BITTINGER | KEEDY | ELLENBOGEN

Elementary Algebra

Concepts and Applications

FOURTH EDITION

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MARVIN L. BITTINGER
Indiana University–Purdue University at Indianapolis

MERVIN L. KEEDY
Purdue University

DAVID ELLENBOGEN
St. Michael's College

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PREFACE

Appropriate for a one-term course in elementary algebra, this text is intended for students who have a firm background in arithmetic. It is the first of two texts in an algebra series that also includes *Intermediate Algebra: Concepts and Applications*, Fourth Edition, by Bittinger/Keedy/Ellenbogen. *Elementary Algebra: Concepts and Applications*, Fourth Edition, is a significant revision of the Third Edition with respect to design, contents, pedagogy, and an expanded supplements package. This series is designed to prepare students for any mathematics course at the college algebra level.

APPROACH

Our approach, which has been developed over many years, is designed to help today's students both learn and retain mathematical concepts. Our goal in preparing this revision was to address the major challenges for teachers of developmental mathematics courses that we have seen emerging during the early 1990s. The first challenge is to prepare students of developmental mathematics to make the transition from "skills-oriented" elementary and intermediate algebra courses to the more "concept-oriented" presentation of college algebra or other college-level mathematics courses. The second is to teach these same students critical-thinking skills: to reason mathematically, to communicate mathematically, and to solve mathematical problems. The third challenge is to reduce the amount of content overlap between elementary algebra and intermediate algebra texts.

Following are some aspects of the approach that we have used in this revision to help meet the challenges we all face teaching developmental mathematics.

PROBLEM SOLVING

One distinguishing feature of our approach is our treatment of and emphasis on problem solving. We use problem solving and applications to motivate the material wherever possible, and we include real-life applications and problem-solving techniques throughout the text. We feel that problem solving encourages students to think about how mathematics can be used. It also challenges students and helps to prepare them for more difficult material in later courses.

- In Chapter 2, we introduce the five-step process for solving problems: (1) Familiarize, (2) Translate, (3) Carry out, (4) Check, and (5) State the answer. These steps are used throughout the text whenever we encounter a problem-solving situation. Repeated use of this algorithm gives students a sense that they have a starting point for any type of problem they encounter, and frees them to focus on the mathematics necessary to successfully translate the problem situation. (See pages 84–86, 302, 303, 341, and 342.)

CONTENT

Chapter 1 includes a brief review of arithmetic topics and then moves quickly into elementary algebra topics. This allows instructors sufficient time to cover the topics necessary to prepare students for intermediate algebra.

- Chapter 3 contains an intuitive introduction to graphing, a topic that is integrated throughout the text. This helps students to visualize the mathematics of many concepts while at the same time allowing them to develop facility with graphing throughout the course.

PEDAGOGY

Skill Maintenance Exercises and *Cumulative Reviews.* Retention of skills is critical to the future success of our students. In nearly all exercise sets, we include carefully chosen exercises that review skills and concepts from preceding chapters of the text. Each chapter test includes Skill Maintenance Exercises selected from the three or four text sections that are identified at the beginning of each chapter. After Chapters 3, 6, and 10, we have also included a Cumulative Review, which reviews skills and concepts from all preceding chapters of the text. (See pages 114, 227, 315, 316, and 363.)

Synthesis Exercises. Each exercise set ends with a set of synthesis exercises. These problems can offer opportunities for students to synthesize skills and concepts from earlier sections with the present material, or can provide students with deeper insights into the current topic. Synthesis exercises are generally more challenging than those in the main body of the exercise set. (See pages 21, 79, 141, and 233.)

Verbalization Skills. Wherever appropriate throughout the text, we have discussed how mathematical terms are used in language. The Summary and Review sections emphasize key terms and important properties and formulas. In addition, thinking and writing exercises are included in the Synthesis Exercises. These encourage students to verbalize mathematical concepts, leading to better understanding. (See pages 45 and 253.)

WHAT'S NEW IN THE FOURTH EDITION?

We have rewritten many key topics in response to user and reviewer feedback and have made significant improvements in design and pedagogy. Detailed information about the content changes is available in the form of a Conversion Guide. Please ask your local Addison-Wesley sales representative for more information. Following is a list of the major changes in this revision.

New Design

- The new design is more open and readable. Pedagogical use of color makes it easier to see where exercises, explanations, and examples begin and end.
- The entire art program is new for this edition. We have ensured the accuracy of the graphical art through the use of computer-generated graphs. Color in the graphical art is used pedagogically and precisely to help the student visualize the mathematics. (See pages 114 and 338.)

Technology Connections

- These features integrate technology, increase the understanding of concepts through visualization, encourage exploration, and motivate discovery learning. Optional Technology Connection exercises occur in many exercise sets. (See page 330.)

Writing Exercises

- Nearly every set of Synthesis Exercises begins with two writing exercises. These exercises are usually not as difficult as other synthesis exercises, but require written answers that aid in student comprehension, critical thinking, and conceptualization. Because some instructors may collect answers to writing exercises, and because more than one answer may be correct, answers to writing exercises are provided only for the chapter reviews. (See pages 30 and 41.)

Content Changes. A variety of content changes have been made. Some of the more significant changes are listed below.

- Graphing is now introduced in Chapter 3 and used as a tool for solving problems there and in later chapters.
- Negative exponents are now covered at the *end* of Chapter 4. This change simplifies the development of the properties of exponents and enables instructors to cover negative exponents at their option.
- Although our fear of students performing "illegal" cancellations is as acute as ever, we now use canceling as a way to simplify rational expressions. We do so in recognition of the fact that we use canceling when working on our own. Whenever canceling is used, we point out that we are effectively "removing" a factor of 1. (See pages 260–261.)
- Differences of squares are now factored *after* polynomials of the type $x^2 + bx + c$ and $ax^2 + bx + c$. This change should better enable students to view differences of squares as a special type of quadratic polynomial.
- Because so many students remain convinced that they "cannot do word problems," we have made increased use of guessing as a means of familiarizing oneself with a problem-solving situation. By checking to see if a guess is correct, students can more easily discover an algebraic translation of the problem. (See pages 88–91.)
- The use of geometry as a way of visualizing concepts has been expanded (see

Sections 4.4 and 4.5). Similar triangles are now covered in Section 6.8, "Problem Solving: Rational Equations and Proportions."

- Throughout the text, we have included a variety of new applications that appeal to a large cross section of the student population. By emphasizing applications that students and faculty find interesting, we hope that we have made the text enjoyable to use. (See pages 114, 116, and 297).

SUPPLEMENTS FOR THE INSTRUCTOR

INSTRUCTOR'S SOLUTIONS MANUAL
by Judith A. Penna

This supplement contains worked-out solutions to all exercises in the text.

INSTRUCTOR'S RESOURCE GUIDE
by Donna DeSpain

This supplement contains the following:

- Extra practice problems for challenging topics in the text
- Black-line masters of grids and number lines for transparency masters or test preparation
- Videotape index and section cross references to the tutorial software packages available with this text
- Conversion guide from the Third Edition to the Fourth Edition

PRINTED TEST BANK
by Donna DeSpain

This supplement contains the following:

- Six alternative test forms for each chapter and six final examinations
- Two multiple-choice versions of each chapter test

All test forms have been completely rewritten.

COMPUTERIZED TESTING

Omnitest II (for IBM and Macintosh). This computerized test bank allows you to create up to 99 versions of a customized test with just a few keystrokes, and allows the option of choosing items by chapter, section, or objective. It contains over 400 multiple-choice and open-ended algorithms. You may enter your own test items, edit existing items, and determine the level of difficulty of problems.

SUPPLEMENTS FOR THE STUDENT

STUDENT'S SOLUTIONS MANUAL
by Judith A. Penna

This manual contains completely worked-out solutions with step-by-step annotations for all the odd-numbered exercises in the text, and answers for all even-numbered exercises in the text.

VIDEOTAPES

Developed especially for the Bittinger/Keedy/Ellenbogen texts, these videotapes feature an engaging team of lecturers presenting material from each section of the text in an interactive format that includes a group of students. The lecturers' presentations often incorporate slides, sophisticated computer-generated graphics, and a white board to support an approach that emphasizes visualization and problem solving.

TUTORIAL SOFTWARE

THE MATHLAB+ (IBM and Macintosh). This software combines a unique combination of drill and practice modules with an interactive and easy-to-use graphing tool. The drill and practice segments feature feedback for wrong answers and detailed record keeping. The graphing tool allows students to graph and explore a wide variety of two-dimensional functions.

Algebra Problem Solver (IBM). After selecting a topic and an exercise type, students can enter their own exercises or request an exercise from the computer. In each case, the student is given detailed, annotated, step-by-step solutions.

ACKNOWLEDGMENTS

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This book's sponsoring editor, Melissa Acuña, performed admirably in coordinating the many intricacies of this project; Lenore Parens, as developmental editor; contributed many useful ideas and suggestions; George and Brian Morris of Scientific Illustrators generated a remarkable set of graphs and illustrations that are both precise and easily understood; and Leo Harrington drew the many fine sketches that enhance our exercises and examples. Geri Davis and Martha Morong of Quadrata, Inc., provided design, editorial, and production services second to none, ensuring that every last detail has been taken care of. To all of these people, we offer our deepest thanks.

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Finally, a special thank you to all those who so generously agreed to discuss their professional uses for mathematics in our chapter openers. These dedicated people, none of whom we knew prior to writing this text, all share a desire to make math more meaningful to students. We cannot imagine a finer set of role models.

M.L.B.
M.L.K.
D.J.E.

CONTENTS

1 Introduction to Algebra and Algebraic Expressions

2 Equations, Inequalities, and Problem Solving

10 Quadratic Equations

CHAPTER 1

Introduction to Algebra and Algebraic Expressions

AN APPLICATION

In the course of one four-month period, the water level of Lake Champlain went down 2 ft, up 1 ft, down 5 ft, and up 3 ft. How much had the lake level changed at the end of the four months?

This problem appears as Example 7 in Section 1.5.

Sharon Meyer
TELEVISION WEATHER FORECASTER

"Whether it's in calculating the number of heating degree days, the probability of precipitation, or changes in lake level, a forecaster's career relies heavily on a solid understanding of mathematics. I use math every single day as part of my job."

Problem solving is the focus of this text. In this chapter we discuss some preliminaries that are needed for the problem-solving approach that we develop and begin to use in Chapter 2. We also review some arithmetic, discuss real numbers and their properties, and examine how real numbers are added, subtracted, multiplied, divided, and raised to powers.

1.1 Introduction to Algebra

Algebraic Expressions • Translating to Algebraic Expressions • Translating to Equations

This section introduces some basic concepts of algebra. Since equation solving is central to the study of algebra, we concentrate on the expressions that appear in equations and some important words for translating English to mathematics.

Algebraic Expressions

Probably the greatest difference between arithmetic and algebra is the extensive use of *variables* in algebra. When a letter is used to stand for any number chosen from a variety of numbers, we call the number a **variable.** For example, if n represents the number of students registered for a college's 8 AM section of Elementary Algebra, the number n will vary from semester to semester, if not from day to day. If each student in the 8:00 AM Elementary Algebra section paid \$500 to take the class, the college would collect a total of $500 \cdot n$ dollars. Since the cost per student, \$500, is the same regardless of how many students are registered, the number 500 is called a **constant.** The number of registered students n can vary, so n is a variable.

Cost Per Student (in Dollars)	Number of Students Registered	Total Collected (in Dollars)
500	n	$500 \cdot n$

The expression $500 \cdot n$ is called a **variable expression** because its value varies with the choice of n. Of course, the total amount collected, $500 \cdot n$, will grow as the number of students registered grows. We now replace n with a variety of values and compute the total amount collected. In doing so, we say that we are **evaluating** the expression $500 \cdot n$.

Cost Per Student (in Dollars) 500	Number of Students Registered n	Total Collected (in Dollars) $500 \cdot n$
500	20	10,000
500	25	12,500
500	30	15,000

Variable expressions are examples of *algebraic expressions*. An **algebraic expression** consists of variables, numerals, operation signs, and/or grouping symbols. Examples of algebraic expressions are

$$x + 35, \quad 7 \cdot t, \quad 13a - b, \quad 15 \div z, \quad \tfrac{9}{7}, \quad \text{and} \quad 3x(a + b).$$

Note that a fraction bar is a division symbol: $\frac{9}{7}$ means $9 \div 7$. Similarly, multiplication can be written in several ways. For example, "7 times t" can be written as $7 \cdot t$, $7 \times t$, $7(t)$, or simply $7t$.

To evaluate an algebraic expression, we **substitute** a number for each variable in the expression.

EXAMPLE 1

Evaluate the expression for the given values: **(a)** $x + y$ when $x = 37$ and $y = 28$; **(b)** $5ab$ when $a = 2$ and $b = 3$.

Solution

a) We substitute 37 for x and 28 for y and carry out the addition:

$$x + y = 37 + 28 = 65.$$

The number 65 is called the **value** of the expression.

b) We substitute 2 for a and 3 for b and multiply:

$$5ab = 5 \cdot 2 \cdot 3 = 10 \cdot 3 = 30.$$

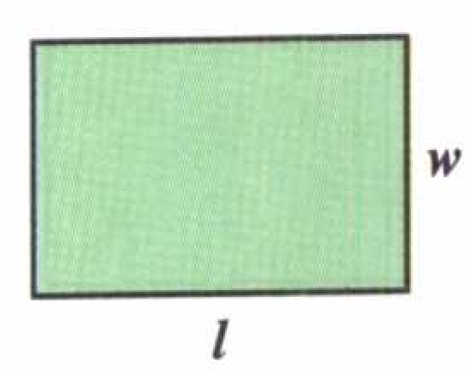
❑

EXAMPLE 2

The area A of a rectangle of length l and width w is given by the formula $A = lw$. Find the area when l is 24.5 in. and w is 10 in.

Solution We evaluate, using 24.5 in. for l and 10 in. for w and carry out the multiplication:

$$\begin{aligned} A = lw &= (24.5 \text{ in.})(10 \text{ in.}) \\ &= (24.5)(10)(\text{in.})(\text{in.}) \\ &= 245 \text{ in}^2, \quad \text{or 245 square inches.} \end{aligned}$$



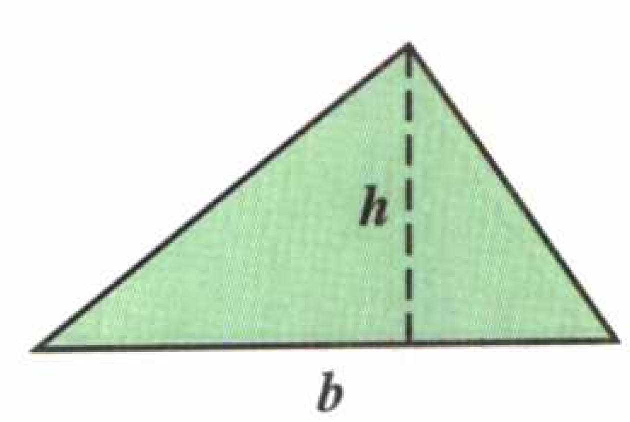
Note that (in.)(in.) = in^2. Exponents are discussed in detail in Section 1.8. ❑

EXAMPLE 3

The area of a triangle with a base of length b and a height of length h is given by the formula $A = \frac{1}{2}bh$. Find the area when b is 8 m and h is 6.4 m.

Solution We substitute 8 m for b and 6.4 m for h and carry out the multiplication:

$$\begin{aligned} A = \tfrac{1}{2}bh &= \tfrac{1}{2}(8 \text{ m})(6.4 \text{ m}) \\ &= \tfrac{1}{2}(8)(6.4)(\text{m})(\text{m}) \\ &= 4(6.4) \text{ m}^2 \\ &= 25.6 \text{ m}^2, \quad \text{or 25.6 square meters.} \end{aligned}$$



❑

Translating to Algebraic Expressions

Before attempting to translate problems to equations, we need to be able to translate certain phrases to algebraic expressions.

KEY WORDS

Addition (+)	**Subtraction** (−)	**Multiplication** (·)	**Division** (÷)
add	subtract	multiply	divide
sum	difference	product	quotient
plus	minus	times	divided by
more than	less than	twice	ratio of
increased by	decreased by	of	
greater than	take from		

EXAMPLE 4

Translate each phrase to an algebraic expression.

a) Twice (or two times) some number
b) Seven less than some number
c) Eighteen more than a number
d) A number divided by five

Solution

a) Think of some number, say 8. What number is twice 8? It is 16. To get 16, you multiplied by 2. Now consider a variable. Use y to represent "some number" and multiply by 2. The expression

$$y \times 2, \qquad 2 \times y, \qquad 2 \cdot y, \quad \text{or} \quad 2y$$

is the translation of "Twice (or two times) some number."

b) We let x represent "some number." Now if the number were 23, then the translation would be $23 - 7$. If the number were 345, the translation would be $345 - 7$. If the number is x, the translation of "Seven less than some number" is

$$x - 7.$$

c) If we knew the number to be 10, the translation would be $10 + 18$, or $18 + 10$. We let t represent "a number," so the translation of "Eighteen more than a number" is

$$t + 18, \quad \text{or} \quad 18 + t.$$

d) We let m represent "a number." If the number were 9, the translation would be $9 \div 5$, or $\frac{9}{5}$. Thus our translation of "A number divided by five" is

$$m \div 5, \quad \text{or} \quad \frac{m}{5}.$$

❑

CAUTION! Because the order in which we subtract and divide affects the answer, answering $7 - x$ or $5 \div m$ in Examples 4(b) and 4(d) is incorrect.

EXAMPLE 5

Translate each of the following.

a) Some number, increased by 5
b) Half of a number
c) Five more than three times some number
d) Six less than the product of two numbers
e) Seventy-six percent of some number