Fundamental Theories of Physics 181

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Quantum Theory: Informational Foundations and Foils

Springer

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ISSN 0168-1222 ISSN 2365-6425 (electronic)
Fundamental Theories of Physics
ISBN 978-94-017-7302-7 ISBN 978-94-017-7303-4 (eBook)
DOI 10.1007/978-94-017-7303-4

Library of Congress Control Number: 2015945141

Springer Dordrecht Heidelberg New York London

Printed on acid-free paper

Springer Science+Business Media B.V. Dordrecht is part of Springer Science+Business Media
(www.springer.com)

Fundamental Theories of Physics

Volume 181

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Introduction

Giulio Chiribella and Robert W. Spekkens

The foundations of Quantum Mechanics are experiencing a golden age. In a timespan of less than two decades, an astonishing number of new results, ideas, and frameworks have revolutionized the way we think about the subject. A new research community is emerging worldwide, attracting scientists from a diverse spectrum of disciplines including physics, computer science, and mathematics. The keyword "foundations" is now included in the strategic priorities of many research institutions and funding agencies, and it regularly features as one of the hot topics in international conferences.

The abundance of ideas, approaches, and resources that have emerged poses some challenges however. For one, having a global vision of the field and reflecting on its high level goals is becoming increasingly difficult. For another, the sheer number of different frameworks that have been put forward risks creating a tower of Babel effect, fragmenting the community into smaller cliques that are unable to talk to one another. In addition, researchers who are joining the field have to cope with a fast-moving landscape where it can be hard to identify stable reference points.

These considerations led us to the project of this book, which aims to showcase the state of the art in quantum foundations. The book provides a collection of articles that deal with influential ideas in the field today, revealing the diversity of approaches on the one hand, and highlighting the common threads among them on the other.

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G. Chiribella and R.W. Spekkens (eds.), *Quantum Theory: Informational Foundations and Foils*, Fundamental Theories of Physics 181,
DOI 10.1007/978-94-017-7303-4_1

1 Characteristics of the New Wave of Quantum Foundations

We start by outlining what is distinctive about the foundational research that this book aims to portray.

1.1 A Pragmatic Perspective

It is useful to distinguish between what one might call *dynamicist* and *pragmatist* traditions in physics. Within the dynamicist tradition, the physicist's job is to describe the natural dynamical behaviour of a system, without reference to human agents or their purposes. In the pragmatic approach, on the other hand, the laws of physics are characterized in terms of the extent to which we can learn and control the behaviour of physical systems. The distinction between the dynamicist and pragmatist points of view is nicely represented in competing formulations of the second law of thermodynamics. One that is clearly in the dynamicist tradition is Clausius's original statement:

> Heat can never pass from a colder to a warmer body without some other change, connected therewith, occurring at the same time [1].

On the other hand, the version of the Kelvin-Planck statement that is found in most textbooks is clearly pragmatic:

> It is impossible to devise a cyclically operating device, the sole effect of which is to absorb energy in the form of heat from a single thermal reservoir and to deliver an equivalent amount of work [2].

Quantum theory has always partaken in both traditions. Indeed, Schrödinger's wave mechanics and Heisenberg's matrix mechanics were distinguished in part by the fact that Schrödinger, following de Broglie's lead, sought to provide a description of the motion of particles, while Heisenberg, following Bohr's lead, espoused an operational philosophy and took his formalism to merely describe what would be observed in certain experimental circumstances. The new foundational work represents a renewed interest in exploring quantum theory within the pragmatic tradition.

1.2 Quantum Foundations in the Light of Quantum Information

The newfound popularity of the pragmatic tradition is tightly connected with the rise of quantum information theory. The real innovation of the recent foundational work is in the way researchers conceive the difference between quantum and classical theories [3]. Historically, quantum theory was taken to consist entirely of *restrictions* on our information-gathering ability; think for instance of the restriction imposed by the uncertainty principle. The quantum information revolution overturned this notion: a quantum world in fact holds new possibilities for information-processing tasks—in particular, communication tasks, cryptographic tasks and computational tasks—that could not be accomplished in classical physics.

Milestone applications of quantum information, such as secure quantum key distribution [4, 5], ultrafast quantum algorithms [6, 7], teleportation [8], and dense coding [9], stimulated the imagination of quantum theorists, and led them to ask questions that moved beyond the usual topics of foundational discussions: *Which principles of quantum theory can account for its information-processing advantages? Does the possibility of achieving one kind of information-processing advantage imply the possibility of achieving others? Is quantum theory the* only *theory where these advantages arise?* These questions were at the center of an influential research programme, launched by Fuchs [10, 11] and Brassard [12], that aimed to understand quantum theory in the light of quantum information. More specifically, the idea was to take certain facts about the information-processing features of a quantum world, for instance, the possibility of secure key distribution and the impossibility of secure bit commitment, and derive the quantum formalism from these. This line of inquiry gave birth to a new breed of foundational research with more pragmatic ambitions, with practitioners that split their time between developing novel practical applications of quantum information and achieving a deeper foundational understanding of quantum theory, with each activity informing the other.

1.3 The Shift from Interpretation to Reconstruction

Traditionally, the focus of many quantum foundations researchers was the *interpretation* of quantum theory. In most such works, the formalism of quantum theory was taken as given, and the goal was to infer from this formalism the correct story to tell about the nature of reality—typically, a story of dynamicist flavour. The Everett interpretation [13] and the deBroglie-Bohm interpretation [14] are examples. Models incorporating physical collapses [15, 16] are also proposed in an effort to secure a dynamicist story about quantum theory.

By contrast, the focus of the new wave is the *reconstruction* of quantum theory from physical principles. Contemporary researchers are looking for an answer to Wheeler's famous question *"Why the quantum?"* [17] and are driven to understand the origin of the formalism itself. Textbook postulates such as "a physical system is described by a complex Hilbert space", "pure states are described by unit vectors", "outcome probabilities are given by the Born rule", and "systems combine by the tensor product rule" are now regarded as abstract mathematical statements in need of a more fundamental explanation. Such an explanation would be akin in spirit to Einstein's derivation of the Lorentz transformations from the light postulate and the principle of relativity.

The goal is to find a compelling set of axioms that singles out quantum theory from among all possible theories. Finding an appealing axiomatization is a problem that has a long tradition, starting with the work of Birkhoff and von Neumann [18] and continuing through the works of Mackey [19], Ludwig [20], and Piron [21] and the tradition of quantum logic [22, 23]. What distinguishes the axiomatic work being pursued today is the use of notions inspired by quantum information theory, the emphasis on composite systems, the focus on finite-dimensional Hilbert spaces, and an insistence on axioms that are operationally meaningful.

1.4 The Operational Framework

Any question of the form "why *this*?" is implicitly asking "why not *that*?". Therefore, to tackle Wheeler's question, one first of all needs to be able to conceive of alternatives to quantum theory, ways the world *might have been*. In short, one requires a framework for describing a broad range of physical theories, including quantum and classical theories, but allowing more exotic alternatives as well.

One way to achieve such a framework is to focus on a strictly operational formulation of physical theories. An operational formulation is one wherein the primitive concepts are preparation procedures, transformation procedures, and measurement procedures, each understood as a specification of a list of instructions for an experimentalist, spelled out in sufficient detail that they could be implemented by any technician, as with a good recipe. The theory specifies a mathematical algorithm that fixes the probability distribution over outcomes for every possible measurement given every possible preparation and intervening transformation. When phyical theories are operationally formulated, therefore, the only relevant differences between them are differences in the sorts of experimental statistics that they allow.

The operational approach encourages one to focus on a characterization of quantum theory in terms of experimental facts, and to consequently avoid, as much as is possible, making claims that go beyond what is strictly required to describe these facts. This sort of exercise can be very useful for freeing the mind from all the baggage of classical preconceptions and previous attempts to interpret the quantum formalism. For many researchers, adopting this approach is not a rejection of the need for providing a dynamicist account of quantum theory, nor is it necessarily an endorsement of the notion that a physical theory is *nothing more* than an algorithm for predicting experimental statistics. Rather, it is considered an effective methodological tool for making progress on questions about the origin of the quantum formalism.

1.5 Foil Theories

A distinctive characteristic of contemporary foundations is the exploration of alternatives to quantum theory, that is, *foil theories*. A foil to X is something that helps to highlight the distinctive characteristics of X by contrasting with it.[1] Given a framework of possible theories that includes quantum theory, every nonquantum point in the landscape is a foil theory. Each such theory specifies a way the world might have been had it not been quantum.

We use the term 'foil' to highlight the attitude that is taken towards these theories: they are *not* being proposed as empirical competitors to quantum theory, with grand ambitions of usurping its throne. Rather, they serve to clarify what is distinctive about quantum theory. For instance, if one can identify a foil theory that shares some set of features with quantum theory, then that set of features cannot possibly be a complete set of axioms for quantum theory. Likewise, constructing foil theories is an essential step for proving the independence of a set of axioms: if one axiom is independent from another, then one should be able to devise a foil theory that satisfies the former but violates the latter.

1.6 Goals

One of the ambitions of researchers in quantum foundations is that the insights coming from their work will help with some of the big challenges of contemporary physics, such as the formulation of a quantum theory of gravity. Another ambition is to find alternatives to quantum theory that *could* eventually become empirical competitors. Given an axiomatic derivation of quantum theory, it suffices to modify a single axiom in order to get a consistent alternative. Furthermore, this approach can be used to avoid an important pitfall of more ad hoc approaches to developing alternatives to quantum theory, namely, that the latter may inadvertently violate fundamental principles that one would prefer not to abandon. A good example is the nonlinear modification of quantum theory proposed by Weinberg [24] which was subsequently shown to allow for superluminal signalling [25] and also to violate the second law of thermodynamics for the normal definition of entropy [26]. In the axiomatic approach, the fundamental principles that one wants to uphold can be built in from the outset.

A more practical application of this foundational work is to advance quantum technologies. Indeed, such work is beginning to clarify how information-processing capabilities can arise from foundational principles. For instance, cryptography based on Bell-inequality violations [5, 27] can be shown to be secure even if the devices used in the protocol are supplied by the adversary, as long as it is presumed that the adversary cannot signal superluminally [28, 29]. This idea, which originated from foundational works, led to an entire field of *device-independent cryptography* [28–32].

[1] "Whenever I marry," she continued after a pause which none interrupted, "I am resolved my husband shall not be a rival, but a foil to me."—from Jane Eyre, by Charlotte Brontë.

2 Frameworks for Operational Theories

It is worth spending a few words on the specific frameworks that have been developed in an attempt to achieve the aims described above. Because existing frameworks were found insufficient, many researchers opted to construct a new canvas for their portrait of quantum theory, with quantum information processing serving as their muse. The emphasis posed on the development of such frameworks is itself a distinctive trait of the new wave of foundational research.

To the outsider, it is hard to appreciate the importance of constructing the framework. But it is in fact a highly non-trivial task, where one is forced to make fundamental choices as to what is considered "general" (i.e. part of the notion of a physical theory) and what is considered "specific" (and hence a possible candidate for an axiom that identifies quantum theory). In a sense, what is a stake in the choice of a framework is the very definition of a physical theory.

Note that having a framework for operational theories is not only useful as an instrument for axiomatizations, but also as a playground for experimenting with alternative models of information processing. Such frameworks are increasingly being used to attempt to describe nonclassical phenomena in a language that does not presume the correctness of quantum theory. Not only is this pursued for the question of Bell inequality violations [33–36], but also for a number of applications to computer science and physics, including the study of communication complexity [37, 38], nonlocal computation [39], measurement-based computation [40–44], games and interactive proof systems [45–50], randomness amplification [51–54], causal networks [55–57], computability [58], complexity [59], key distribution [60], bit commitment [61–63], complementarity [64, 65], no cloning [63, 66, 67], teleportation [63, 68, 69], state discrimination [70–72], entropy [73–75], thermodynamics [76–78], general resource theories [79], and spacetime physics [80, 81]. This long list provides a good illustration of how fertile the development of new frameworks has been. In the following we identify the main directions along which the framework-building activity has developed so far.

2.1 The Framework of Convex Operational Theories

A particularly popular framework is that of *convex operational theories*, where preparations, transformations, and measurements are represented by elements of suitable convex sets, the dimension of which is fixed by the nature of the physical systems involved in the experiment.

The framework of convex operational theories is the contemporary descendant of the frameworks used in the tradition of operational quantum logic, in particular those introduced by Mackey [19], Ludwig [20], and Davis and Lewis [82]. In the new wave of quantum foundations, the first elaboration of this framework appeared in Hardy's 2001 axiomatization of quantum theory [83]. With respect to earlier works in quantum logic, Hardy's framework distinguishes itself by being more manageable and intuitive, partly because of its focus on finite-dimensional systems. This approach was brought to completion through a series of works by a number of other authors [66, 69, 84, 85].

2.2 *The Category-Theoretic Framework*

Due to the long tradition of using convex sets to represent the state spaces of physical systems, there is a strong temptation to identify the operational approach with the framework of convex operational theories. However, a substantial part of what defines a physical theory has nothing to do with convex sets, or even with probabilities. For example, operational notions such as composing two systems in parallel (this *and* that) and composing two physical processes in a sequence (do this *and then* do that) are more primitive than the notion of probability. Such notions of composition are the focus of the *category-theoretic framework* initiated by Abramsky and Coecke [68, 86–88]. In this framework, the mathematical structure describing a general physical theory, in particular the two notions of composition and how they interact, is that of a strict symmetric monoidal category. One of the characteristic features of the category-theoretic framework is that all the relations of interest can be encoded in diagrams, similar to those used in the representation of quantum circuits.

2.3 *The Framework of Operational-Probabilistic Theories*

The lesson of the category-theoretic framework is that the composition of systems and processes is fundamental to the operational structure of a theory and that one can talk about information processing without even having to mention probabilities. On the other hand, the precise probabilistic predictions of an operational theory are sometimes a feature of interest. If one is interested in *both* the compositional and the probabilistic features of a theory, then the framework of *operational-probabilistic theories*, recently developed by Chiribella et al. [63, 89, 90] and Hardy [91, 92], provides a supplementation of the category-theoretic framework with probabilistic structure.

In this framework, the category-theoretic notions are used to define circuits of physical processes. An experiment is represented by a closed circuit, starting from the preparation of a system and ending with a measurement having a particular outcome. The probabilistic structure is added on top of the circuit framework by introducing a rule that assigns probabilities to these closed circuits. The result of this construction is that states, transformations, and measurements are represented by elements of suitable vector spaces, as they are in the framework of convex-operational theories. However, the framework of operational-probabilistic theories allows one to describe also theories where the state space is not convex, such as Spekkens' toy theory [93].

In addition, it allows one to treat causality as an emergent feature in a broader class of physical theories where causality is not assumed as part of the framework [63].

When we wish to refer to a framework that can describe features of experimental probabilities, while remaining noncommital about whether it is the framework of convex operational theories or the more general framework of operational-probabilistic theories, we shall speak simply of the framework of *generalized probabilistic theories* (GPTs).

2.4 The Device-Independent Framework

Another popular framework is the *device-independent framework* [28, 29, 94, 95]. Here an experiment is not parsed into preparations, transformations and measurements, with a physical system of a particular dimension acting as a causal mediary between these. Rather, the experiment is treated as a black box, characterized completely by how it maps classical inputs to classical outputs. The roots of this approach can also be traced back to the quantum information revolution: considering input-output black boxes is a natural approach to the design of cryptographic protocols that are secure even if the functioning of the devices is not trusted. In this context, proving the security of a protocol independently of the inner workings of its black box components is desirable because the components may have been designed by one's adversary.

The device-independent framework is apt to capture the *device-independent features* of quantum theory. The paradigmatic example of a device-independent quantum feature is the Tsirelson bound [96], which can be viewed as an upper bound on the probability that two cooperating players win a game, known as the *CHSH* game after the seminal work of Clauser et al. [97]. In the CHSH game, the inputs are the questions asked by a referee to the two players, and the outputs are their answers. While playing the game, the players are allowed to share arbitrary entangled states and are allowed to perform arbitrary local measurements on their systems. Still, their winning probability is upper bounded, independently of the states they prepare and of the measurements they perform. The bound is device-independent, in that it depends only on the validity of quantum theory.

The CHSH game is the problem that got the device-independent approach started, when Popescu and Rohrlich [94] and Rastall [98] came up with a foil theory that is *more nonlocal than quantum theory*, i.e., it guarantees to the players a higher winning probability in the CHSH game. Nevertheless, any other game would define a device-independent feature of quantum theory. The ultimate device-independent feature is the specification of the full set of correlations (i.e. the conditional probability of the outputs given the inputs) that are achievable by local quantum measurements on a bipartite quantum state. This is known as *the quantum set*.

A particularly active line of research in recent years has been the problem of *deriving* device-independent features of quantum theory from information-theoretic principles. The ultimate dream of researchers working in this area is to derive the specific shape of the quantum set by using only device-independent axioms, that is, axioms that refer only to the conditional input-output probabilities. Although the study of information processing in generalized probabilistic theories and the study of device-independent features have developed on separate tracks until now, the time is ripe for uncovering connections between them. On the one hand, the tools developed in the study of axioms for generalized probabilistic theories may help to achieve a characterization of the quantum set, a project that is notoriously difficult. On the other hand, device-independent features may provide candidates for new axioms. A detailed discussion of the connections between the device-independent framework and the framework of general probabilistic theory can be found in Ref. [99].

3 Book Synopsis

The information-theoretic characterization of quantum theory is a general direction that unites the efforts of the new quantum foundationalists, although below this umbrella there is an exceptional variety of different approaches and goals. The book aims to provide a panoramic view of the field, including some of the most promising directions that have emerged in the past decade. It is divided into four sections, corresponding to the following themes:

1. Foil theories (Chaps. 1–3)
2. Axiomatizations (Chaps. 4–8)
3. Categories and convex sets (Chaps. 9–10)
4. Quantum versus super-quantum correlations (Chaps. 11–15)

This subdivision is meant as an aid for readers who are approaching the field for the first time and want to have an idea of the big picture. Many other organizational schemes would have worked just as well, and we therefore encourage readers to explore other paths through the various contributions. In the following, we provide a synopsis of the book through its four sections.

3.1 Foil Theories

We open the book with three examples of foil theories.

Wootters (Chap. 1) considers *real quantum theory* [100–102], which is the foil theory that results from replacing the complex field with the real field in the standard formalism of quantum theory. He considers the information transfer from a preparation to a measurement and shows that for certain natural ways of quantifying this transfer—for instance, the mutual information between the angle of a polarizer that prepares a photon's polarization and the relative frequency of outcomes in a measurement of polarization—the information transfer is optimized for real quantum theory and not for complex quantum theory. He further considers the question of whether some *other* notion of information transfer might pick out complex quantum theory rather than its real counterpart.

Schumacher and Westmoreland (Chap. 2) present and develop *modal quantum theory* [103], which replaces the complex field with a finite field. This necessitates a more dramatic modification of the quantum formalism than is required to replace the complex field with the real field. The foil theory that they construct is *possibilistic* rather than *probabilistic*: it does not specify the probabilities of different measurement outcomes, but only which outcomes are possible and which are impossible. Despite the fact that modal quantum theory is rather minimalist in the scope of states and measurements that it permits, it nonetheless reproduces a surprising number of qualitative features of quantum theory.

Spekkens (Chap. 3) considers a family of foil theories that arise from taking a classical statistical theory and imposing an epistemic restriction, that is, a restriction on the amount of knowledge any observer can have about the physical state of a classical system [93]. Depending on the type of degree of freedom being considered, the resulting foil theory either describes a subset of the preparations, transformations and measurements allowed in the full quantum theory for that type of degree of freedom, or it describes a distortion of such a subset that is inequivalent in its predictions to quantum theory. Both types are shown to reproduce a large number of phenomena that are usually taken to be distinctively quantum, but to lack others, thereby suggesting a distinction between weak and strong notions of nonclassicality.

3.2 *Axiomatizations*

This part of the book presents three different axiomatizations of quantum theory (Chaps. 4–6) along with two contributions on themes that are closely related to the axiomatic endeavour (Chaps. 7–8). For reasons of space, all of the axiomatization chapters confine themselves to presenting an outline of the main ideas behind the derivation of the Hilbert space formalism, while omitting the technicalities that go into the mathematical derivations (these can be found, of course, by referring to the original research articles).

Masanes and Müller (Chap. 4) present their 2011 axiomatization of quantum theory [104]. We start our lineup of axiomatization here because this work is a direct descendant of Hardy's seminal 2001 axiomatization, from which it inherits some of its axioms. With respect to Hardy 2001, the main progress here is in the elimination of one axiom, called the "Simplicity Axiom", which, compared to the others, seemed to be less motivated. Within both the Hardy 2001 and the Masanes-Müller 2011 axiomatizations, the feature that distinguishes quantum from classical theory is the fact that every two pure states are connected by a *continuous* path of pure states.

Chiribella, D'Ariano and Perinotti (Chap. 5) present their axiomatization [89] next. The central axiom here is the Purification Postulate, stating that every mixed