MATHEMATICS RESEARCH DEVELOPMENTS

Generalized Fractional Programming

Gaston M. N'Guérékata ▪ Ram Verma

NOVA

This monograph is aimed at presenting smooth and unified generalized fractional programming (or a program with a finite number of constraints). Under the current interdisciplinary computer-oriented research environment, these programs are among the most rapidly expanding research areas in terms of its multi-facet applications and empowerment for real world problems that can be handled by transforming them into generalized fractional programming problems. Problems of this type have been applied for the modeling and analysis of a wide range of theoretical as well as concrete, real world, practical problems. More specifically, generalized fractional programming concepts and techniques have found relevance and worldwide applications in approximation theory, statistics, game theory, engineering design (earthquake-resistant design of structures, design of control systems, digital filters, electronic circuits, etc.), boundary value problems, defect minimization for operator equations, geometry, random graphs, graphs related to Newton flows, wavelet analysis, reliability testing, environmental protection planning, decision making under uncertainty, geometric programming, disjunctive programming, optimal control problems, robotics, and continuum mechanics, among others. It is highly probable that among all industries, especially for the automobile industry, robots are about to revolutionize the assembly plants forever. That would change the face of other industries toward rapid technical innovation as well.

The main focus of this monograph is to empower graduate students, faculty and other research enthusiasts for more accelerated research advances with significant applications in the interdisciplinary sense without borders. The generalized fractional programming problems have a wide range of real-world problems, which can be transformed in some sort of a generalized fractional programming problem. Consider fractional programs that arise from management decision science; by analyzing system efficiency in an economical sense, it is equivalent to maximizing system efficiency leading to fractional programs with occurring objectives:

Maximizing productivity
Maximizing return on investment
Maximizing return/ risk
Minimizing cost/time
Minimizing output/input

The authors envision that this monograph will uniquely present the interdisciplinary research for the global scientific community (including graduate students, faculty, and general readers). Furthermore, some of the new concepts can be applied to duality theorems based on the use of a new class of multi-time, multi-objective, variational problems as well.

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Preface

This book is aimed at presenting a smooth and unified generalized fractional programming (or program with a finite number of constraints). Under the current interdisciplinary computer-oriented research environment, these programs are among the most rapidly expanding research areas in terms of its multi-facet applications-empowerment for real-world problems that can be handled by transforming them into generalized fractional programming problems. Problems of this type have been applied for the modeling and analysis of a wide range of theoretical as well as concrete, real-world, practical problems. More specifically, generalized fractional programming concepts and techniques have found relevance and worldwide applications in approximation theory, statistics, game theory, engineering design (earthquake-resistant design of structures, design of control systems, digital filters, electronic circuits, etc.), boundary value problems, defect minimization for operator equations, geometry, random graphs, graphs related to Newton flows, wavelet analysis, reliability testing, environmental protection planning, decision making under uncertainty, geometric programming, disjunctive programming, optimal control problems, robotics, and continuum mechanics, among others. It is highly probable that among all industries, especially for the automobile industry, the robots are about to revolutionize the assembly plants forever. That would change the face of other industries toward rapid technical innovation as well.

The main focus of this book is to empower graduate students, faculty and other research enthusiasts for more accelerated research advances with significant applications in the interdisciplinary sense without borders. The generalized fractional programming problems have a wide range of real-world problems which can be transformed in some sort of generalized fractional programming problems.

Let us consider fractional programs that arise from management decision science. If we consider a system efficiency in economical sense, it is equivalent to maximizing system efficiency leading to fractional programs with occurring objectives:

– Maximizing productivity
– Maximizing return on investment
– Maximizing return/risk
– Minimizing cost/time
– Minimizing output/input

We envision that this book is a unique presentation for interdisciplinary research for the worldwide scientific community (including graduate students, faculty, and general readers). Furthermore, some of the new concepts can be applied to duality theorems based on using a new class of multitime multiobjective variational problems as well.

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Ram U. Verma

September 1, 2017

Contents

Introduction

There exists a remarkable possibility that one can master a subject mathematically, without really understanding its essence.

- Albert Einstein

Human mind is the most valuable treasure.

- Unknown

In mathematical programming (or program), for a fractional programming (or program), the objective function is the ratio of two functions which are generally nonlinear on a feasible set with a finite number of constraints. Sometimes, this type of fractional programming problems are referred to as *generalized fractional programs*. When the status of ratio functions is not specified (i.e., if both functions are affine, and the feasible set is a convex polyhedron), it leads to a *linear fractional program*. For more details on mathematical programming and related literature, we refer the reader [1] - [97], while for details on applications, we refer the publications and surveys by Schaible and Shi [53] - [60].

On the other hand, a fractional programming problem with a finite number of variables and infinitely many constraints is referred to as a *semiinfinite programming problem* in the literature, especially the publications of Zalmai [89], [90] - [94]. Problems of this type have been applied for the modeling and analysis of a wide range of theoretical as well as concrete, real-world, practical problems. More specifically, semiinfinite programming concepts and techniques (introduced in [89], [91] - [95]) have found relevance and worldwide applications in approximation theory, statistics, game theory, engineering design (earthquake-resistant design of structures, design of control systems, digital filters, electronic circuits, etc.), boundary value problems, defect minimization for operator equations, geometry, random graphs, graphs related to Newton flows, wavelet analysis, reliability testing, environmental protection planning, decision making under uncertainty, semidefinite programming, geometric programming, disjunctive programming, optimal control problems, robotics, and continuum mechanics, among others. For more suitable details pertaining to various aspects of semiinfinite programming, including areas of significant applications, optimality conditions, duality relations, and numerical algorithms, we refer the reader [26], [42], [50], [62] - [64], [88], [91] - [93], [95] - [96].

Generalized Programming Problems

We begin with considering a generalized minimization programming problem as follows:

$$(P_1) \qquad \text{Minimize} \qquad f(x)$$

subject to constraints $g_j(x) \leq 0$ for all $j \in \underline{m} = \{1, 2, \cdots, m\}$, $x \in X \subset \mathbb{R}^n$, and $f_i, i \in \underline{p}$ and $g_j, j \in \underline{m}$ are real-valued continuous functions on X.

Role of Constraint Qualifications

Consider the following programming problem [37]

$$(P^*) \qquad \text{Minimize} \qquad f(x)$$

subject to constraints $g_j(x) \leq 0$, and $h(x) = 0$, with the feasible set $\mathbb{F}$ defined by

$$\mathbb{F} = \{x \in \mathbb{R}^n : g(x) \leq 0, h(x) = 0\},$$

where $f : \mathbb{R}^n \rightarrow \mathbb{R}^p$, $g : \mathbb{R}^n \rightarrow \mathbb{R}^m$ and $h : \mathbb{R}^n \rightarrow \mathbb{R}^r$are twice continuously differentiable functions on $\mathbb{R}^n$.

We begin with the well-known necessary conditions for $x^* \in \mathbb{F}$ to be a local minimum solution to (P^*).

The Fritz John Condition

There exist multipliers $\lambda_0 \in \mathbb{R}$, $\lambda \in \mathbb{R}^m$ and $\mu \in \mathbb{R}^k$, not all zero, such that (for $x^* \in \mathbb{F}$)

$$\lambda_0 \nabla f(x^*) + \sum_{i=1}^{m} \lambda_i \nabla g_i(x^*) + \sum_{j=1}^{k} \mu_j \nabla h_j(x^*) = 0,$$

$$\lambda_0 \geq 0, \lambda = (\lambda_1, \cdots, \lambda_m) \geq 0, \textit{ and } \lambda_i = 0 \, \forall \, i \notin A \cup \{0\}, \tag{1}$$

where

$$A = \{i : g_i(x^*) = 0\}.$$

The Kuhn-Tucker Condition

There exist multipliers $\lambda_0 \in \mathbb{R}$, $\lambda \in \mathbb{R}^m$ and $\mu \in \mathbb{R}^k$, not all zero, such that

$$\lambda_0 f(x^*) + \sum_{i=1}^{m} \lambda_i \nabla g_i(x^*) + \sum_{j=1}^{k} \mu_j \nabla h_j(x^*) = 0,$$

$$\lambda_0 > 0, \lambda = (\lambda_1, \cdots, \lambda_m) \geq 0, \textit{ and } \lambda_i = 0 \, \forall \, i \notin A \cup \{0\}, \tag{2}$$

where

$$A = \{i : g_i(x^*) = 0\}.$$

Furthermore, we need some more constraints, that are referred to as Constraint Qualifications/(CQ) in the literature.

Abadie's Constraint Qualifications

$$(ACQ) \qquad L_1 = T_1, \tag{3}$$

where

$$L_1 := \{y \in \mathbb{R}^n : \nabla g_A(x^*)y \leq 0, \nabla h(x^*) = 0\}$$

and

$$T_1 := \{y \in \mathbb{R}^n : \exists\, x_n \in \mathbb{F}, \exists\, t_n \to +\infty \ \textit{such that}\ x_n = x^* + t_n y + o(t_n)\}$$

and $o(t_n)$ is a vector satisfying $\frac{\|o(t_n)\|}{t_n} \to 0$.

Guignard's Constraint Qualification

$$(GCQ) \qquad L_1 = \bar{co}\, T_1, \tag{4}$$

where $\bar{co}$ is the closure of the convex hull of T_1.

Linear Constraint Qualification

(LCQ) The active constraints are all affine.

Mangasarian-Fromovitz Constraint Qualification

(MFCQ) $\nabla h_1(x^*), \cdots, \nabla h_k(x^*)$ are all linearly independent and there is some direction v such that
$\nabla g_A(x^*)v < 0, \nabla h(x^*)v = 0.$