

SECOND  
EDITION

# COULSON & RICHARDSON'S CHEMICAL ENGINEERING

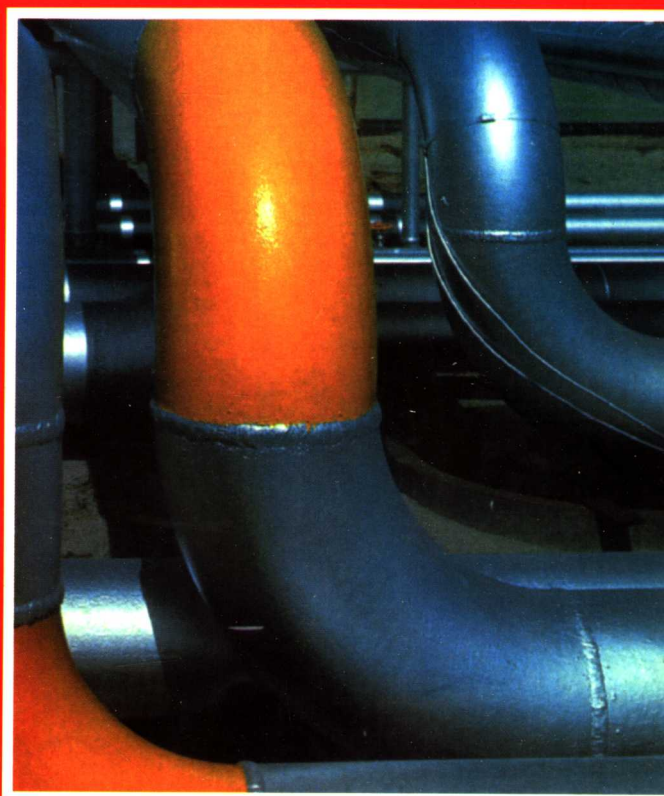
## 化 学 工 程

J R Backhurst & J H Harker

第 5 卷 第 2 版

VOLUME 5

5



**Solutions to the Problems in Volumes 2 & 3**

Butterworth  
Heinemann

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Coulson & Richardson's  
**CHEMICAL ENGINEERING**

**VOLUME 5**  
**SECOND EDITION**

**J. M. COULSON and J. F. RICHARDSON**

*Solutions to the Problems in Chemical Engineering*  
*Volumes 2 and 3*

*By*

**J. R. BACKHURST and J. H. HARKER**

*University of Newcastle upon Tyne*

**B**UTTERWORTH  
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## Preface to the Second Edition

IT IS always a great joy to be invited to prepare a second edition of any book and on two counts. Firstly, it indicates that the volume is proving useful and fulfilling a need, which is always gratifying and secondly, it offers an opportunity of making whatever corrections are necessary and also adding new material where appropriate. With regard to corrections, we are, as ever, grateful in the extreme to those of our readers who have written to us pointing out, mercifully minor errors and offering, albeit a few of what may be termed 'more elegant solutions'. It is important that a volume such as this is as accurate as possible and we are very grateful indeed for all the contributions we have received which, please be assured, have been incorporated in the preparation of this new edition.

With regard to new material, this new edition is now in line with the latest edition, that is the Fourth, of Volume 2 which includes new sections, formerly in Volume 3 with, of course, the associated problems. The sections are: 17, Adsorption; 18, Ion Exchange; 19, Chromatographic Separations and 20, Membrane Separation Processes and we are more than grateful to Professor Richardson's colleagues at Swansea, J. H. Bowen, J. R. Conder and W. R. Bowen, for an enormous amount of very hard work in preparing the solutions to these problems. A further and very substantial addition to this edition of Volume 5 is the inclusion of solutions to the problems which appear in *Chemical Engineering, Volume 3—Chemical & Biochemical Reactors & Process Control* and again, we are greatly indebted to the authors as follows:

- 3.1 Reactor Design—J. C. Lee
- 3.2 Flow Characteristics of Reactors—J. C. Lee
- 3.3 Gas-Solid Reactions and Reactors—W. J. Thomas and J. C. Lee
- 3.4 Gas-Liquid and Gas-Liquid-Solid Reactors—J. C. Lee
- 3.5 Biological Reaction Engineering—M. G. Jones and R. L. Lovitt
- 3.6 Process Control—A. P. Wardle

and also of course, to Professor Richardson himself, who, with a drive and enthusiasm which seems to be getting ever more vigorous as the years proceed, has not only arranged for the preparation of this material and overseen our efforts with his usual meticulous efficiency, but also continues very much in master-minding this whole series. We often reflect on the time when, in preparing 150 solutions for the original edition of Volume 4, the worthy Professor pointed out that we had only 147 correct, though rather reluctantly agreed that we might still just merit first class honours! Whatever, we always have and we are sure that we always will owe him an enormous debt of gratitude.

We must also offer thanks to our seemingly ever-changing publishers for their drive, efficiency and encouragement and especially to the present staff at Butterworth-Heinemann



for not inconsiderable efforts in locating the manuscript for the present edition which was apparently lost somewhere in all the changes and chances of the past months.

We offer a final thought as to the future where there has been a suggestion that the titles Volume 4 and Volume 5 may find themselves hijacked for new textural volumes, coupled with a proposal that the solutions offered here hitherto may just find a new resting place on the Internet. Whatever, we will continue with our efforts in ensuring that more and more solutions find their way into the text in Volumes 1 and 2 and, holding to the view expressed in the Preface to the First Edition of Volume 4 that '... worked examples are essential to a proper understanding of the methods of treatment given in the various texts', that the rest of the solutions are accessible to the widest group of students and practising engineers as possible.

*Newcastle upon Tyne, 1997*

J. R. BACKHURST  
J. H. HARKER



## *Preface to the First Edition*

IN THE preface to the first edition of *Chemical Engineering*, Volume 4, we quoted the following paragraph written by Coulson and Richardson in their preface to the first edition of *Chemical Engineering*, Volume 1:

'We have introduced into each chapter a number of worked examples which we believe are essential to a proper understanding of the methods of treatment given in the text. It is very desirable for a student to understand a worked example before tackling fresh practical problems himself. Chemical Engineering problems require a numerical answer, and it is essential to become familiar with the different techniques so that the answer is obtained by systematic methods rather than by intuition.'

It is with these aims in mind that we have prepared Volume 5, which gives our solutions to the problems in the third edition of *Chemical Engineering*, Volume 2. The material is grouped in sections corresponding to the chapters in that volume and the present book is complementary in that extensive reference has been made to the equations and sources of data in Volume 2 at all stages. The book has been written concurrently with the revision of Volume 2 and SI units have been used.

In many ways these problems are more taxing and certainly longer than those in Volume 4, which gives the solutions to problems in Volume 1, and yet they have considerable merit in that they are concerned with real fluids and, more importantly, with industrial equipment and conditions. For this reason we hope that our efforts will be of interest to the professional engineer in industry as well as to the student, who must surely take some delight in the number of tutorial and examination questions which are attempted here.

We are again delighted to acknowledge the help we have received from Professors Coulson and Richardson in so many ways. The former has the enviable gift of providing the minimum of data on which to frame a simple key question, which illustrates the crux of the problem perfectly, whilst the latter has in a very gentle and yet thorough way corrected our mercifully few mistakes and checked the entire work. Our colleagues at the University of Newcastle upon Tyne have again helped us, in many cases unwittingly, and for this we are grateful.

Newcastle upon Tyne, 1978

J. R. BACKHURST  
J. H. HARKER



# Factors for conversion of SI units

<i>mass</i>		<i>pressure</i>	
1 lb	0.454 kg	1 lbf/in <sup>2</sup>	6.895 kN/m <sup>2</sup>
1 ton	1016 kg	1 atm	101.3 kN/m <sup>2</sup>
		1 bar	100 kN/m
<i>length</i>		1 ft water	2.99 kN/m <sup>2</sup>
1 in	25.4 mm	1 in water	2.49 N/m <sup>2</sup>
1 ft	0.305 m	1 in Hg	3.39 kN/m <sup>2</sup>
1 mile	1.609 km	1 mm Hg	133 N/m <sup>2</sup>
<i>time</i>		<i>viscosity</i>	
1 min	60 s	1 P	0.1 N s/m <sup>2</sup>
1 h	3.6 ks	1 lb/ft h	0.414 mN s/m <sup>2</sup>
1 day	86.4 ks	1 stoke	10 <sup>-4</sup> m <sup>2</sup> /s
1 year	31.5 Ms	1 ft <sup>2</sup> /h	0.258 cm <sup>2</sup> /s
<i>area</i>		<i>mass flow</i>	
1 in <sup>2</sup>	645.2 mm <sup>2</sup>	1 lb/h	0.126 g/s
1 ft <sup>2</sup>	0.093 m <sup>2</sup>	1 ton/h	0.282 kg/s
		1 lb/h ft <sup>2</sup>	1.356 g/s m <sup>2</sup>
<i>volume</i>		<i>thermal</i>	
1 in <sup>3</sup>	16,387.1 mm <sup>3</sup>	1 Btu/h ft <sup>2</sup>	3.155 W/m <sup>2</sup>
1 ft <sup>3</sup>	0.0283 m <sup>3</sup>	1 Btu/h ft <sup>2</sup> °F	5.678 W/m <sup>2</sup> K
1 UK gal	4546 cm <sup>3</sup>	1 Btu/lb	2.326 kJ/kg
1 US gal	3786 cm <sup>3</sup>	1 Btu/lb °F	4.187 kJ/kg K
		1 Btu/h ft °F	1.731 W/m K
<i>force</i>		<i>energy</i>	
1 pdl	0.138 N	1 kWh	3.6 MJ
1 lb	4.45 N	1 therm	106.5 MJ
1 dyne	10 <sup>-5</sup> N		
<i>energy</i>		<i>calorific value</i>	
1 ft lb	1.36 J	1 Btu/ft <sup>3</sup>	37.26 kJ/m <sup>3</sup>
1 cal	4.187 J	1 Btu/lb	2.326 kJ/kg
1 erg	10 <sup>-7</sup> J		
1 Btu	1.055 kJ		
<i>power</i>		<i>density</i>	
1 h.p.	745 W	1 lb/ft <sup>3</sup>	16.02 kg/m <sup>3</sup>
1 Btu/h	0.293 W		



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## SECTION 2-1

# *Particulate Solids*

**Problem 1.1** The size analysis of a powdered material on a mass basis is represented by a straight line from 0% mass at 1  $\mu\text{m}$  particle size to 100% mass at 101  $\mu\text{m}$  particle size. Calculate the mean surface diameter of the particles constituting the system.

**Solution** From equation 1.15 the surface mean diameter is given by:  $d_s = 1/\Sigma(x_i/d_i)$   
 Since the size analysis is represented by the continuous line:  $d = 100x + 1$  (Fig. 1.7)

$$\begin{aligned} d_s &= \frac{1}{\int_0^1 dx/d} = \frac{1}{\int_0^1 dx/(100x + 1)} \\ &= 100/\ln 101 = \underline{\underline{21.7\mu\text{m}}} \end{aligned}$$

**Problem 1.2** The equations giving the number distribution curve for a powdered material are  $dn/dd = d$  for the size range 0 to 10  $\mu\text{m}$  and  $dn/dd = 100000/d^4$  for the size range 10 to 100  $\mu\text{m}$ . Sketch the number, surface, and mass distribution curves. Calculate the surface mean diameter for the powder. Explain briefly how the data for the construction of these curves would be obtained experimentally.

**Solution** For the range,  $d = 0 - 10 \mu\text{m}$ ,  $dn/dd = d$

On integration:  $n = 0.5d^2 + c_1$  (i)

where  $c_1$  is the constant of integration.

For the range,  $d = 10 - 100 \mu\text{m}$ ,  $dn/dd = 10^5 d^{-4}$

On integration:  $n = c_2 - (0.33 \times 10^5 d^{-3})$  (ii)

Where  $c_2$  is the constant of integration.

When  $d = 0$ ,  $n = 0$ , and from (i):  $c_1 = 0$ .

When  $d = 10 \mu\text{m}$ , in (i):  $n = (0.5 \times 10^2) = 50$

and in (ii):  $50 = c_2 - (0.33 \times 10^5 \times 10^{-3})$ , and  $c_2 = 83.0$ .



Thus for  $d = 0 - 10 \mu\text{m}$ ,  $n = 0.5d^2$

and for  $d = 10 - 100 \mu\text{m}$ ,  $n = 83.0 - (0.33 \times 10^5 d^{-3})$

Using these equations, values of  $n$  are obtained as follows:

$d(\mu\text{m})$	$n$	$d(\mu\text{m})$	$n$
0	0	10	50.0
2.5	3.1	25	80.9
5.0	12.5	50	82.7
7.5	28.1	75	82.9
10.0	50.0	100	83.0

and these data are plotted in Fig. 1a.

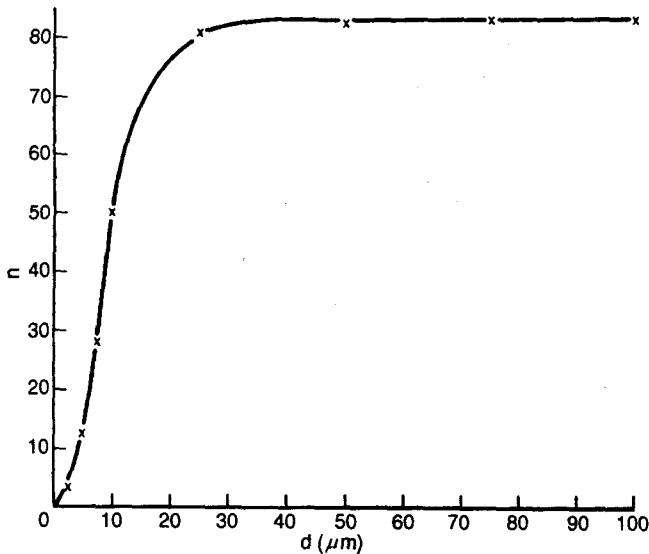


Figure 1a. Plot of data for Problem 1.2

From this plot, values of  $d$  are obtained for various values of  $n$  and hence  $n_1$  and  $d_1$  are obtained for each increment of  $n$ . Values of  $n_1 d_1^2$  and  $n_1 d_1^3$  are calculated and the totals obtained. The surface area of the particles in the increment is then given by:

$$s_1 = n_1 d_1^2 / \Sigma n_1 d_1^2$$

and  $s$  is then found as  $\Sigma s_1$ . Similarly the mass of the particles,  $x = \Sigma x_1$  where:

$$x_1 = n_1 d_1^3 / \Sigma n_1 d_1^3$$



The results are:

$n$	$d$	$n_1$	$d_1$	$n_1 d_1^2$	$n_1 d_1^3$	$s_1$	$s$	$x_1$	$x$
0	0								
20	6.2	20	3.1	192	596	0.014	0.014	0.001	0.001
40	9.0	20	7.6	1155	8780	0.085	0.099	0.021	0.022
50	10.0	10	9.5	903	8573	0.066	0.165	0.020	0.042
60	11.4	10	10.7	1145	12250	0.084	0.249	0.029	0.071
65	12.1	5	11.75	690	8111	0.057	0.300	0.019	0.090
70	13.6	5	12.85	826	10609	0.061	0.0361	0.025	0.115
72	14.7	2	14.15	400	5666	0.029	0.390	0.013	0.128
74	16.0	2	15.35	471	7234	0.035	0.425	0.017	0.145
76	17.5	2	16.75	561	9399	0.041	0.466	0.022	0.167
78	19.7	2	18.6	692	12870	0.057	0.517	0.030	0.197
80	22.7	2	21.2	890	18877	0.065	0.582	0.044	0.241
81	25.5	1	24.1	581	14000	0.043	0.625	0.033	0.274
82	31.5	1	28.5	812	23150	0.060	0.685	0.055	0.329
83	100	1	65.75	4323	284240	0.316	1.000	0.670	1.000
				13641	424355				

Values of  $s$  and  $x$  are plotted as functions of  $d$  in Fig. 1b.

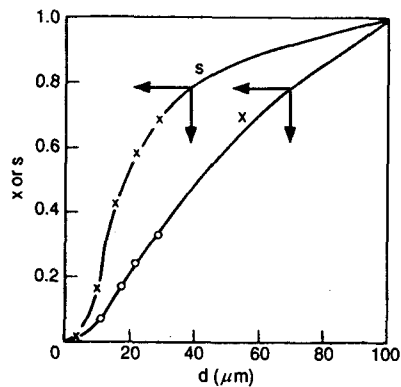


Figure 1b. Calculated data for Problem 1.2



The surface mean diameter,  $d_s = \Sigma(n_i d_i^3) / \Sigma(n_i d_i^2) = 1 / \Sigma(x_i / d_i)$

$$= \int d^3 dn / \int d^2 dn \quad (\text{equations 1.14 and 1.15})$$

For  $0 < d < 10 \mu\text{m}$ ,

$$dn = d \, dd$$

For  $10 < d < 100 \mu\text{m}$ ,

$$dn = 10^5 d^{-4} dd$$

$$\begin{aligned} \therefore d_s &= \left( \int_0^{10} d^4 dd + \int_{10}^{100} 10^5 d^{-1} dd \right) / \left( \int_0^{10} d^3 dd + \int_{10}^{100} 10^5 d^{-2} dd \right) \\ &= ([d^5/5]_0^{10} + 10^5 [\ln d]_{10}^{100}) / ([d^4/4]_0^{10} + 10^5 [-d^{-1}]_{10}^{100}) \\ &= (2 \times 10^4 + 2.303 \times 10^5) / (2.5 \times 10^3 + 9 \times 10^3) = \underline{\underline{21.8 \mu\text{m}}} \end{aligned}$$

The size range of a material is determined by sieving for relatively large particles and by sedimentation methods for particles which are too fine for sieving. The use of such a method is described in Volume 2, Chapter 1.

**Problem 1.3** The fineness characteristic of a powder on a cumulative basis is represented by a straight line from the origin to 100% undersize at particle size  $50 \mu\text{m}$ . If the powder is initially dispersed uniformly in a column of liquid, calculate the proportion by mass which remains in suspension at a time interval from commencement of settling such that a  $40 \mu\text{m}$  particle would fall the total height of the column.

**Solution** For flow under streamline conditions, the velocity is proportional to the diameter squared and hence the time taken for a  $40 \mu\text{m}$  particle to fall a height  $h$  m is:  $h/40^2 k$  s where  $k$  is a constant.

During this time, a particle of diameter  $d \mu\text{m}$  has fallen a distance:  $kd^2 h/40^2 k = hd^2/40^2 \text{m}$ . The proportion of particles of size  $d$  which are still in suspension  $= (1 - d^2/40^2)$  and the fraction by mass of particles which are in suspension  $= \int_0^{40} (1 - d^2/40^2) dw$

Since  $dw/dd = 1/50$ , the mass fraction  $= (1/50) \int_0^{40} (1 - d^2/40^2) dd$   
 $= (1/50)[d - d^3/4800]_0^{40} = 0.533$  or 53.3% of the particles remain in suspension

**Problem 1.4** In a mixture of quartz, specific gravity 2.65, and galena, specific gravity 7.5, the sizes of the particles range from 0.0052 to 0.025 mm. On separation in a hydraulic classifier under free settling conditions, three fractions are obtained, one consisting of quartz only, one a mixture of quartz and galena, and one of galena only. What are the ranges of sizes of particles of the two substances in the mixed portion?

**Solution** Use is made of equation 3.23, which may be written as:  $u_0 = kd^2(\rho_s - \rho)$ , where  $k (= g/18\mu)$  is a constant.

For large galena,  $u_0 = k \times 0.025^2(7.5 - 1.0) = 0.00406k \text{ mm/s}$

For small galena,  $u_0 = k \times 0.0052^2(7.5 - 1.0) = 0.000175k \text{ mm/s}$

For large quartz,  $u_0 = k \times 0.025^2(2.65 - 1.0) = 0.00103k \text{ mm/s}$

For small quartz,  $u_0 = k \times 0.0052^2(2.65 - 1.0) = 0.000045k \text{ mm/s}$



If the time of settling was such that particles with a velocity equal to  $0.00103k$  mm/s settled, then the bottom product would contain quartz. This is not so and hence the maximum size of galena particles still in suspension is given by:

$$0.00103k = kd^2(7.5 - 1.0) \text{ and } d = 0.0126 \text{ mm}$$

Similarly if the time of settling was such that particles with a velocity equal to  $0.000175k$  mm/s did not start to settle, then the top product would contain galena. This is not the case and hence the minimum size of quartz in suspension is given by:

$$0.000175k = kd^2(2.65 - 1.0) \text{ or } d = 0.0103 \text{ mm}$$

It may therefore be concluded that, assuming streamline conditions, the fraction of material in suspension, that is containing quartz and galena, is made up of particles of sizes in the range: 0.0103–0.0126 mm

**Problem 1.5** It is desired to separate into two pure fractions a mixture of quartz and galena of a size range from 0.015 mm to 0.065 mm by the use of a hindered settling process. What is the minimum apparent density of the fluid that will give this separation? How will the viscosity of the bed affect the minimum required density? (Density of galena =  $7500 \text{ kg/m}^3$ , density of quartz =  $2650 \text{ kg/m}^3$ .)

**Solution** Assuming that the shapes of the galena and quartz particles are similar, then from equation 1.31, the required density of fluid for viscous conditions is given by:

$$(0.065/0.015) = [(7500 - \rho)/(2650 - \rho)]^{0.5} \text{ or } \rho = 2377 \text{ kg/m}^3$$

From equation 3.31, the required density for fully turbulent conditions is given by:

$$(0.065/0.015) = (7500 - \rho)/(2650 - \rho) \text{ and } \rho = 1196 \text{ kg/m}^3$$

Thus the minimum density of the fluid to effect the separation is  $1196 \text{ kg/m}^3$ . This assumes that fully turbulent conditions prevail. As the viscosity is increased, the value of the Reynolds group will decrease and the required density will rise to the value of  $2377 \text{ kg/m}^3$  necessary for viscous conditions.

**Problem 1.6** The following table gives the size distribution of a dust as measured by the microscope. Convert these figures to obtain the distribution on a mass basis, and calculate the specific surface, assuming spherical particles of specific gravity 2.65.

Size range in $\mu\text{m}$	Number of particles in range
0–2	2000
2–4	600
4–8	140
8–12	40
12–16	15
16–20	5
20–24	2



**Solution** The determination of a particle size distribution by microscopic and sedimentation techniques is discussed in Section 1.2.2. In simple terms, sedimentation would be adopted for very small particles which are too fine for classification by either sieving or visual examination.

From equation 1.4, the mass fraction of particles of size  $d_1$  is given by:  $x_1 = n_1 k_1 d_1^3 \rho_s$ , where  $k_1$  is a constant,  $n_1$  the number of particles of size  $d_1$ , and  $\rho_s$  the density of the particles = 2650 kg/m<sup>3</sup>.

$\Sigma x_1 = 1$  and hence the mass fraction is:  $x_1 = n_1 k_1 d_1^3 \rho_s / \Sigma n k d^3 \rho_s$

In this case:

$d$	$n$	$k d^3 n \rho_s$	$x$
1	200	5,300,000 $k$	0.011
3	600	42,930,000 $k$	0.090
6	140	80,136,000 $k$	0.168
10	40	106,000,000 $k$	0.222
14	15	109,074,000 $k$	0.229
18	5	77,274,000 $k$	0.162
22	2	56,434,400 $k$	0.118
		$\Sigma = 477,148,400k$	$\Sigma = 1.0$

The surface mean diameter is given by equation 1.14:  $d_s = \Sigma(n_1 d_1^3) / \Sigma(n_1 d_1^2)$  and hence:

$d$	$n$	$n d^2$	$n d^3$
1	2000	2000	2000
3	600	5400	16,200
6	140	5040	30,240
10	40	4000	40,000
14	15	2940	41,160
18	5	1620	29,160
22	2	968	21,296
		$\Sigma = 21,968$	$\Sigma = 180,056$

$$d_s = (180,056 / 21,968) = 8.20 \mu\text{m}$$

This is the size of particle with the same specific surface as the mixture.

Volume of a particle 8.20  $\mu\text{m}$  diameter =  $(\pi/6) \times 8.20^3 = 288.7 \mu\text{m}^3$ .

Surface area of a particle of 8.20  $\mu\text{m}$  diameter =  $(\pi \times 8.20^2) = 211.2 \mu\text{m}^2$  and hence the specific surface =  $(211.2/288.7) = 0.731 \mu\text{m}^2/\mu\text{m}^3$  or  $0.731 \times 10^6 \text{m}^2/\text{m}^3$

**Problem 1.7** The performance of a solids mixer has been assessed by calculating the variance occurring in the mass fraction of a component amongst a selection of samples withdrawn from the mixture. The quality was tested at intervals of 30 s and the results are:



Sample variance	0.025	0.006	0.015	0.018	0.019
Mixing time (s)	30	60	90	120	150

If the component analysed is estimated to represent 20% of the mixture by mass and each of the samples removed contained approximately 100 particles, comment on the quality of the mixture produced and present the data in graphical form showing the variation of mixing index with time.

**Solution** For a completely unmixed system,

$$s_0^2 = p(1 - p) = 0.20(1 - 0.20) = 0.16 \quad (\text{equation 1.23})$$

For a completely random mixture,

$$s_r^2 = p(1 - p)/n = 0.20(1 - 0.20)/100 = 0.0016 \quad (\text{equation 1.22})$$

The degree of mixing  $M$  is given by equation 1.24 as:  $M = (s_0^2 - s^2)/(s_0^2 - s_r^2)$

In this case,  $M = (0.16 - s^2)/(0.16 - 0.0016) = 1.01 - 6.313s^2$

The calculated data are therefore:

$t(s)$	30	60	90	120	150
$s^2$	0.025	0.006	0.015	0.018	0.019
$M$	0.852	0.972	0.915	0.896	0.890

These data are plotted in Fig. 1c and it is clear that the degree of mixing is a maximum at  $t = 60s$ .

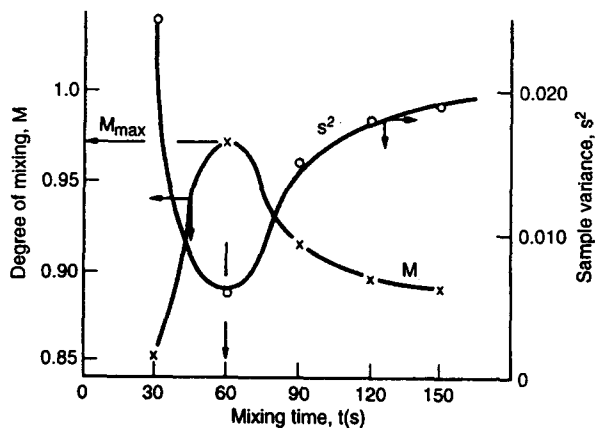


Figure 1c. Problem 1.7—degree of mixing as a function of mixing time



## SECTION 2-2

# Size Reduction of Solids

**Problem 2.1** A material is crushed in a Blake jaw crusher such that the average size of particle is reduced from 50 mm to 10 mm with a consumption of energy at the rate of 13.0 kW/(kg/s). What will be the consumption of energy needed to crush the same material of average size 75 mm to an average size of 25 mm:

- (a) assuming Rittinger's law applies?
- (b) assuming Kick's law applies?

Which of these results would be regarded as being more reliable and why?

**Solution** (a) *Rittinger's Law* This is given by equation 2.2:  $E = K_R f_c \left( \frac{1}{L_2} - \frac{1}{L_1} \right)$

or:  $13.0 = K_R f_c ((1/10) - (1/50))$

$\therefore K_R f_c = (13.0 \times 50/4) = 162.5 \text{ kW s/kg mm}$

Thus the energy required to crush 75 mm material to 25 mm is:

$$E = 162.5((1/25) - (1/75)) = \underline{\underline{4.33 \text{ kJ/kg}}}$$

(b) *Kick's Law* This is given by equation 2.3:  $E = K_K f_c \ln(L_1/L_2)$

or:  $13.0 = K_K f_c \ln(50/10)$

$\therefore K_K f_c = (13.0/1.609) = 8.08 \text{ kW/(kg/s)}$

Thus the energy required to crush 75 mm material to 25 mm is given by:

$$E = 8.08 \ln(75/25) = \underline{\underline{8.88 \text{ kJ/kg}}}$$

The size range involved may be classified as coarse crushing, and because Kick's law more closely relates to the energy required to effect elastic deformation before fracture occurs, this would be taken as giving the more accurate result.

**Problem 2.2** A crusher was used to crush a material whose compressive strength was 22.5 MN/m<sup>2</sup>. The size of the feed was *minus* 50 mm, *plus* 40 mm, and the energy required was 13.0 kJ/kg. The screen analysis of the product was as follows:



Size of aperture (mm)	Per cent of product
Through 6.00	100
On 4.00	26
On 2.00	18
On 0.75	23
On 0.50	8
On 0.25	17
On 0.125	3
Through 0.125	5

What would be the power required to crush 1 kg/s of a material of compressive strength 45 MN/m<sup>2</sup> from a feed *minus* 45 mm, *plus* 40 mm to a product of average size 0.50 mm?

**Solution** Firstly, it is necessary to obtain a dimension representing the mean size of the product. Using Bond's method of taking the size of opening through which 80% of the material will pass, a value of just over 4.00 mm is indicated by the data. Alternatively, calculations may be made as follows:

Size of aperture (mm)	Mean $d_1$ (mm)	$n_1$	$nd_1$	$nd_1^2$	$nd_1^3$	$nd_1^4$
6.00	5.00	0.26	1.3	6.5	32.5	162.5
4.00	3.00	0.18	0.54	1.62	4.86	14.58
2.00	1.375	0.23	0.316	0.435	0.598	0.822
0.75	0.67	0.08	0.0536	0.0359	0.0241	0.0161
0.50	0.37	0.17	0.0629	0.0233	0.0086	0.00319
0.25	0.1875	0.03	0.0056	0.00105	0.00020	0.000037
0.125	0.125	0.05	0.00625	0.00078	0.000098	0.000012
			2.284	8.616	37.991	177.92

From equation 1.11, the mass mean diameter,  $d_v = \Sigma n_1 d_1^4 / \Sigma n_1 d_1^3$

$$= (177.92/37.991) = 4.683 \text{ mm}$$

From equation 1.14, the surface mean diameter,  $d_s = \Sigma n_1 d_1^3 / \Sigma n_1 d_1^2$

$$= (37.991/8.616) = 4.409 \text{ mm}$$

From equation 1.18, the length mean diameter,  $d_l = \Sigma n_1 d_1^2 / \Sigma n_1 d_1$

$$= (8.616/2.284) = 3.772 \text{ mm}$$

From equation 1.19, the mean length diameter,  $d'_l = \Sigma n_1 d_1 / \Sigma n_1$

$$= (2.284/1.0) = 2.284 \text{ mm}$$