Texts and
Monographs
in Physics

Rug[illegible] Maria Santilli

Foundations of Theoretical Mechanics I

The Inverse Problem in Newtonian Mechanics

Springer-Verlag
New York Heidelberg Berlin

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Editors:

ISBN 0-387-08874-1 Springer-Verlag New York
ISBN 3-540-08874-1 Springer-Verlag Berlin Hiedelberg

Library of Congress Cataloging in Publication Data

Santilli, Ruggero Maria.
Foundations of theoretical mechanics.
(Texts and monographs in physics)
Bibliography: p.
Includes index.
1. Mechanics. 2. Inverse problems
(Differential equations) I. Title.

QA808.S26 531 78-9735

Printed in the United States of America.

9 8 7 6 5 4 3 2 1

Texts and Monographs in Physics

Questo volume è dedicato a mia moglie

Carla

con infinito amore

Preface

The objective of this monograph is to present some methodological foundations of theoretical mechanics that are recommendable to graduate students prior to, or jointly with, the study of more advanced topics such as statistical mechanics, thermodynamics, and elementary particle physics.

A program of this nature is inevitably centered on the methodological foundations for Newtonian systems, with particular reference to the central equations of our theories, that is, Lagrange's and Hamilton's equations. This program, realized through a study of the analytic representations in terms of Lagrange's and Hamilton's equations of generally nonconservative Newtonian systems (namely, systems with Newtonian forces not necessarily derivable from a potential function), falls within the context of the so-called Inverse Problem, and consists of three major aspects:

1. The study of the necessary and sufficient conditions for the existence of a Lagrangian or Hamiltonian representation of given equations of motion with arbitrary forces;
2. The identification of the methods for the construction of a Lagrangian or Hamiltonian from the given equations of motion; and
3. The analysis of the significance of the underlying methodology for other aspects of Newtonian Mechanics, e.g., transformation theory, symmetries, and first integrals for nonconservative Newtonian systems.

This first volume is devoted to the foundations of the Inverse Problem, with particular reference to aspects 1 and 2. The second volume deals with some generalizations and applications of the Inverse Problem, with particular reference to aspect 3, and the problem of the construction of equivalent forms of the equations of motion that satisfy the conditions for the existence of a Lagrangian or Hamiltonian representation.

I had several motivations for undertaking this task. The first motivation came to me as a teacher. Indeed, the decision to study the analytic representations of systems with arbitrary Newtonian forces grew out of my uneasiness in teaching a graduate course in classical mechanics in the conventional manner. Typically, an articulated body of interrelated methodological formulations (i.e., analytic, variational, algebraic, geometrical, etc.) is presented; but in the final analysis, in view of the lack of knowledge of the methods for computing a Lagrangian for systems with more general Newtonian forces, these formulations are nowadays applicable only to systems with forces derivable from a potential function (basically, conservative systems). My uneasiness was ultimately due to the fact that, strictly speaking, conservative systems do not exist in our Newtonian environment. As a result, the Lagrangian representation of conservative Newtonian systems is, in general, only a crude approximation of physical reality.

A few remarks are sufficient to illustrate this point. For instance, the entire conventional theory of the Lagrangian representation in the space of the generalized coordinates of conservative Newtonian systems subject to holonomic constraints, is based on the often tacit assumption that the constraints are frictionless. But in practice, holonomic constraints are realized by mechanical means, e.g., hinges, rods, etc. Therefore, the presence of frictional forces is inevitable whenever holonomic constraints occur and, in turn, a Lagrangian representation that does not reflect this dissipative nature can only be considered a first approximation of the systems considered.

Owing to the fundamental nature of the knowledge of a Lagrangian or Hamiltonian, the above limitation of the conventional approach to Newtonian Mechanics is present at virtually all levels of the theory. For instance, the theory of canonical transformations for the one-dimensional harmonic oscillator is well known. But the extension of this theory to the more realistic case of the damped oscillator is not treated in currently available textbooks, again because of the lack of methods for constructing a Hamiltonian when damping forces are present. Similarly, the Hamilton–Jacobi theory of the frictionless spinning top is well known, but its extension to the system which actually occurs in our environment, namely, the spinning top with damping torque, is unknown at this time to the best of my knowledge. Therefore, the analysis presented in this monograph, the analytic representations of nonconservative Newtonian systems, grew out of my attempts to more closely represent Newtonian reality.

Other motivations for undertaking this task came to me as a theoretical physicist. As we all know, the significance of Newtonian Mechanics goes beyond the pragmatic aspect of merely studying Newtonian systems, because its methodological foundations apply, apart from technical rather than conceptual modifications, to several branches of physics, such as quantum mechanics and elementary particle physics. As soon as I became aware of new methodological prospectives within the context of purely Newtonian systems, I became intrigued by their possible significance for other branches of physics.

Predictably, it will take a considerable amount of time and effort by more than one researcher to ascertain the possible significance of the Inverse Problem for non-Newtonian frameworks. Nevertheless, to stimulate research along these lines, a few remarks are presented in the Introduction.

Owing to the lack of recent accounts of the Inverse Problem in both the mathematical and the physical literature, one of the most time-consuming parts of my program has been the identification of the prior state of the art. Indeed, it was only after a laborious library search, which I conducted over a three-year period by moving backward in time to the beginning of the past century, that I came to realize that the methodological foundations of the Inverse Problem were fully established in the mathematical literature by the first part of this century within the context of the calculus of variations. This was the result of the contributions of several authors, such as Jacobi (1837), Helmholtz (1887), Darboux (1891), Mayer (1896), Hirsh (1898), Bohem (1900), Könisberger (1901), Hamel (1903), Kurshak (1906), and others. The most comprehensive account of which I am aware is the thesis of D. R. Davis in 1926 at the Department of Mathematics of the University of Chicago, under the supervision of G. A. Bliss, subsequently expanded and published in three articles in 1928, 1929, and 1931 (see References). Since that time, regrettably, the problem remained largely ignored in both the mathematical and physical literature, with only a few exceptions known to me, which I shall indicate in the Introduction.

In this volume I present the results of my search in specialized mathematical and physical literature, and of my efforts on aspects such as the use of the Converse of the Poincaré Lemma for the proof of the central theorem on the necessary and sufficient conditions for the existence of a Lagrangian, the methods for the construction of a Lagrangian from the given equations of motion, the independent treatment of the Inverse Problem for phase space formulations without the prior knowledge of a Lagrangian, and the algebraic or geometrical significance of the necessary and sufficient conditions for the existence of a Hamiltonian. However, owing to the vast accumulation of literature in classical mechanics, calculus of variations, and other disciplines over the centuries, I make no claim to originality.

I make no claim to mathematical rigor, either. I concentrated my efforts primarily on presenting and illustrating the basic concepts in as simple a manner as possible. In essence, by specific intent, this volume should be readable by first- or second-year graduate students without major difficulties. In writing this monograph, I have also attempted to render it self-sufficient—extensive reference study is needed only for certain complementary aspects, such as for certain problems of the theory of differential equations or for certain geometrical interpretations, and a sound knowledge of undergraduate mechanics is the only prerequisite.

I have also made an effort to adopt the most widely used notations and symbols. When necessary, new notations are identified by footnotes.

Equations are referred to by notation of the form (1.2.3a), where 1, 2, and 3a indicate the chapter, the section, and the equations therein, respectively.

The references are listed at the end of the volume in chronological and then alphabetical order. My list of textbooks must be considered as purely representative, though incompletely so, of contributions in theoretical mechanics and related disciplines. However, for specialized topics not treated in currently available textbooks, I have listed all the relevant references of which I am aware.

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January 3, 1978

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Acknowledgments

I simply have no words to express my gratitude to A. Shimony. It is a truism to say that, without his encouragement, support, and advice, this work would not have been completed.

I would also like to express my sincere gratitude to A. C. Hurst, S. Shanmugadhasan, and P. L. Huddleston for carefully reading an earlier version of the manuscript and for numerous suggestions.

I am also sincerely grateful to my graduate students J. Eldridge and A. Sen for carefully studying an earlier version. Their various suggestions were very valuable in the overall improvement of the manuscript.

Correspondence on several historical or technical points with H. Corben, P. Dedecker, D. G. B. Edelen, G. M. Ewing, A. Harvey, P. Havas, R. Herman, M. R. Hestenes, C. B. Morrey, L. A. Pars, E. G. Saletan, D. C. Spencer, P. Stehle and K. R. Symon must be acknowledged.

Special thanks must be acknowledged to:

R. Mertens, E. Engels, F. Cantrijn, and W. Sarlet. Their frequent assistance has simply been invaluable for the entire project.

H. Rund and D. Lovelock. Their advice on the use of the calculus of differential forms has been invaluable for the proof of the main theorems.

A. Thellung and J. Kobussen. Their suggestions and comments have also been invaluable.

L. Y. Bahar and R. Brooks. Their critical reading of the manuscript has been invaluable.

C. N. Ktorides. His penetrating critical comments have been equally invaluable.

It is a pleasure to acknowledge support from the U.S. Department of Energy under contract no. ER-78-S-02-47420.A000.

It is a pleasure to thank F. E. Low, H. Feshbach, and R. Jackiw for their hospitality at the Center for Theoretical Physics of the Massachusetts Institute of Technology in 1976–1977, where part of this project was conducted.

It is a pleasure to thank S. Weinberg, M. Tinkham, and H. Georgi for their hospitality at the Lyman Laboratory of Physics, where this project was completed. In particular, I would like to express my appreciation for the opportunity of delivering an informal seminar course on the Inverse Problem at the Lyman Laboratory during the fall of 1977, which proved to be invaluable for the finalization of this project.

D. Nordstrom gave generously of his time and experience in assisting with the editorial preparation of the manuscript.

I would like also to express my gratitude to the Editorial Staff of Springer-Verlag for their invaluable assistance in the finalization of this project.

Almost needless to say, I am solely responsible for the content of this volume, including several modifications implemented in the final version of the manuscript.

Volume Organization

In the Introduction, I formulate the Inverse Problem of the calculus of variations, point out its reduction to a Newtonian context, and indicate all the relevant references on such problem of which I am aware.

In Chapter 1, I outline the rudiments of three disciplines, ordinary differential equations, calculus of differential forms, and calculus of variations, which are prerequisites for the methodology of the Inverse Problem.

In Chapter 2, I introduce the central mathematical tool of the analysis, the so-called variational approach to self-adjointness, and I specialize it to the most important forms of Newtonian systems.

In Chapter 3, I work out the central objectives of this monograph, which consist of the necessary and sufficient conditions for the existence of a Lagrangian or, independently, a Hamiltonian, the methods for their computation from given equations of motion, and an analysis of those Newtonian forces that are admissible by a Lagrangian or Hamiltonian representation.

In Appendix A, I review those concepts of Newtonian Mechanics that are useful for the analysis of the main text, to avoid excessive reference to the existing literature.

The presentation is organized into a main text, a series of charts, a set of examples, and problems. In the main text, I treat the essential concepts and formulations of the approach. In the charts, I present those complementary aspects which, even though not essential for the basic lines of the approach, are valuable for a deeper insight, and I touch on topics of more advanced nature for subsequent study by the interested reader. The examples are intended to illustrate the basic concepts introduced in the text only. The problems are intended to test the student's understanding of the given methodology and then his capability to work out specific applications.

The generalization of methodology of the Inverse Problem for the construction of a Lagrangian or Hamiltonian representation of systems of ordinary differential equations which, as given, violate the integrability conditions, is treated in Santilli (1979).

Use Suggestions

This book can be used as a textbook for a one-term graduate course on the Inverse Problem or on Nonconservative Newtonian Mechanics.

For the use of this book as a reference book for a section of a regular graduate course in classical mechanics devoted to the Inverse Problem, the instructor is recommended to work out a summary presentation of Chapters 2 and 3.

For graduate students in physics, I recommend first reading the main text, verifying the illustrative examples, and working out the problems. Subsequent study should then incorporate the charts and quoted references. A prior inspection of the Appendix A might be recommendable.

For graduate students and instructors in mathematics, this book can be complemented by currently available treatises on the calculus of variations, optimal control theory, differential geometry, and other topics to formulate the Inverse Problem in these disciplines.

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