

DYNAMICS OF CONSTRAINED MECHANICAL SYSTEMS

约束力学系统动力学 (英文版)

Mei Fengxiang Wu Huibin

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内 容 简 介

本书系统地阐述了约束力学系统的变分原理、运动方程、相关专门问题的理论与应用、积分方法、对称性与守恒量等内容,具有很高的学术价值,为方便国际学术交流,译成英文出版。全书共分为六个部分:

第一部分:约束力学系统的基本概念。本部分包含6章,介绍分析力学的主要基本概念;第二部分:约束力学系统的变分原理。本部分有5章,阐述微分变分原理、积分变分原理以及Pfaff-Birkhoff原理;第三部分:约束力学系统的运动微分方程。本部分共11章,系统介绍完整系统、非完整系统的各类运动方程;第四部分:约束力学系统的专门问题。本部分有8章,讨论运动稳定性和微扰理论、刚体定点转动、相对运动动力学、可控力学系统动力学、打击运动动力学、变质量系统动力学、机电系统动力学、事件空间动力学等内容;第五部分:约束力学系统的积分方法。本部分有6章,介绍降阶方法、动力学代数与Poisson方法、正则变换、Hamilton-Jacobi方法、场方法、积分不变量;第六部分:约束力学系统的对称性与守恒量。本部分共10章,讨论Noether对称性、Lie对称性、形式不变性,以及由它们导致的各种守恒量。

本书的出版必将引起国内外同行的关注,对该领域的发展将起到重要的推动作用。

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Preface

This book is entitled *Dynamics of Constrained Mechanical Systems*. The constrained mechanical systems, in my opinion, contain the three kinds of the systems, i.e. the holonomic systems, the nonholonomic systems and the Birkhoff systems. The book covers the following six parts.

Part I Fundamental Concepts in Constrained Mechanical Systems. The part has 6 chapters: Constraints and their classification, Generalized coordinates, Quasi-velocities and quasi-coordinates, Virtual displacements, Ideal constraints, Transpositional relations of differential and variational operators.

Part II Variational Principles in Constrained Mechanical Systems. It covers 5 chapters: Differential variational principles, Integral variational principles in terms of generalized coordinates for holonomic systems, Integral variational principles in terms of quasi-coordinates for holonomic systems, Integral variational principles for nonholonomic systems, Pfaff-Birkhoff principle.

Part III Differential Equations of Motion of Constrained Mechanical Systems. It covers 11 chapters: Lagrange equations of holonomic systems, Lagrange equations with multiplier for nonholonomic systems, Mac Millan equations for nonholonomic systems, Volterra equations for nonholonomic systems, Chaplygin equations for nonholonomic systems, Boltzmann-Hamel equations for nonholonomic systems, Euler-Lagrange equations for higher order nonholonomic systems, Nielsen equations, Appell equations, Equations of motion of mixed type, Canonical equations.

Part IV Special Problems in Constrained Mechanical Systems. It covers 8 chapters: Stability of motion and theory of small oscillations, Dynamics of rigid body with fixed point, Dynamics of relative motion, Dynamics of controllable mechanical systems, Dynamics of impulsive motion, Dynamics of variable mass systems, Dynamics of electromechanical systems, Dynamics in event space.

Part V Integration Methods in Constrained Mechanical Systems. It covers 6 chapters: Methods of reduction of order, Dynamics algebra and Poisson method, Canonical transformations, Hamilton-Jacobi method, Field method, Integral invariants.

Part VI Symmetries and Conserved Quantities in Constrained Mechanical Systems. The part has 10 chapters: Noether symmetries and conserved quantities, Lie symmetries and Hojman conserved quantities, Form invariance and new conserved quantities, Noether symmetries and Hojman conserved quantities, Noether symmetries and new conserved quantities, Lie symmetries and Noether conserved quantities, Lie symmetries and new conserved quantities, Form invariance and Noether conserved quantities, Form invariance and Hojman conserved quantities, Unified symmetries and conserved quantities.

The emphatic discussions in the book are: the equations of motion of nonholonomic systems; the theory and application of special problems; the integral methods and the symmetries and conservative quantities.

The mathematics tools in the book are simple. Advanced mathematics, such as modern differential geometry, are not involved. Interested readers can read some relative reference books.

The book is addressed to graduate students, professors and researchers in the areas of applied mechanics, applied mathematics and applied physics.

In spring of 1963, I began to learn the knowledge of nonholonomic mechanics and finished my graduate dissertation under the direction of Professor Lu Yiling in Peking University. In autumn of 1981 I visited the Laboratoire de Mécanique Théorique in Université de Besançon, presented my work on the nonholonomic mechanics, and obtained the direction of Professor Capodanno. P. In spring of 1982, I finished my Thèse de Doctorat d'Etat under the direction of Professor Pironneau I in ENSM at Nantes France.

The great majority of my papers and my books are published in Chinese. In order to exchange experience of study on dynamics of constrained mechanical systems, I decide to write this English book. This book is written by myself and my colleague Dr Wu Huibin.

Acknowledgments

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Mei Fengxiang
November 2008
in Beijing

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