2006 年日本数学家伊藤清先生荣膺首届高斯奖，法国 W. Werner 获得 Fields 奖， 2007 年印度裔美国数学家 S. R. S. Varadhan 获得 Abel 奖。这些奖项只是冰山一角，在其背后是概率论在过去半个多世纪的蓬勃发展。如同其他数学分支一样，概率论对所有科学有深刻的影响，还间接影响了技术、商业和日常生活。以伊藤的研究为例，从上世纪四十年代开始，伊藤发展了一套全新的数学理论，称之为随机分析。它使得数学家们可以将随机力和确定性的力混在一个所谓的随机微分方程中表达，甚至可以在某种意义下解这个方程。伊藤的理论十分抽象， 可以应用到许多完全不同的领域中去。金融市场中的股票价格遵从随机力量， 就像在布朗运动中起作用的那样。为抵消波动影响，银行家们发现他们被迫在"连续时间"进行交易， 至少在理论上是这样的。由伊藤的思想发展出一套连续交易的策略， 最终可用公式计算期权价格。今天，Black-Scholes 公式成为几乎所有跟期权和期货有关的经济交易的基础。此外，它也为这两位发明人赢得了 1997 年的诺贝尔经济学奖。伊藤的理论还应用于生物的种群大小、基因库中某种等位基因的出现频率，甚至更复杂的生物量。因为伊藤的工作，生物学家可以判断某基因支配全种群或某物种存活的概率。

在过去半个多世纪里，概率论得到迅速发展，内容极大丰富。 这些变化也反映在研究生的教材里。每位教师都希望在有限的教学时间内教给学生最有价值的内容，这就需要一本能够精选内容、反映时代要求的教科书。为此国内外学者不断编写教材，其中影响较大的当推美国斯坦福大学钟开莱教授所著 A Course in Probability Theory。该书编写严谨，习题难度大。 该书的第二版于 1974 年出版，距今也有三十多年了。 进入上世纪九十年代，美国康奈尔大学 Rick Durrett 教授所著 Probability: Theory and Examples 一书因选材恰当、编排合理、难度适中、兼顾理论与应用、契合

当今研究生教学的实际情况而为美国多所著名高校选为研究生教材。在有限篇幅内，作者不仅很好地保留了传统教科书的经典内容，也吸收了许多最新研究成果；作者注意分散难点，避免冗长证明。自 1998 年以来，我在北京大学讲授研究生课程《高等概率论》和《随机过程论》各三次，均以该书为参考书。

我国开展研究生教育的时间并不太长，与国际最高水平相比尚有很大差距。为了更好地提高我国研究生的培养水平，我们必须多方面借鉴世界先进水平，包括引进国外优秀教材。纵观科学发展历史，所有学科都是在传承、交流中发展起来的。还是以 Durrett 书为例，Durrett 早年就读于斯坦福大学，学的就是钟开莱所写的书。Durrett 一度称钟为 academic god-father。之后 Durrett 在 UCLA 任教，Durrett 书的体例安排与 UCLA 研究生课程的教案相当吻合。Durrett 的书反映了美国半个世纪以来在概率论教育实践中所积累的经验，是值得我们认真学习借鉴的。在吸收消化的同时，我们还应当注意总结自.己的心得体会，不断创新，应当“对人类做出较大的贡献”。这几年我国工业界许多行业就是从引进开始，经过消化创新，再走向世界的。随着我国综合国力的提升，今后由我国学者编撰的教材在美国得以采用也绝非天方夜谭。

此书宜为概率统计专业一年级研究生《高等概率论》和《随机过程论》的教材。对于学过概率论的学者而言，这也不失为一本好的参考书。据传，美国一位刚得学位的年轻博士曾云游南美一年，除了生活必需品，随身携带的只有 Durrett 这本书。这位博士现已是美国一所常青藤大学的教授。同学们在学习概率论时要注意培养自己的直观判断能力和严谨推理能力，应以探索的精神去理解定义、命题和定理之间的逻辑关系，而不应迷信书中的每一句话。Durrett 在其第二版的前言称已消灭了第一版中 500 个笔误或排版错误，之后网上又出现了有关第二版的勘误表，长达 6 页之多。我们相信第三版已有改进，同时也有理由相信第三版中仍会有谬误之处。瑕不掩瑜，总的来说，这是一本优秀的研究生教材，我热忱向国内读者推荐此书。

陈大岳
北京大学数学科学学院
2007 年 8 月 27 日

Preface

Once in a while you get shown the light,
In the strangest of places if you look at it right.
Grateful Dead

The first and most obvious use for this book is as a textbook for a one year graduate course in probability taught to students who are familiar with measure theory. An Appendix, which gives complete proofs of the results from measure theory we need, is provided so that the book can be used whether or not the students are assumed to be familiar with measure theory.

The title of the book indicates that as we develop the theory, we will focus our attention on examples. Hoping that the book would be a useful reference for people who apply probability in their work, we have tried to emphasize the results that can be used to solve problems.

Exercises are integrated into the text because they are an integral part of it. In general, the exercises embedded in the text can be done immediately using the material just presented, and the reader should do these exercises to check her understanding and prepare for later developments. Exercises at the end of the section present extensions of the results and various complements.

Changes in the Second and Third Edition. The second edition published in 1995 brought four major changes: (i) More than 500 typographical errors were corrected. (ii) More details were added to many proofs to make them easier to understand. For example, Chapter 1 grew from 63 to 78 pages. (iii) Some sections were rearranged or divided into subsections. (iv) Last, and most important, I worked all the problems and prepared a solutions manual.

While the second edition was an improvement over the first, several hundred, mostly minor, typos remained. In this the third edition, I have concentrated on correcting errors, and adding a few lines here and there where I couldn't figure out what the author had in mind. I am grateful to Antal Jarai, who sorted through a two-inch-thick folder of emails, and made the first typo list that was posted on my web page in the summer of 2000.

With the third edition, the book enters the Duxbury Classics series, where it will hopefully live long, prosper, and be reasonably inexpensive. I would like to express my appreciation to my editor Carolyn Crockett for her help in making this happen. I don't plan on doing a fourth edition, but small errors can be corrected in future reprints so keep those emails coming to rtd1@cornell.edu.

Acknowledgements. I am always grateful to the many people who sent me comments and typos. Helping to correct the first edition were David Aldous, Ken Alexander, Daren Cline, Ted Cox, Robert Dalang, Joe Glover, David Griffeath, Phil Griffin, Joe Horowitz, Olav Kallenberg, Jim Kuelbs, Robin Pemantle, Yuval Peres, Ken Ross, Byron Schmuland, Steve Samuels, Jon Wellner, and Ruth Williams.

The third edition benefitted from input from Manel Baucells, Eric Blair, Zhen-Qing Chen, Ted Cox, Bradford Crain, Winston Crandall, Finn Christensen, Amir Dembo, Neil Falkner, Changyong Feng, Brighten Godfrey, Boris Granovsky, Jan Hannig, Andrew Hayen, Martin Hildebrand, Kyoungmun Jang, Anatole Joffe, Daniel Kifer, Steve Krone, Greg Lawler, T.Y. Lee, Shlomo Levental, Torgny Lindvall, Arif Mardin, Carl Mueller, Robin Pemantle, Yuval Peres, Mark Pinsky, Ross Pinsky, Boris Pittel, David Pokorny, Vinayak Prabhu, Brett Presnell, Jim Propp, Yossi Schwarzfuchs, Rami Shakarchi, Lian Shen, Marc Shivers, Rich Sowers, Bob Strain, Tsachy Weissman, and Hao Zhang.

Family Update. Turning to the home front, where the date is March 2003, David and Greg are now 16 and 14. Life is the same and it is different. The game console (now the Nintendo Game Cube) and the computer games have changed, in the latter case from being exclusively on their PCs to primarily being found on or played over the Internet, but as I write this we are again waiting for the latest Legend of Zelda game to be released. High school brings new challenges to David and Greg inside and outside the classroom. David works on the Ithaca High School paper and has an internship at our local paper, the *Ithaca Journal.* Greg plays clarinet and golf with his father. I am looking forward to this summer when he can teach me how to program in *Java.*

Most of Greg and David's achievements would not be possible without their mother, who drives them to their lessons and jobs, makes sure they do their homework, and helps clear up problems when they arise. In between, she frets about the damage to her trees from this January's ice storm and bides her time waiting for Spring by having the kitchen redone. It is impossible to encapsulate 23 years of married life into a ten-second sound bite. However, the recent melt down of two 20+ year marriages involving people we know well has given me a new appreciation for her many fine quantities. The three editions of this book (including the courses I taught in their preparation) represent a total of four to five years of my life. I hope you enjoy and learn from this the "final" version.

Rick Durrett

Introductory Lecture

As Breiman should have said in his preface, "Probability theory has a right and a left hand. On the left is the rigorous foundational work using the tools of measure theory. The right hand 'thinks probabilistically,' reduces problems to gambling situations, coin-tossing, and motions of a physical particle." We have interchanged Breiman's hands in the quote because we learned in a high school English class that the left hand is sinister and the right is dexterous. While measure theory does not, as the dictionary says, "threaten harm, evil or misfortune," it is an unfortunate fact that we will need four sections of definitions before we come to the first interesting result. To motivate the reader for this necessary foundational work, we will now give some previews of coming attractions.

For a large part of the first two chapters, we will be concerned with the laws of large numbers and the central limit theorem. To introduce these theorems and to illustrate their use, we will begin by giving their interpretation for a person playing roulette. In doing this, we will use some terms (e.g. independent, mean, variance) without explaining them. If some of the words that we use are unfamiliar, don't worry. There will be more than enough definitions when the time comes.

A roulette wheel has 38 slots - 18 red, 18 black, and 2 green ones that are numbered 0 and 00. Thus, if our gambler bets \$1 on red coming up, he wins \$1 with probability 18/38 and loses \$1 with probability 20/38. Let $X_1, X_2, \ldots$ be the outcomes of the first, second, and subsequent bets. If the house and gambler are honest, $X_1, X_2, \ldots$ are independent random variables and each has the same distribution, namely $P(X_i = 1) = 9/19$ and $P(X_i = -1) = 10/19$. The gambler's main interest in what we can tell him about the amount he has won at time n: $S_n = X_1 + \cdots + X_n$.

The first facts we can tell him are that (i) the average amount of money he will win on one play (= the mean of X_1 and denoted EX_1) is

$$(9/19) \cdot \$1 + (10/19) \cdot (-\$1) = -\$1/19 = -\$.05263$$

and (ii) on the average after n ways his winnings will be $ES_n = nEX_1 = -\$n/19$. For most values of n the probability of having lost exactly $n/19$ dollars is zero, so the next question to be answered is: How close will his experience be to the average? The first answer is provided by the

Weak Law of Large Numbers. If $X_1, X_2, \ldots$ are independent and identically distributed random variables with mean $EX_1 = \mu$, then for all $\epsilon > 0$

$$P(|S_n/n - \mu| > \epsilon) \to 0 \text{ as } n \to \infty$$

Less formally, if n is large then S_n/n is close to μ with high probability.

This result provides some information but leaves several questions unanswered. The first one is: If our gambler was statistically minded and wrote down the values of S_n/n, would the resulting sequence of numbers converge to $-1/19$? The answer to this question is given by the

Strong Law of Large Numbers. If $X_1, X_2, \ldots$ are independent and identically distributed random variables with mean $EX_i = \mu$ then with probability one, S_n/n converges to μ.

An immediate consequence of the last result of interest to our gambler is that with probability one $S_n \to -\infty$ as $n \to \infty$. That is, the gambler will eventually go bankrupt no matter how much money he starts with.

The laws of large numbers tell us what happens in the long run but do not provide much information about what will happens over the short run. That gap is filled by the

Central Limit Theorem. If $X_1, X_2, \ldots$ are independent and identically distributed random variables with mean $EX_i = \mu$ and variance $\sigma^2 = E(X_i - \mu)^2$ then for any y

$$P\left(\frac{S_n - n\mu}{\sigma n^{1/2}} \le y\right) \to \mathcal{N}(y)$$

where $\mathcal{N}(y) = \int_{-\infty}^{y} (2\pi)^{-1/2} e^{-x^2/2}\, dx$ is the (standard) normal distribution.

If we let χ denote a random variable with a normal distribution, then the last conclusion can be written informally as

$$S_n \approx n\mu + \sigma n^{1/2} \chi$$

In the example, we have been considering $\mu = -1/19$ and

$$\sigma^2 = \frac{9}{19}(1 + 1/19)^2 + \frac{10}{19}(-1 + 1/19)^2 = 1 - (1/19)^2 = .9972$$

If we use $\sigma^2 \approx 1$ to simplify the arithmetic, then the central limit theorem tells us

$$S_n \approx -n/19 + n^{1/2}\chi$$

or when $n = 100$,

$$S_{100} \approx -5.26 + 10\chi$$

If we are interested in the probability $S_{100} \geq 0$, this is

$$P(-5.26 + 10\chi \geq 0) = P(\chi \geq .526) \approx .30$$

from the table of the normal distribution at the back of the book.

The last result shows that after 100 plays the negative drift is not too noticeable. The gambler has lost \$5.26 on the average and has a probability .3 of being ahead. To see why casinos make money, suppose there are 100 gamblers playing 100 times and set $n = 10,000$ to get

$$S_{10,000} \approx -526 + 100\chi$$

Now $P(\chi \leq 2.3) = .99$ so with that probability $S_{10,000} \leq -296$, that is, the casino is slowly but surely making money.

Contents

1 Laws of Large Numbers

In the first three sections, we will recall some definitions and results from measure theory. Our purpose is not only to review that material but also to introduce the terminology of probability theory, which differs slightly from that of measure theory. In Section 1.4, we introduce the crucial concept of independence and explore its properties. In Section 1.5, we prove the weak law of large numbers and give several applications. In Section 1.6, we prove some Borel-Cantelli lemmas to prepare for the proof of the strong law of large numbers in Section 1.7. In Section 1.8, we investigate the convergence of random series that leads to estimates on the rate of convergence in the law of large numbers. Finally, in Section 1.9, we show that in nice situations convergence in the weak law occurs exponentially rapidly.

1.1. Basic Definitions

Here and throughout the book, terms being defined are set in **boldface**. We begin with the most basic quantity. A **probability space** is a triple $(\Omega, \mathcal{F}, P)$ where Ω is a set of "outcomes," $\mathcal{F}$ is a set of "events," and $P : \mathcal{F} \to [0,1]$ is a function that assigns probabilities to events. We assume that $\mathcal{F}$ is a **σ-field** (or **σ-algebra**), i.e., a (nonempty) collection of subsets of Ω that satisfy

(i) if $A \in \mathcal{F}$ then $A^c \in \mathcal{F}$, and

(ii) if $A_i \in \mathcal{F}$ is a countable sequence of sets then $\cup_i A_i \in \mathcal{F}$.

Here and in what follows, **countable** means finite or countably infinite. Since $\cap_i A_i = (\cup_i A_i^c)^c$, it follows that a σ-field is closed under countable intersections. We omit the last property from the definition to make it easier to check.

Without P, $(\Omega, \mathcal{F})$ is called a **measurable space**, i.e., it is a space on which we can put a measure. A **measure** is a nonnegative countably additive set function; that is, a function $\mu : \mathcal{F} \to \mathbf{R}$ with

(i) $\mu(A) \geq \mu(\emptyset) = 0$ for all $A \in \mathcal{F}$, and