The Physics of Plasmas

T. J. M. Boyd and J. J. Sanderson

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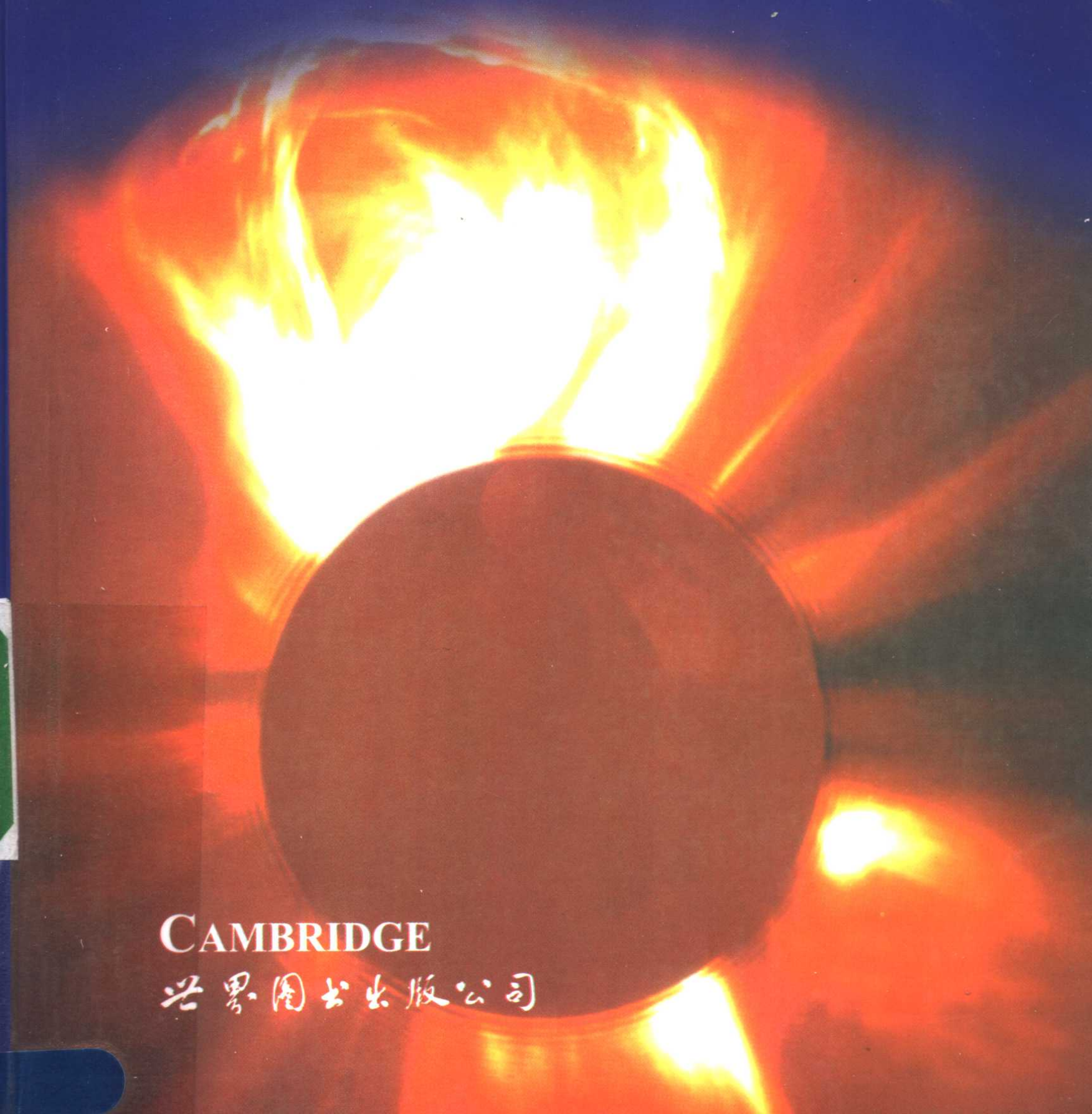

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Preface

The present book has its origins in our earlier book *Plasma Dynamics* published in 1969. Many who used *Plasma Dynamics* took the trouble to send us comments, corrections and criticism, much of which we intended to incorporate in a new edition. In the event our separate preoccupations so delayed this that we came to the conclusion that we should instead write another book, that might better reflect changes of emphasis in the subject since the original publication. In writing we had two aims. The first was to describe topics that have a place in any core curriculum for plasma physics, regardless of subsequent specialization and to do this in a way that, while keeping physical understanding firmly in mind, did not compromise on a proper mathematical framework for developing the subject. At the same time we felt the need to go a step beyond this and illustrate and extend this basic theory with examples drawn from topics in fusion and space plasma physics.

In developing the subject we have followed the traditional approach that in our experience works best, beginning with particle orbit theory. This combines the relative simplicity of describing the dynamics of a single charged particle, using concepts familiar from classical electrodynamics, before proceeding to a variety of magnetohydrodynamic (MHD) models. Some of the intrinsic difficulties in getting to grips with magnetohydrodynamics stem from the persistent neglect of classical fluid dynamics in most undergraduate physics curricula. To counter this we have included in Chapter 3 a brief outline of some basic concepts of fluid dynamics before characterizing the different MHD regimes. This leads on to a detailed account of ideal MHD in Chapter 4 followed by a selection of topics illustrating different aspects of resistive MHD in Chapter 5. Plasmas support a bewildering variety of waves and instabilities and the next two chapters are given over to classifying the most important of these. Chapter 6 continues the MHD theme, dealing with waves which can be described macroscopically. In contrast to normal fluids, plasmas are characterized by modes which have to be described microscopically, i.e. in terms of kinetic theory, because only particular particles in the distribution interact with the modes in question. An introduction to plasma kinetic theory is included in Chapter 7 along with a full discussion of the basic modes, the physics of which is governed largely by wave–particle interactions. The development of kinetic theory is continued in Chapter 8 but with a change of emphasis. Whereas the effect of collisions between plasma particles is disregarded in Chapter 7, these move centre stage in Chapter 8 with an introduction to another key topic, plasma transport theory.

A thorough grounding in plasma physics is provided by a selection of topics from the first eight chapters, which make up a core syllabus irrespective of subsequent specialization. The remaining chapters develop the subject and provide a basis for more specialized courses, although arguably Chapter 9 on plasma radiation is properly part of any core syllabus. This chapter, which discusses the principal sources of plasma radiation, excepting bound–bound transitions, along with an outline of radiative transport and the scattering of radiation by laboratory plasmas, provides an introduction to a topic which underpins a number of key plasma diagnostics. Chapters 10 and 11 deal in turn and in different ways with aspects of non-linear plasma physics and with effects in inhomogeneous plasmas. Both subjects cover such a diversity of topics that we have been limited to a discussion of a number of examples, chosen to illustrate the methodology and physics involved. In Chapter 10 we mainly follow a tutorial approach, outlining a variety of important non-linear effects, whereas in Chapter 11 we describe in greater detail a few particular examples by way of demonstrating the effects of plasma inhomogeneity and physical boundaries. The book ends with a chapter on the classical theory of plasmas in which we outline the comprehensive mathematical structure underlying the various models used, highlighting how these relate to one another.

An essential part of getting to grips with any branch of physics is working through exercises at a variety of levels. Most chapters end with a selection of exercises ranging from simple quantitative applications of basic results on the one hand to others requiring numberical solution or reference to original papers.

We are indebted to many who have helped in a variety of ways during the long period it has taken to complete this work. For their several contributions, comments and criticism we thank Hugh Barr, Alan Cairns, Angela Dyson, Pat Edwin, Ignazio Fidone, Malcolm Haines, Alan Hood, Gordon Inverarity, David Montgomery, Ricardo Ondarza-Rovira, Sean Oughton, Eric Priest, Bernard Roberts, Steven Schwartz, Greg Tallents, Alexey Tatarinov and Andrew Wright. We are indebted to Dr J.M. Holt for permission to reproduce Fig. 9.16. Special thanks are due to Andrew Mackwood who prepared the figures and to Misha Sanderson who shared with Andrew the burden of producing much of the LaTeX copy. Finally, we thank Sally Thomas, our editor at CUP, for her ready help and advice in bringing the book to press.

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Introduction

1.1 Introduction

The plasma state is often referred to as the *fourth* state of matter, an identification that resonates with the element of *fire*, which along with earth, water and air made up the elements of Greek cosmology according to Empedocles.† Fire may indeed result in a transition from the gaseous to the plasma state, in which a gas may be fully or, more likely, partially ionized. For the present we identify as *plasma* any state of matter that contains enough free charged particles for its dynamics to be dominated by electromagnetic forces. In practice quite modest degrees of ionization are sufficient for a gas to exhibit electromagnetic properties. Even at 0.1 per cent ionization a gas already has an electrical conductivity almost half the maximum possible, which is reached at about 1 per cent ionization.

The outer layers of the Sun and stars in general are made up of matter in an ionized state and from these regions winds blow through interstellar space contributing, along with stellar radiation, to the ionized state of the interstellar gas. Thus, much of the matter in the Universe exists in the plasma state. The Earth and its lower atmosphere is an exception, forming a plasma-free oasis in a plasma universe. The upper atmosphere on the other hand, stretching into the ionosphere and beyond to the magnetosphere, is rich in plasma effects.

Solar physics and in a wider sense cosmic electrodynamics make up one of the roots from which the physics of plasmas has grown; in particular, that part of the subject known as magnetohydrodynamics – MHD for short – was established largely through the work of Alfvén. A quite separate root developed from the physics of gas discharges, with glow discharges used as light sources and arcs as a means of cutting and welding metals. The word *plasma* was first used by Langmuir in 1928 to describe the ionized regions in gas discharges. These origins

† Empedocles, who lived in Sicily in the shadow of Mount Etna in the fifth century BC, was greatly exercised by fire. He died testing his theory of buoyancy by jumping into the volcano in 433BC.

are discernible even today though the emphasis has shifted. Much of the impetus for the development of plasma physics over the second half of the twentieth century came from research into controlled thermonuclear fusion on the one hand and astrophysical and space plasma phenomena on the other.

To a degree these links with 'big science' mask more bread-and-butter applications of plasma physics over a range of technologies. The use of plasmas as sources for energy-efficient lighting and for metal and waste recycling and their role in surface engineering through high-speed deposition and etching may seem prosaic by comparison with fusion and space science but these and other commercial applications have laid firm foundations for a new plasma technology. That said, our concern throughout this book will focus in the main on the physics of plasmas with illustrations drawn where appropriate from fusion and space applications.

1.2 Thermonuclear fusion

While thermonuclear fusion had been earlier indentified as the source of energy production in stars it was first discussed in detail by Bethe, and independently von Weizsäcker, in 1938. The chain of reactions proposed by Bethe, known as the carbon cycle, has the distinctive feature that after a sequence of thermonuclear burns involving nitrogen and oxygen, carbon is regenerated as an end product enabling the cycle to begin again. For stars with lower central temperatures the proton–proton cycle

$$\begin{aligned}
{}_1\mathrm{H}^1 + {}_1\mathrm{H}^1 &\rightarrow {}_1\mathrm{D}^2 + \mathrm{e}^+ + \nu && (1.44\,\mathrm{MeV})\\
{}_1\mathrm{D}^2 + {}_1\mathrm{H}^1 &\rightarrow {}_2\mathrm{He}^3 + \gamma && (5.49\,\mathrm{MeV})\\
{}_2\mathrm{He}^3 + {}_2\mathrm{He}^3 &\rightarrow {}_2\mathrm{He}^4 + 2\,{}_1\mathrm{H}^1 && (12.86\,\mathrm{MeV})
\end{aligned}$$

where e^+, ν and γ denote in turn a positron, neutrino and gamma-ray, is more important and is in fact the dominant reaction chain in lower main sequence stars (see Salpeter (1952)). Numbers in brackets denote the energy per reaction. In the first reaction in the cycle, the photon energy released following positron–electron annihilation (1.18 MeV) is included; the balance (0.26 MeV) carried by the neutrino escapes from the star. The third reaction in the cycle is only possible at temperatures above about 10^7 K but accounts for almost half of the total energy release of 26.2 MeV. The proton–proton cycle is dominant in the Sun, the transition to the carbon cycle taking place in stars of slightly higher mass. The energy produced not only ensures stellar stability against gravitational collapse but is the source of luminosity and indeed all aspects of the physics of the outer layers of stars.

The reaction that offers the best energetics for controlled thermonuclear fusion in the laboratory on the other hand is one in which nuclei of deuterium and tritium fuse to yield an alpha particle and a neutron:

$$_1\mathrm{D}^2 + {}_1\mathrm{T}^3 \rightarrow {}_2\mathrm{He}^4 + {}_0\mathrm{n}^1 \quad (17.6\,\mathrm{MeV})$$

The total energy output $\Delta E = 17.6\,\mathrm{MeV}$ is distributed between the alpha particle which has a kinetic energy of about 3.5 MeV and the neutron which carries the balance of the energy released. The alpha particle is confined by the magnetic field containing the plasma and used to heat the fuel, whereas the neutron escapes through the wall of the device and has to be contained by a neutron-absorbing blanket.

1.2.1 The Lawson criterion

Although the D–T reaction rate peaks at temperatures of the order of 100 keV it is not necessary for reacting nuclei to be as energetic as this, otherwise controlled thermonuclear fusion would be impracticable. Thanks to quantum tunnelling through the Coulomb barrier, the reaction rate for nuclei with energies of the order of 10 keV is sufficiently large for fusion to occur. A simple and widely used index of thermonuclear gain is provided by the Lawson criterion. For equal deuterium and tritium number densities, $n_{\mathrm{D}} = n_{\mathrm{T}} = n$, the thermonuclear power generated by a D–T reactor per unit volume is $P_{\mathrm{fus}} = \frac{1}{4}n^2\langle\sigma v\rangle\Delta E$, where $\langle\sigma v\rangle$ denotes the reaction rate, σ being the collisional cross-section and v the relative velocity of colliding particles. For a D–T plasma at a temperature of 10 keV, $\langle\sigma v\rangle \sim 1.1 \times 10^{-22}\,\mathrm{m^3\,s^{-1}}$ so that $P_{\mathrm{fus}} \sim 7.7 \times 10^{-35}n^2\,\mathrm{W\,m^{-3}}$. About 20% of this output is alpha particle kinetic energy which is available to sustain the fuel at thermonuclear reaction temperatures, the balance being carried by the neutrons which escape from the plasma. Thus the power absorbed by the plasma is $P_\alpha = \frac{1}{4}\langle\sigma v\rangle n^2 E_\alpha$ where $E_\alpha = 3.5\,\mathrm{MeV}$. This is the heat added to unit volume of plasma per unit time as a result of fusion.

We have to consider next the energy lost through radiation, in particular as bremsstrahlung from electron–ion collisions. We shall find in Chapter 9 that bremsstrahlung power loss from hot plasmas may be represented as $P_{\mathrm{b}} = \alpha n^2 T^{1/2}$, where α is a constant and T denotes the plasma temperature. Above some critical temperature the power absorbed through alpha particle heating outstrips the bremsstrahlung loss. Other energy losses besides bremsstrahlung have to be taken into consideration. In particular, heat will be lost to the wall surrounding the plasma at a rate $3nk_{\mathrm{B}}T/\tau$ where τ is the containment time and k_{B} is Boltzmann's constant. Balancing power gain against loss we arrive at a relation for $n\tau$. Lawson (1957) introduced an efficiency factor η to allow power available for heating to be expressed in terms of the total power leaving the plasma. The *Lawson criterion* for power

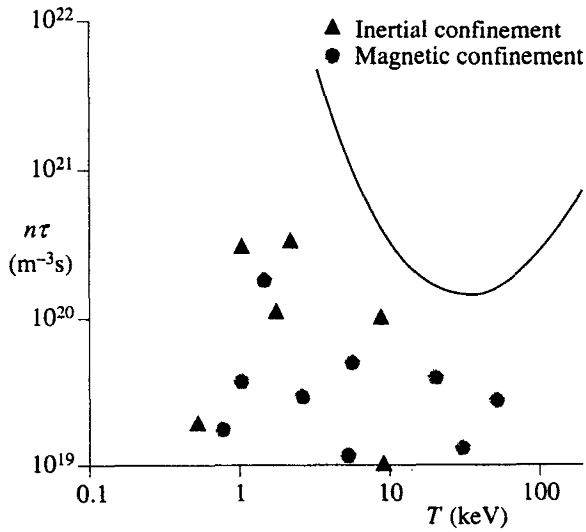

Fig. 1.1. The Lawson criterion for ignition of fusion reactions. Data points correspond to a range of magnetic and inertial confinement experiments showing a progression towards the Lawson curve.

gain is then

$$n\tau > \frac{3k_B T}{\left[\frac{\eta}{4(1-\eta)}\langle\sigma v\rangle \Delta E - \alpha T^{1/2}\right]} \tag{1.1}$$

This condition is represented in Fig. 1.1. Using Lawson's choice for $\eta = 1/3$ (which with hindsight is too optimistic), the power-gain condition reduces to $n\tau > 10^{20}\,\mathrm{m}^{-3}\,\mathrm{s}$. The data points shown in Fig. 1.1 are $n\tau$ values from a range of both magnetically and inertially contained plasmas over a period of about two decades, showing the advances made in both confinement schemes towards the Lawson curve.

1.2.2 Plasma containment

Hot plasmas have to be kept from contact with walls so that from the outset magnetic fields have been used to contain plasma in controlled thermonuclear fusion experiments. Early devices such as Z-pinches, while containing and pinching the plasma radially, suffered serious end losses. Other approaches trapped the plasma in a magnetic bottle or used a closed toroidal vessel. Of the latter the *tokamak*, a contraction of the Russian for *toroidal magnetic chamber*, has been the most successful. Its success compared with competing toroidal containment schemes is

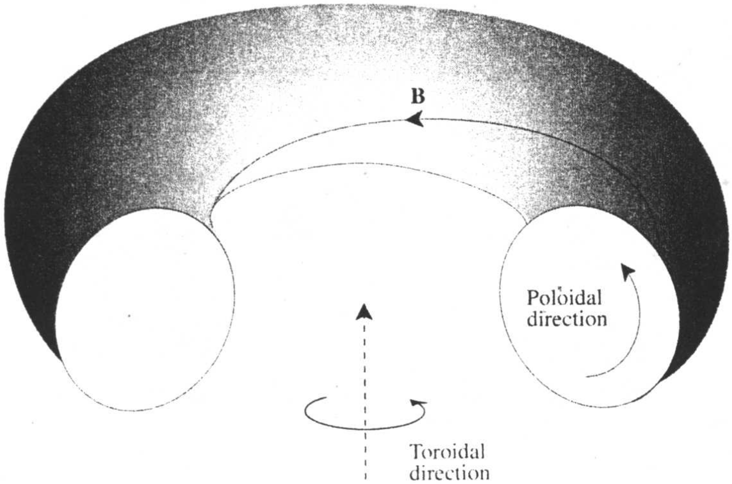

Fig. 1.2. Tokamak cross-section.

attributable in large part to the structure of the magnetic field used. Tokamak fields are made up of two components, one *toroidal*, the other *poloidal*, with the resultant field winding round the torus as illustrated in Fig. 1.2. The toroidal field produced by currents in external coils is typically an order of magnitude larger than the poloidal component and it is this aspect that endows tokamaks with their favourable stability characteristics. Whereas a plasma in a purely toroidal field drifts towards the outer wall, this drift may be countered by balancing the outward force with the magnetic pressure from a poloidal field, produced by currents in the plasma. Broadly speaking, the poloidal field maintains toroidal stability while the toroidal field provides radial stability. For a typical tokamak plasma density the Lawson criterion requires containment times of a few seconds.

Inertial confinement fusion (ICF) offers a distinct alternative to magnetic containment fusion (MCF). In ICF the plasma, formed by irradiating a target with high-power laser beams, is compressed to such high densities that the Lawson criterion can be met for confinement times many orders of magnitude smaller than those needed for MCF and short enough for the plasma to be confined inertially. The ideas behind inertial confinement are represented schematically in Fig. 1.3(a) showing a target, typically a few hundred micrometres in diameter filled with a D–T mixture, irradiated symmetrically with laser light. The ionization at the target surface results in electrons streaming away from the surface, dragging ions in their wake. The back reaction resulting from ion blow-off compresses the target and the aim of inertial confinement is to achieve compression around 1000 times

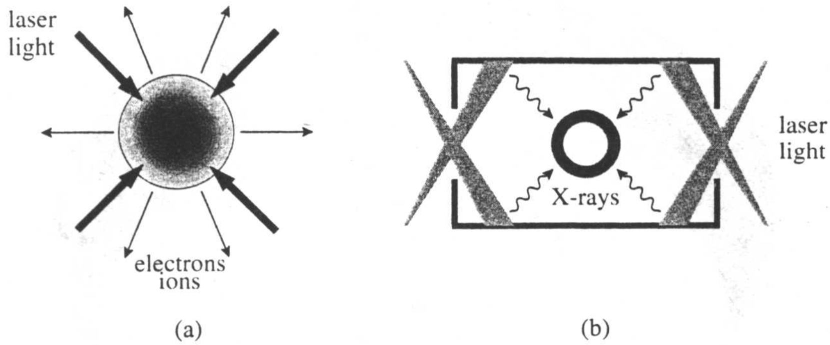

Fig. 1.3. Direct drive (a) and indirect drive (hohlraum) (b) irradiation of targets by intense laser light.

liquid density with minimal heating of the target until the final phase when the compressed fuel is heated to thermonuclear reaction temperatures. An alternative to the *direct drive* approach illustrated in Fig. 1.3(a) is shown in Fig. 1.3(b) in which the target is surrounded by a *hohlraum*. Light enters the hohlraum and produces X-rays which in turn provide target compression and *indirect drive* implosion.

1.3 Plasmas in space

Thermonuclear burn in stars is the source of plasmas in space. From stellar cores where thermonuclear fusion takes place, keV photons propagate outwards towards the surface, undergoing energy degradation through radiation–matter interactions on the way. In the case of the Sun the surface is a black body radiator with a temperature of 5800 K. Photons propagate outwards through the radiation zone across which the temperature drops from about 10^7 K in the core to around 5×10^5 K at the boundary with the convection zone. This boundary is marked by a drop in temperature so steep that radiative transfer becomes unstable and is supplanted as the dominant mode of energy transport by the onset of convection.

Just above the convection zone lies the photosphere, the visible 'surface' of the Sun, in the sense that photons in the visible spectrum escape from the photosphere. UV and X-ray surfaces appear at greater heights. Within the photosphere the Sun's temperature falls to about 4300 K and then unexpectedly begins to rise, a transition that marks the boundary between photosphere and chromosphere. At the top of the chromosphere temperatures reach around 20 000 K and heating then surges dramatically to give temperatures of more than a million degrees in the corona.

The surface of the Sun is characterized by magnetic structures anchored in the photosphere. Not all magnetic field lines form closed loops; some do not close