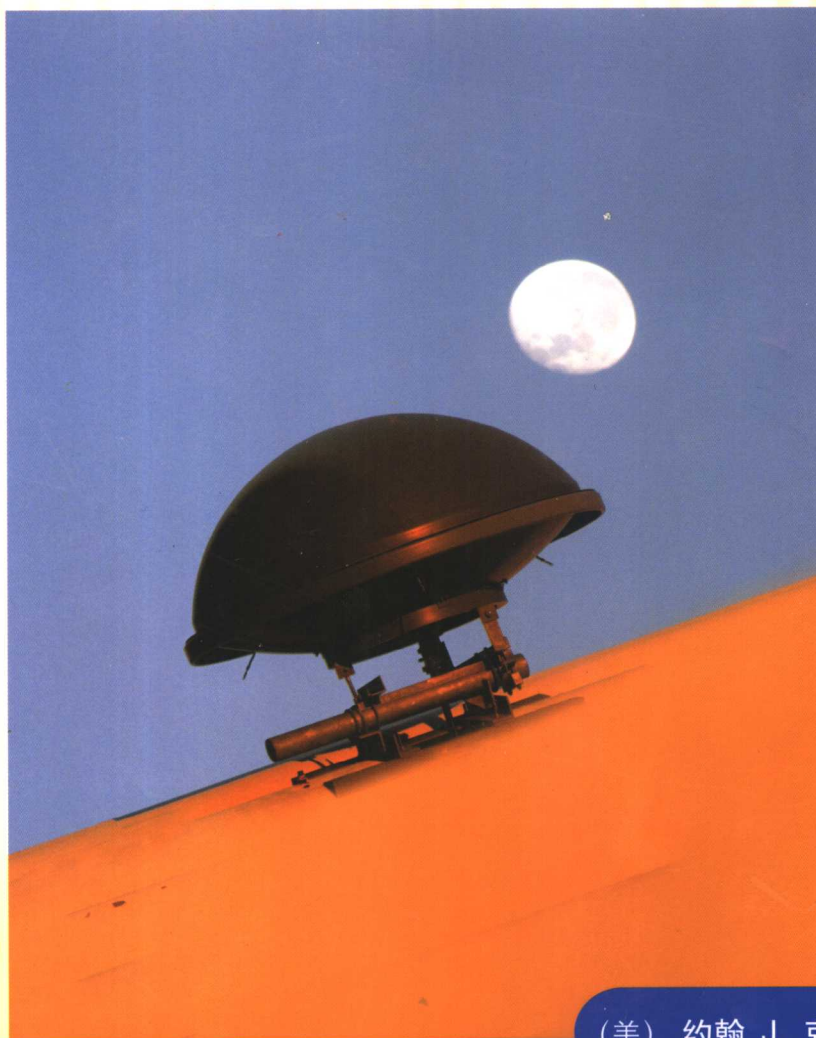


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机器人学导论

(英文版·第3版)



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机械工业出版社
China Machine Press

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江苏工业学院图书馆
藏书章

(美) 约翰 J. 克拉格 著



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出版者的话

文艺复兴以降，源远流长的科学精神和逐步形成的学术规范，使西方国家在自然科学的各个领域取得了垄断性的优势；也正是这样的传统，使美国在信息技术发展的六十多年间名家辈出、独领风骚。在商业化的进程中，美国的产业界与教育界越来越紧密地结合，计算机学科中的许多泰山北斗同时身处科研和教学的最前线，由此而产生的经典科学著作，不仅擘划了研究的范畴，还揭集了学术的源变，既遵循学术规范，又自有学者个性，其价值并不会因年月的流逝而减退。

近年，在全球信息化大潮的推动下，我国的计算机产业发展迅猛，对专业人才的需求日益迫切。这对计算机教育界和出版界都既是机遇，也是挑战；而专业教材的建设在教育战略上显得举足轻重。在我国信息技术发展时间较短、从业人员较少的现状下，美国等发达国家在其计算机科学发展的几十年间积淀的经典教材仍有许多值得借鉴之处。因此，引进一批国外优秀计算机教材将对我国计算机教育事业的发展起积极的推动作用，也是与世界接轨、建设真正的世界一流大学的必由之路。

机械工业出版社华章图文信息有限公司较早意识到“出版要为教育服务”。自1998年开始，华章公司就将工作重点放在了遴选、移译国外优秀教材上。经过几年的不懈努力，我们与Prentice Hall, Addison-Wesley, McGraw-Hill, Morgan Kaufmann等世界著名出版公司建立了良好的合作关系，从它们现有的数百种教材中甄选出Tanenbaum, Stroustrup, Kernighan, Jim Gray等大师名家的一批经典作品，以“计算机科学丛书”为总称出版，供读者学习、研究及度藏。大理石纹理的封面，也正体现了这套丛书的品位和格调。

“计算机科学丛书”的出版工作得到了国内外学者的鼎力襄助，国内的专家不仅提供了中肯的选题指导，还不辞劳苦地担任了翻译和审校的工作；而原书的作者也相当关注其作品在中国的传播，有的还专诚为其书的中译本作序。迄今，“计算机科学丛书”已经出版了近百个品种，这些书籍在读者中树立了良好的口碑，并被许多高校采用为正式教材和参考书籍，为进一步推广与发展打下了坚实的基础。

随着学科建设的初步完善和教材改革的逐渐深化，教育界对国外计算机教材的需求和应用都步入一个新的阶段。为此，华章公司将加大引进教材的力度，在“华章教育”的总规划之下出版三个系列的计算机教材：除“计算机科学丛书”之外，对影印版的教材，则单独开辟出“经典原版书库”；同时，引进全美通行的教学辅导书“Schaum's Outlines”系列组成“全美经典学习指导系列”。为了保证这三套丛书的权威性，同时也为了更好地为学校和老师提供服务，华章公司聘请了中国科学院、北京大学、清华大学、国防科技大学、复旦大学、上海交通大学、南京大学、浙江大学、中国科技大学、哈尔滨工业大学、西安交通大学、中国人民大学、北京航空航天大学、北京邮电大学、中山大学、解放军理工大学、郑州大学、湖北工学院、中国国

家信息安全测评认证中心等国内重点大学和科研机构在计算机的各个领域的著名学者组成“专家指导委员会”，为我们提供选题意见和出版监督。

这三套丛书是响应教育部提出的使用外版教材的号召，为国内高校的计算机及相关专业的教学度身订造的。其中许多教材均已为M. I. T., Stanford, U.C. Berkeley, C. M. U. 等世界名牌大学所采用。不仅涵盖了程序设计、数据结构、操作系统、计算机体系结构、数据库、编译原理、软件工程、图形学、通信与网络、离散数学等国内大学计算机专业普遍开设的核心课程，而且各具特色——有的出自语言设计者之手、有的历经三十年而不衰、有的已被全世界的几百所高校采用。在这些圆熟通博的名师大作的指引之下，读者必将在计算机科学的宫殿中由登堂而入室。

权威的作者、经典的教材、一流的译者、严格的审校、精细的编辑，这些因素使我们的图书有了质量的保证，但我们的目标是尽善尽美，而反馈的意见正是我们达到这一终极目标的重要帮助。教材的出版只是我们的后续服务的起点。华章公司欢迎老师和读者对我们的工作提出建议或给予指正，我们的联系方式如下：

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前言

科学家常会感到通过自己的研究工作在不断地认识自我,物理学家在工作中认识到了这一点,同样,心理学家和化学家也认识到了这一点。在机器人学的研究中,研究领域和研究者自身之间的关系尤为明显。同时,与一门仅追求分析的自然科学不同,当前机器人学所追求的是偏重于综合的工程学科。也许正是这个原因,这个领域才使我们当中的许多人着迷。

机器人学研究的是怎样综合运用机械、传感器、驱动器和计算机来实现人类某些方面的功能。显然,这是一项庞大的任务,它必然需要运用各种“传统”领域的研究思想。

现今,机器人学诸方面的研究工作都是由不同领域的专家们进行的。通常没有一个人能够完全掌握机器人领域的所有知识。因此,自然有必要对这个研究领域进行划分。在更高的层次上,可把机器人学划分为四个主要领域:机械操作、移动、计算机视觉和人工智能。

本书介绍机械操作的理论和工程知识,这是机器人学的分学科,它是建立在几个传统学科基础之上的。主要的相关学科有力学、控制理论、计算机科学。在本书中,第1章至第8章包括机械工程和数学的主题,第9章至第11章为控制理论的题材,第12章和第13章属于计算机科学的内容。另外,本书自始至终强调通过计算解决问题;例如,与力学密切相关的每一章都有一节简要介绍计算方面的问题。

本书源于斯坦福大学1983~1985年秋季学期的“机器人学导论”的讲义。第1版和第2版在1986~2002年间被许多大学采用。第3版得益于这些应用,并且根据多方面的反馈意见做了修改和改进。在此,向那些对本书作者提出修正意见的人表示感谢。

本书适用于高年级本科生或者低年级研究生课程。选修此课程的学生如果学过静力学和动力学这两门基础课程之一,同时学习过线性代数,并且能够使用计算机高级语言编程,这将有助于他们的学习。此外,虽然不必先修控制理论方面的入门课程,但学过这门课程也是有益的。本书的目标之一是以简单、直观的方式介绍机器人学的知识。特别需要指出的是,虽然本书很多内容选自机械领域,但本书的读者不必是机械工程师。在斯坦福大学,很多电气工程师、计算机科学家、数学家都认为本书具有很强的可读性。

虽然本书直接由机器人系统的研发工程师使用,但是对于任何将要从事机器人研究工作的人,本书内容应当被看作是重要的背景资料。同样,至少从事过某种硬件的软件开发人员和直接参与机器人的机械和控制的研究人员,应当具备一些本书提供的背景知识。

第3版和第2版相似,分为13章。本书的材料适合于一学期来讲授,如果要在半学期内讲授,需要略去一些章。即便如此,仍然无法深入讲解所有专题。本书在编写时从某些方面考虑了这一点;例如,多数章只采用一种方法解决常见的问题。编写本书的主要问题之一就是尽量在限定的教学时间内为每个主题合理地分配时间。为此,我的办法是只考虑那些直接影响机器人机械操作学习的材料。

在每章的最后都有一组习题。在每道习题题号后的方括号中给出习题的难度系数。难度系

数在[00]到[50]之间。[00]是最简单的题目，[50]是尚未被解决的研究性问题[⊖]。当然，一个人认为困难的问题，另一个人可能认为容易，因此，一些读者会发现那些难度系数在某些情况下会引起误解。不过，这种方法在某种程度上评价了这些习题的难度。

在每章的末尾，有一个编程作业，学生可以把相应章的知识应用到一个简单的三关节平面操作臂中。这个简单的操作臂足以用来证明大多数一般操作臂的所有原理，而不必使学生陷入过于复杂的问题中。每个编程作业都建立在前一个作业的基础上，到课程结束时，学生就会得到一个完整的操作臂软件程序库。

另外，我们在本书第3版中添加了使用MATLAB的习题。第1~9章共有12道使用MATLAB的习题。这些习题由俄亥俄大学的Robert L. Williams II教授编写，我对他所做的贡献深表感谢。这些习题可以配合澳大利亚CSIRO首席研究科学家Peter Corke编写的MATLAB机器人学工具箱[⊗]使用。

第1章是机器人学的介绍，介绍一些背景资料、基本思想和本书所使用的符号，并预览后面各章的内容。

第2章包括描述三维空间中的位置与方法的数学知识。这是极为重要的内容：通过定义机械操作本身与周围空间的移动物体（部件、工具、机器人自身）联系起来。我们需要用一种易于理解并且尽可能直观的方式来描述这些动作。

第3章和第4章讨论机械操作臂的几何性质。介绍机械工程学科中的运动学分支，这个分支研究运动但不考虑引起这种运动的力。在这两章里，我们讨论操作臂运动学，但把研究范围限定在静态定位问题上。

第5章将运动学的研究范围扩展到速度和静力方面。

第6章开始研究引起操作臂运动的力和力矩。这就是操作臂动力学问题。

第7章描述操作臂在空间的运动轨迹。

第8章涉及到许多与操作臂机械设计有关的问题。例如，设计多少关节是适宜的，关节的类型应是什么，它们需如何布局。

第9章和第10章研究操作臂的控制方法（通常利用数字计算机），使其准确地经过预先设定的空间轨迹。第9章研究线性控制方法，第10章将研究拓展到非线性领域。

第11章讨论操作臂的主动力控制。这就是研究如何对操作臂控制力的运用，这种控制模式在操作臂接触周围环境的情况下非常重要，比如操作臂用海绵擦窗户。

第12章概述机器人编程方法，特别是机器人编程系统中所需的基本成分以及与工业机器人编程相关的特殊问题。

第13章介绍离线仿真和编程系统，其中描述人与机器人接口的最新进展。

我非常感谢牺牲宝贵时间协助我完成这本书的许多人。首先，感谢斯坦福大学1983~1985届ME219班的同学们，他们学习了初稿，发现了不少错误，并提出了许多建议。Bernard Roth教授在多方面给予了帮助，不仅对草稿提出了建设性的意见，而且为我提供了完成第1版的环境。在SILMA公司，我得到了很好的仿真环境和资源，从而使我完成了第2版。Jeff Kerr博士写出第8

⊖ 我采用了与D.Knuth所著《The Art of Computer Programming》(Addison-Wesley出版)同样的难度等级。

⊗ 关于MATLAB机器人学工具箱，请访问<http://www.ict.csiro.au/robotics/ToolBox7.htm>。

章的初稿, Robert L. Williams II教授设计了每章最后的MATLAB习题。Peter Corke扩充了他的Robotics Toolbox (机器人学工具箱), 以此支持本书采用的Denavit-Hartenberg符号体系。在此, 我也深深地感谢我在机器人学方面的导师Marc Raibert、Carl Ruoff、Tom Binford 和Bernard Roth。

我还要感谢来自斯坦福大学、SILMA公司、Adept公司和其他地方的许多人, 他们以各种方式对我提供了帮助, 他们是John Mark Agosta、Mike Ali、Lynn Balling、Al Barr、Stephen Boyd、Chuck Bunkley、Joel Burdick、Jim Callan、Brian Carlisle、Monique Craig、Subas Desa、Tri Dai Do、Karl Garcia、Ashitava Ghosal、Chris Goad、Ron Goldman、Bill Hamilton、Steve Holland、Peter Jackson、Eric Jacobs、Johann Jäger、Paul James、Jeff Kerr、Oussama Khatib、Jim Kramer、Dave Lowe、Jim Maples、Dave Marimont、Dave Meer、Kent Ohlund、Madhusudan Raghavan、Richard Roy、Ken Salisbury、Bruce Shimano、Donalda Speight、Bob Tilove、Sandy Wells 和Dave Williams。

斯坦福大学的Roth教授在给2002届的学生们讲授机器人课程时使用了本书第2版, 并指出了许多错误, 这些错误在第3版中做了修订。

最后, 我还要感谢Prentice Hall出版社的Tom Robbins对第1版以及本版给予的指导帮助。

J. J. C

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CHAPTER 1

Introduction

1.1 BACKGROUND

1.2 THE MECHANICS AND CONTROL OF MECHANICAL MANIPULATORS

1.3 NOTATION

1.1 BACKGROUND

The history of industrial automation is characterized by periods of rapid change in popular methods. Either as a cause or, perhaps, an effect, such periods of change in automation techniques seem closely tied to world economics. Use of the **industrial robot**, which became identifiable as a unique device in the 1960s [1], along with computer-aided design (CAD) systems and computer-aided manufacturing (CAM) systems, characterizes the latest trends in the automation of the manufacturing process. These technologies are leading industrial automation through another transition, the scope of which is still unknown [2].

In North America, there was much adoption of robotic equipment in the early 1980s, followed by a brief pull-back in the late 1980s. Since that time, the market has been growing (Fig. 1.1), although it is subject to economic swings, as are all markets.

Figure 1.2 shows the number of robots being installed per year in the major industrial regions of the world. Note that Japan reports numbers somewhat differently from the way that other regions do: they count some machines as robots that in other parts of the world are not considered robots (rather, they would be simply considered “factory machines”). Hence, the numbers reported for Japan are somewhat inflated.

A major reason for the growth in the use of industrial robots is their declining cost. Figure 1.3 indicates that, through the decade of the 1990s, robot prices dropped while human labor costs increased. Also, robots are not just getting cheaper, they are becoming more effective—faster, more accurate, more flexible. If we factor these *quality adjustments* into the numbers, the cost of using robots is dropping even faster than their price tag is. As robots become more cost effective at their jobs, and as human labor continues to become more expensive, more and more industrial jobs become candidates for robotic automation. This is the single most important trend propelling growth of the industrial robot market. A secondary trend is that, economics aside, as robots become more capable they become *able* to do more and more tasks that might be dangerous or impossible for human workers to perform.

The applications that industrial robots perform are gradually getting more sophisticated, but it is still the case that, in the year 2000, approximately 78% of the robots installed in the US were welding or material-handling robots [3].

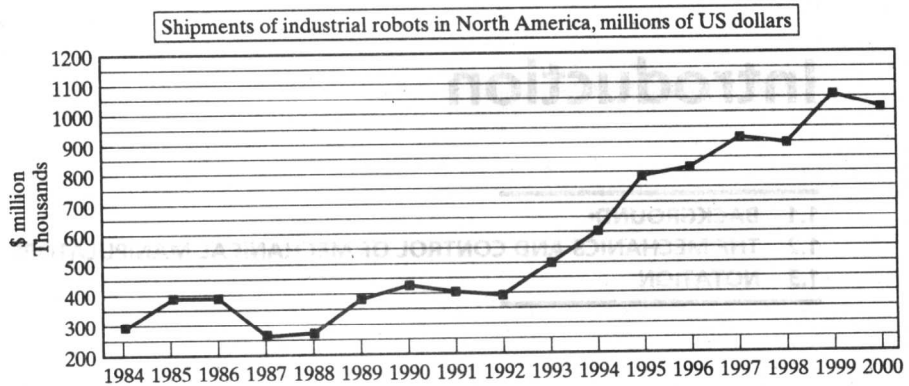


FIGURE 1.1: Shipments of industrial robots in North America in millions of US dollars [3].

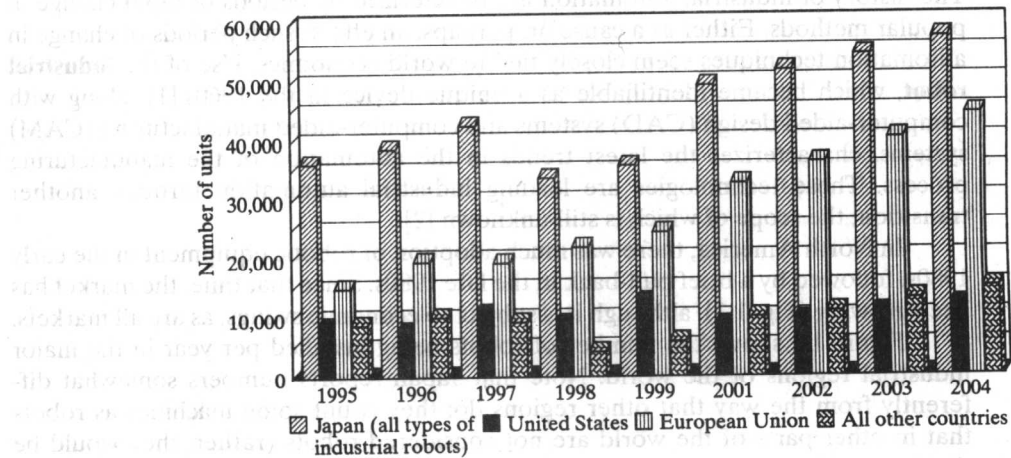


FIGURE 1.2: Yearly installations of multipurpose industrial robots for 1995–2000 and forecasts for 2001–2004 [3].

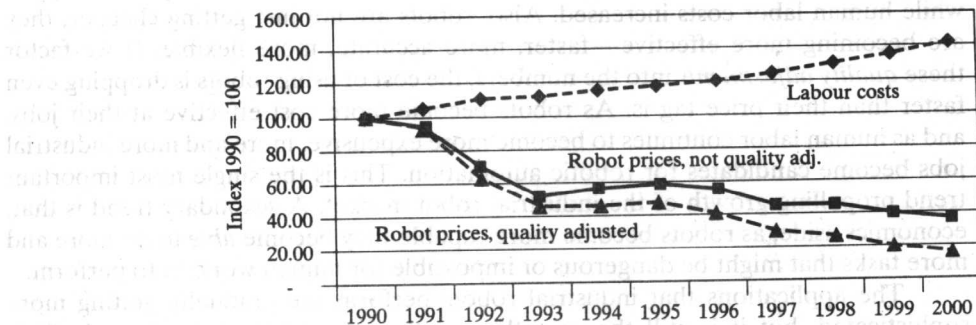


FIGURE 1.3: Robot prices compared with human labor costs in the 1990s [3].

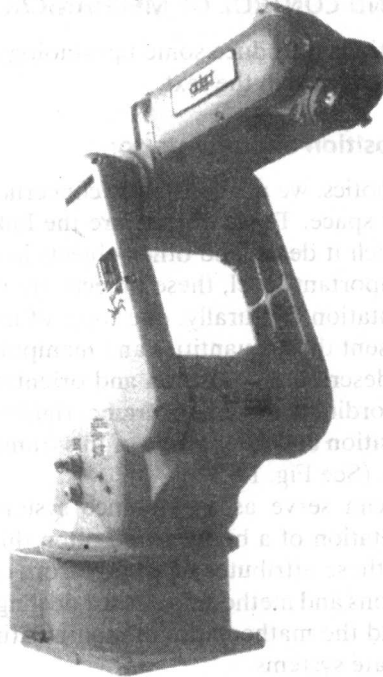


FIGURE 1.4: The Adept 6 manipulator has six rotational joints and is popular in many applications. Courtesy of Adept Technology, Inc.

A more challenging domain, **assembly** by industrial robot, accounted for 10% of installations.

This book focuses on the mechanics and control of the most important form of the industrial robot, the **mechanical manipulator**. Exactly what constitutes an industrial robot is sometimes debated. Devices such as that shown in Fig. 1.4 are always included, while numerically controlled (NC) milling machines are usually not. The distinction lies somewhere in the sophistication of the programmability of the device—if a mechanical device can be programmed to perform a wide variety of applications, it is probably an industrial robot. Machines which are for the most part limited to one class of task are considered **fixed automation**. For the purposes of this text, the distinctions need not be debated; most material is of a basic nature that applies to a wide variety of programmable machines.

By and large, the study of the mechanics and control of manipulators is not a new science, but merely a collection of topics taken from “classical” fields. Mechanical engineering contributes methodologies for the study of machines in static and dynamic situations. Mathematics supplies tools for describing spatial motions and other attributes of manipulators. Control theory provides tools for designing and evaluating algorithms to realize desired motions or force applications. Electrical-engineering techniques are brought to bear in the design of sensors and interfaces for industrial robots, and computer science contributes a basis for programming these devices to perform a desired task.

1.2 THE MECHANICS AND CONTROL OF MECHANICAL MANIPULATORS

The following sections introduce some terminology and briefly preview each of the topics that will be covered in the text.

Description of position and orientation

In the study of robotics, we are constantly concerned with the location of objects in three-dimensional space. These objects are the links of the manipulator, the parts and tools with which it deals, and other objects in the manipulator's environment. At a crude but important level, these objects are described by just two attributes: position and orientation. Naturally, one topic of immediate interest is the manner in which we represent these quantities and manipulate them mathematically.

In order to describe the position and orientation of a body in space, we will always attach a coordinate system, or **frame**, rigidly to the object. We then proceed to describe the position and orientation of this frame with respect to some reference coordinate system. (See Fig. 1.5.)

Any frame can serve as a reference system within which to express the position and orientation of a body, so we often think of *transforming* or *changing the description of* these attributes of a body from one frame to another. Chapter 2 discusses conventions and methodologies for dealing with the description of position and orientation and the mathematics of manipulating these quantities with respect to various coordinate systems.

Developing good skills concerning the description of position and rotation of rigid bodies is highly useful even in fields outside of robotics.

Forward kinematics of manipulators

Kinematics is the science of motion that treats motion without regard to the forces which cause it. Within the science of kinematics, one studies position, velocity,

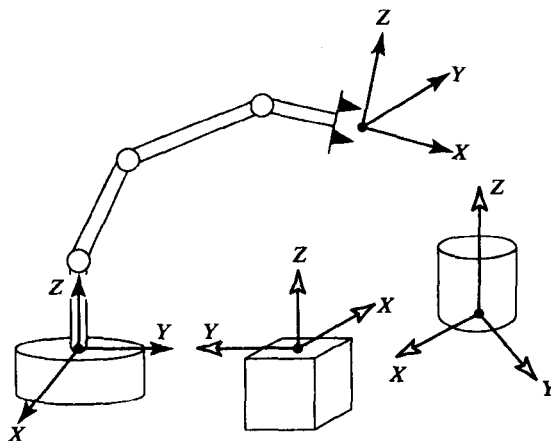


FIGURE 1.5: Coordinate systems or “frames” are attached to the manipulator and to objects in the environment.

acceleration, and all higher order derivatives of the position variables (with respect to time or any other variable(s)). Hence, the study of the kinematics of manipulators refers to all the geometrical and time-based properties of the motion.

Manipulators consist of nearly rigid **links**, which are connected by **joints** that allow relative motion of neighboring links. These joints are usually instrumented with position sensors, which allow the relative position of neighboring links to be measured. In the case of rotary or **revolute** joints, these displacements are called **joint angles**. Some manipulators contain sliding (or **prismatic**) joints, in which the relative displacement between links is a translation, sometimes called the **joint offset**.

The number of **degrees of freedom** that a manipulator possesses is the number of independent position variables that would have to be specified in order to locate all parts of the mechanism. This is a general term used for any mechanism. For example, a four-bar linkage has only one degree of freedom (even though there are three moving members). In the case of typical industrial robots, because a manipulator is usually an open kinematic chain, and because each joint position is usually defined with a single variable, the number of joints equals the number of degrees of freedom.

At the free end of the chain of links that make up the manipulator is the **end-effector**. Depending on the intended application of the robot, the end-effector could be a gripper, a welding torch, an electromagnet, or another device. We generally describe the position of the manipulator by giving a description of the **tool frame**, which is attached to the end-effector, relative to the **base frame**, which is attached to the nonmoving base of the manipulator. (See Fig. 1.6.)

A very basic problem in the study of mechanical manipulation is called **forward kinematics**. This is the static geometrical problem of computing the position and orientation of the end-effector of the manipulator. Specifically, given a set of joint

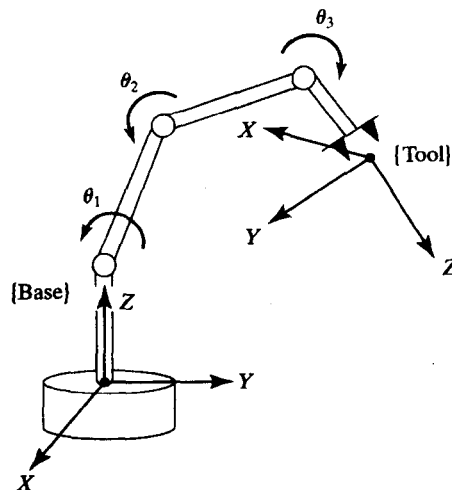


FIGURE 1.6: Kinematic equations describe the tool frame relative to the base frame as a function of the joint variables.

angles, the forward kinematic problem is to compute the position and orientation of the tool frame relative to the base frame. Sometimes, we think of this as changing the representation of manipulator position from a **joint space** description into a **Cartesian space** description.¹ This problem will be explored in Chapter 3.

Inverse kinematics of manipulators

In Chapter 4, we will consider the problem of **inverse kinematics**. This problem is posed as follows: Given the position and orientation of the end-effector of the manipulator, calculate all possible sets of joint angles that could be used to attain this given position and orientation. (See Fig. 1.7.) This is a fundamental problem in the practical use of manipulators.

This is a rather complicated geometrical problem that is routinely solved thousands of times daily in human and other biological systems. In the case of an artificial system like a robot, we will need to create an algorithm in the control computer that can make this calculation. In some ways, solution of this problem is the most important element in a manipulator system.

We can think of this problem as a *mapping* of “locations” in 3-D Cartesian space to “locations” in the robot’s internal joint space. This need naturally arises anytime a goal is specified in external 3-D space coordinates. Some early robots lacked this algorithm—they were simply moved (sometimes by hand) to desired locations, which were then recorded as a set of joint values (i.e., as a location in joint space) for later playback. Obviously, if the robot is used purely in the mode of recording and playback of joint locations and motions, no algorithm relating

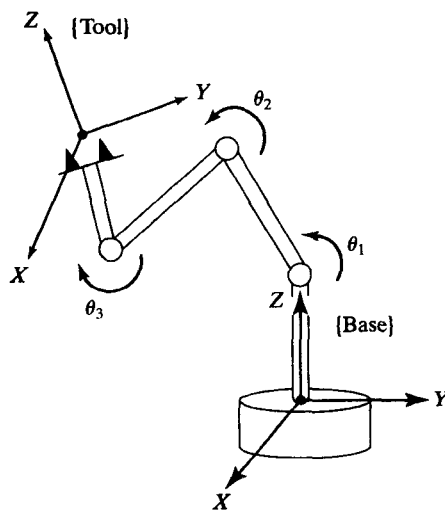


FIGURE 1.7: For a given position and orientation of the tool frame, values for the joint variables can be calculated via the inverse kinematics.

¹By *Cartesian space*, we mean the space in which the position of a point is given with three numbers, and in which the orientation of a body is given with three numbers. It is sometimes called *task space* or *operational space*.

joint space to Cartesian space is needed. These days, however, it is rare to find an industrial robot that lacks this basic inverse kinematic algorithm.

The inverse kinematics problem is not as simple as the forward kinematics one. Because the kinematic equations are nonlinear, their solution is not always easy (or even possible) in a closed form. Also, questions about the existence of a solution and about multiple solutions arise.

Study of these issues gives one an appreciation for what the human mind and nervous system are accomplishing when we, seemingly without conscious thought, move and manipulate objects with our arms and hands.

The existence or nonexistence of a kinematic solution defines the **workspace** of a given manipulator. The lack of a solution means that the manipulator cannot attain the desired position and orientation because it lies outside of the manipulator's workspace.

Velocities, static forces, singularities

In addition to dealing with static positioning problems, we may wish to analyze manipulators in motion. Often, in performing velocity analysis of a mechanism, it is convenient to define a matrix quantity called the **Jacobian** of the manipulator. The Jacobian specifies a **mapping** from velocities in joint space to velocities in Cartesian space. (See Fig. 1.8.) The nature of this mapping changes as the configuration of the manipulator varies. At certain points, called **singularities**, this mapping is not invertible. An understanding of the phenomenon is important to designers and users of manipulators.

Consider the rear gunner in a World War I-vintage biplane fighter plane (illustrated in Fig. 1.9). While the pilot flies the plane from the front cockpit, the rear gunner's job is to shoot at enemy aircraft. To perform this task, his gun is mounted in a mechanism that rotates about two axes, the motions being called azimuth and elevation. Using these two motions (two degrees of freedom), the gunner can direct his stream of bullets in any direction he desires in the upper hemisphere.

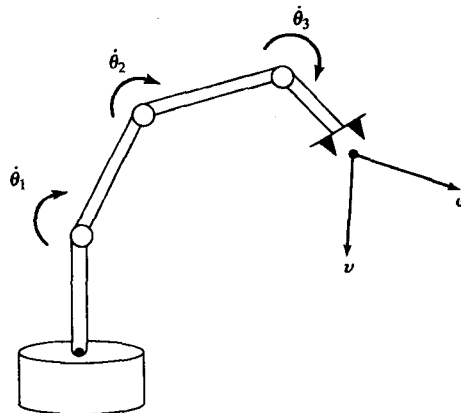


FIGURE 1.8: The geometrical relationship between joint rates and velocity of the end-effector can be described in a matrix called the Jacobian.