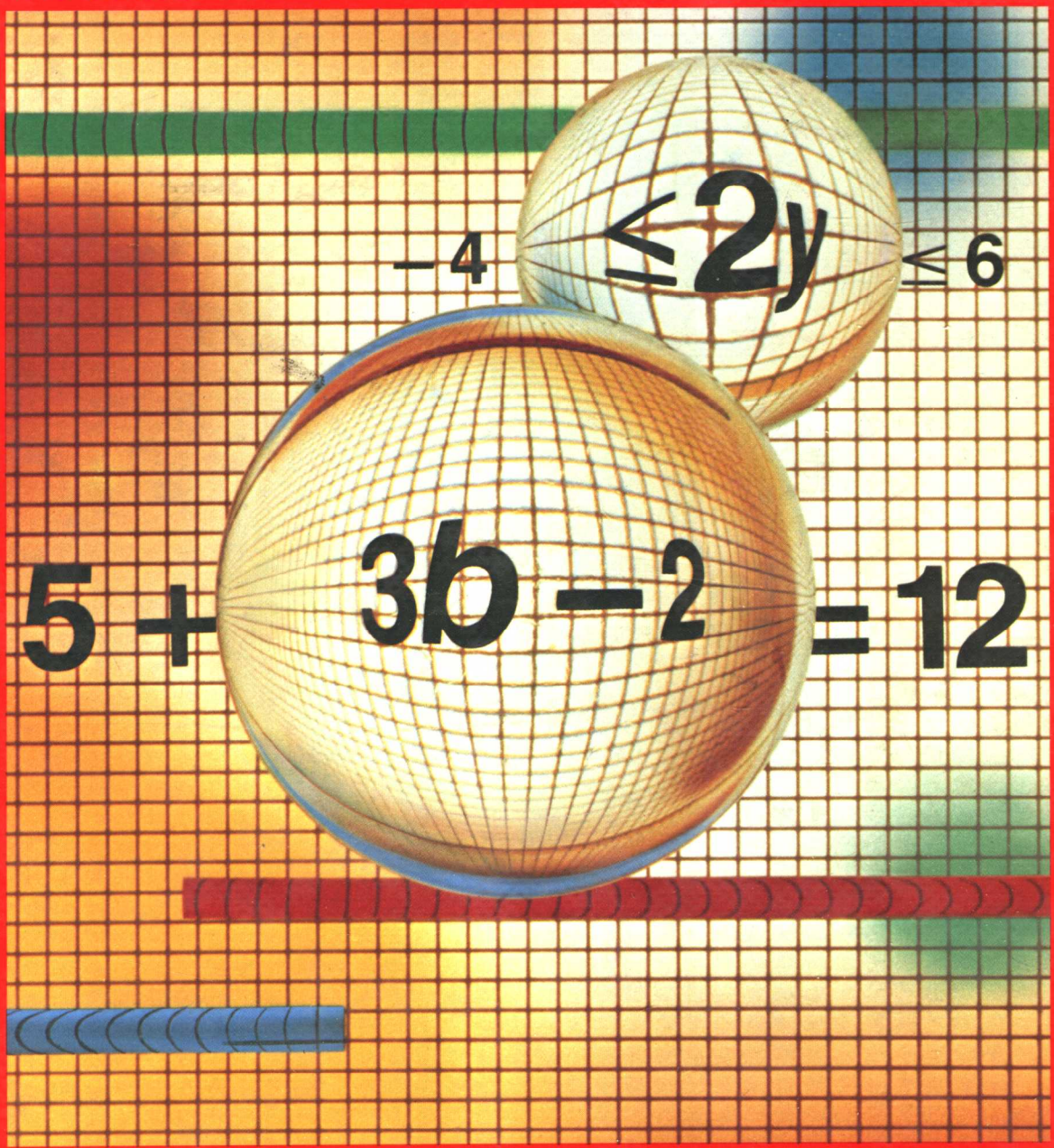
ALGEBRA

Structure and Method

Book 1



Houghton Mifflin

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HOUGHTON MIFFLIN COMPANY · BOSTON

Atlanta Dallas Geneva, Ill. Lawrenceville, N.J. Palo Alto Toronto

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Printed in U.S.A.

ISBN: 0-395-43052-6

FGHIJ-D-943210/89

Reading Your Algebra Textbook

An algebra textbook requires a different type of reading than a novel or a short story. Every paragraph of a mathematical text must be read with great care and concentration. You should not only read the words, but you should think about their meaning. You should read slowly through the explanations in this book. Algebra builds upon itself; the method of multiplying binomials you will study on page 196 will be useful to you on page 531. Read with a pencil in your hand; do calculations, draw sketches, take notes.

Vocabulary

You will be exposed to many new words in algebra. Some, such as *axiom* and *polynomial,* are mathematical in nature, while others, such as *line* and *side,* are used in everyday speech but have different meanings when used in algebra. Important words whose meanings you will learn are printed in heavy type. (See page 10.) Also, they are listed at the beginning of each Self-Test. If you cannot recall the meaning of a word, you can look it up in the Glossary or the Index at the back of the book. The Glossary will give you a definition, and the Index will give you page references for more information.

Symbols

Algebra, and mathematics in general, has its own symbolic language. You must be able to read these symbols in order to understand algebra. For example, $x \in \{-1, 0, 1, 2\}$ means "x is a member of the set whose members are -1, 0, 1, and 2." A list of symbols appears on page xii. If you cannot recall what a symbol means, check this list.

Diagrams

Throughout this book you will find many diagrams. These contain information that will help you to understand the concepts under discussion. Study these diagrams carefully when you read the text that accompanies them.

Displayed Material

Throughout the book important information is displayed in red boxes. This information includes axioms, properties, definitions, methods, and summaries. Be certain to read and understand the material in these boxes. You will also find these boxes useful when reviewing for tests and exams. Be sure to study the worked-out examples, such as the ones on page 218, as they will help you in doing many of the exercises and problems.

Reading Aids

Throughout this book you will find sections called Reading Algebra. These sections deal with such topics as problem-solving strategies and inequalities. Be sure to read these sections, as they will help you to better understand the subjects with which they deal.

SYMBOLS

		Page
$\cdot$	$\times$ (times)	1
$=$	equals, is equal to	2
$\neq$	is not equal to	2
( )	parentheses—a grouping symbol	2
[ ]	brackets—a grouping symbol	5
π	pi, a number approximately equal to $\frac{22}{7}$	8
$\therefore$	therefore	10
$\in$	is a member of, belongs to	10
$\stackrel{?}{=}$	is this statement true?	25
$-$	negative	30
$+$	positive	30
$-a$	opposite or additive inverse of a	34
$\lvert a \rvert$	absolute value of a	35
$<$	is less than	37
$>$	is greater than	37
$\frac{1}{b}$	reciprocal or multiplicative inverse of b	76
$\emptyset$	empty set, null set	112
$a:b$	ratio of a to b	277
(a, b)	ordered pair whose first component is a and second component is b	335
$f(x)$	f of x, the value of f at x	405
$\geq$	is greater than or equal to	449
$\leq$	is less than or equal to	449
$\cap$	the intersection of	465
$\cup$	the union of	465
$\approx$	is approximately equal to	506
$\sqrt{\ }$	principal square root	510
$\overleftrightarrow{AB}$	line AB	576
$\overline{AB}$	segment AB	576
AB	the length of $\overline{AB}$	576
$\overrightarrow{AB}$	ray AB	576
$\angle$	angle	576
$^\circ$	degree(s)	577
$\triangle$	triangle	581
$\sim$	is similar to	584
$\cos A$	cosine of A	587
$\sin A$	sine of A	587
$\tan A$	tangent of A	587
$P(A)$	probability of event A	599

Metric Units of Measure

Length: mm millimeter
cm centimeter
m meter
km kilometer

Area: mm^2 square millimeter
cm^2 square centimeter
m^2 square meter
km^2 square kilometer
ha hectare

Volume: cm^3 cubic centimeter
mL milliliter
L liter

Time: s second
min minute
h hour

Speed: m/s meters per second
km/h kilometers per hour

Mass: mg milligram
g gram
kg kilogram

Temperature: °C degrees Celsius

Reading Algebra / *Symbols*

Close attention is needed to read mathematics because mathematical materials usually involve special symbols as well as words. And, often, those symbols can be read in several ways. Here are some examples.

$7 + 3$

"seven plus three"
"the sum of seven and three"
"*add seven and three*"
"seven increased by three"
"three more than seven"

$8 - 1$

"eight minus one"
"the difference of eight and one"
"*subtract one from eight*"
"eight decreased by one"
"one less than eight"

11×4

"eleven times four"
"the product of eleven and four"
"multiply eleven by four"

$10 \div 2$

"ten divided by two"
"the quotient of ten and two"
"divide ten by two"

$\frac{1}{2} \times 5$

"one half times five"
"one half of five"
"the product of one half and five"

$\frac{5}{2}$

"five halves"
"five divided by two"
"the quotient of five and two"

Exercises

Read each of the following expressions in at least three ways.

1. 4×8 **2.** $15 - 0$ **3.** $2 + 9$ **4.** $\frac{7}{4}$

Translate the given words into mathematical symbols and then write the common numeral for the number described.

Example Four increased by six ***Solution*** $4 + 6$; 10

5. Twelve decreased by five
6. Eleven more than six
7. The product of fifteen and three
8. Five less than thirty
9. *Two thirds of six*
10. Twice seven
11. The difference of fifty and thirty
12. The product of zero and nine

The symbols on the facing page are commonly used in mathematics. In studying this book, you will learn how to read and use them.

Contents

3 Solving Equations and Problems

4 Polynomials

5 Factoring Polynomials

6 Fractions

7 Applying Fractions

8 Linear Equations and Systems

9 Introduction to Functions

Variables and Equations

1-1 Variables

Objective To simplify and to evaluate numerical expressions and variable expressions.

A scientist needed to gather data for an underwater experiment. Diving in sea water, the scientist measured the pressure of the water at different depths, as illustrated in the photograph at the left. These water pressures were then arranged in a table similar to the one below, which shows pressures at depths of 10, 20, 30, and 40 meters.

Depth in meters	Water pressure in atmospheres
10	0.104×10
20	0.104×20
30	0.104×30
40	0.104×40

Each of the expressions for water pressure fits the pattern

$$0.104 \times n$$

where the letter n stands for 10, 20, 30, or 40. We call n a *variable.*

A **variable** is a symbol used to represent one or more numbers. The numbers are called the **values of the variable.**

An expression, such as $0.104 \times n$, that contains a variable is called a **variable expression.** Expressions, such as 0.104×2, that name a particular number are called **numerical expressions** or **numerals.**

When you write a product that contains a variable, you usually omit the multiplication symbol.

$0.104 \times n$ is usually written $0.104n$.

$y \times z$ is usually written yz.

In numerical expressions for products like 0.104×20, you must use a multiplication symbol to avoid confusion. The raised dot $\cdot$ is also used as a multiplication sign.

0.104×20 can be written $0.104 \cdot 20$.

The number named by a numerical expression is called the **value of the expression.** Since the expressions $3 + 5$ and 8 name the same number, they have the same value. To show that these expressions have the same value, you use the *equals sign,* $=$. You write

$$3 + 5 = 8$$

and say "three plus five equals (or is equal to or is) eight." Of course, 8 is the *simplest,* or most common, name for the number eight.

The symbol $\neq$ means is not equal to. You write

$$3 + 5 \neq 7$$

to show that the expressions $3 + 5$ and 7 do not have the same value.

Replacing a numerical expression by the simplest name of its value is called **simplifying the expression.** In simplifying a numerical expression, you use the following principle.

Substitution Principle

Changing the numeral by which a number is named in an expression does not change the value of the expression.

Example 1 Simplify. **a.** $(56 \div 7) + 9$ **b.** $36 \div (9 - 3)$

Solution The parentheses ( ) show how the numerals in the expression are to be grouped. The numerals within parentheses are simplified first.

a. $(56 \div 7) + 9 = 8 + 9 = 17$
b. $36 \div (9 - 3) = 36 \div 6 = 6$

Note that to read the symbols "$(56 \div 7) + 9$," you may say "the sum of the *quantity* fifty-six divided by seven, plus nine."

Replacing each variable in a variable expression by a given value and simplifying the result is called **evaluating the expression** or **finding the value of the expression.**

Example 2 Evaluate $(5x) - (3 + y)$ if $x = 12$ and $y = 9$.

Solution Replace x with 12 and y with 9, and insert the necessary multiplication symbol. Then simplify the result.

$$\begin{aligned}(5x) - (3 + y) &= (5 \times 12) - (3 + 9)\\ &= 60 - 12\\ &= 48\end{aligned}$$

Oral Exercises

State whether or not each statement is true. Give a reason for your answer.

Example 1 $5 \times 4 = 10 + 10$ ***Solution*** True, because the value of both 5×4 and $10 + 10$ is 20.

Example 2 $2 \times 7 = 2 + 7$ ***Solution*** False, because $2 \times 7 = 14$, whereas $2 + 7 = 9$.

1. $9 \times 7 = 7 \times 9$

2. $4 \times 0 = 0 \times 6$

3. $8 \div 1 \neq 1 \div 8$

4. $48 \times \frac{1}{2} \neq 48 \times 0.5$

5. $3 \times (4 \times 9) = (3 \times 4) \times 9$

6. $(14 - 3) - 1 = 14 - (3 - 1)$

7. $\frac{(8 - 2)}{2} = 8 - 1$

8. $0.12 \times 5 = 1.2 \times 0.5$

Simplify each expression.

9. $9 + (5 \times 4)$

10. $(9 + 5) \times 4$

11. $17 - (3 \times 3)$

12. $(17 - 3) \times 3$

13. $\frac{(32 - 7)}{5}$

14. $\frac{(13 + 11)}{(6 - 2)}$

Evaluate each expression if $a = 1$, $b = 2$, and $c = 3$.

15. $4b$

16. $6a$

17. $c - 3$

18. $9 - c$

19. $\frac{2}{b}$

20. $(5c) - 4$

21. $b + (ac)$

22. $(bc) + a$

23. $3(a + 1)$

24. $2(b - 2)$

25. $\frac{a + b}{c}$

26. $\frac{a}{c - b}$

Written Exercises

Simplify each expression.

A

1. $(8 - 3) + 3$

2. $9 + (18 - 2)$

3. $5 \times (11 + 1)$

4. $(13 - 6) \times 7$

5. $(6 + 12) \div 3$

6. $6 + (12 \div 3)$

7. $29 - (0 \times 9)$

8. $5 - (16 \div 4)$

9. $(8 \times 17) + (2 \times 17)$

10. $(12 \times 11) - (2 \times 11)$

11. $(26 + 4) \div (30 \div 2)$

12. $(40 \div 10) \div (1 \times 4)$

Evaluate each expression if $x = 2$, $y = 1$, $z = 9$, $a = 7$, $b = 0$, and $c = 3$.

13. xy

14. ab

15. $z - (xc)$

16. $y + (xa)$

17. $(2c) + (2x)$

18. $(3a) - (5y)$