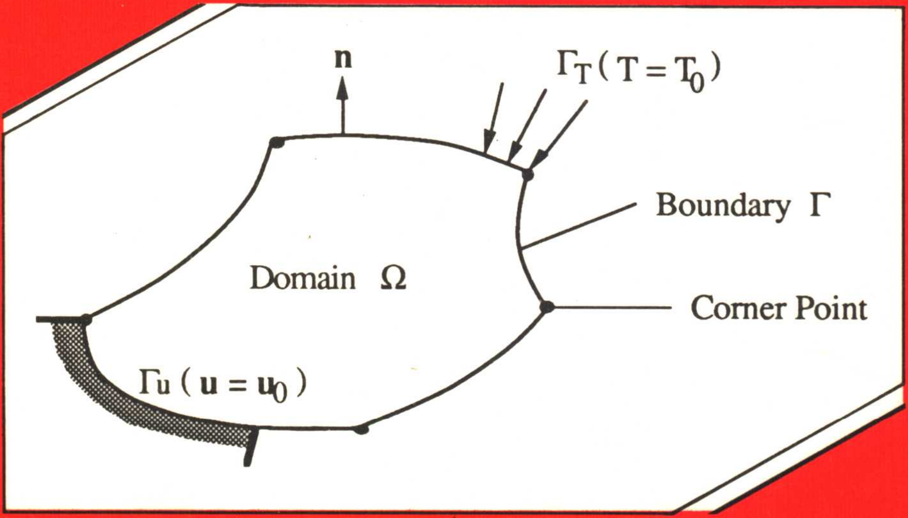
Boundary Elements XII

Vol. 2 Applications in Fluid Mechanics and Field Problems

Editors: M. Tanaka, C.A. Brebbia and T. Honma



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Vol 2: Applications in Fluid Mechanics and Field Problems

Proceedings of the Twelth International Conference on Boundary Elements in Engineering, held at Hokkaido University, Sapporo, Japan, during 24-27 September 1990.

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PREFACE

The Boundary Element Method has become an accepted and powerful method for the solution of many engineering problems. Recent theoretical and computational developments have transformed the BEM from a technique purely for research into an efficient tool for engineering design. These developments are in great part due to the many researchers who have attended this series of conferences since the first was held in Southampton in 1978.

The 12th International Conference on Boundary Elements in Engineering held at Hokkaido University (Sapporo) Japan, during 24-27th September 1990 reviewed the latest developments in the technique and pointed out advanced future trends. A special objective of this Conference was to bring together researchers from all over the world and make them aware of the rapid advances in the Boundary Element Method made by colleagues in Asia. The conference also acted as a link between practising engineers and industrial users of BEM and researchers working on the latest developments of the method.

This book with its companion volume comprises edited versions of the papers presented at the meeting. The present volume includes sections on Fluid Mechanics; Free Surface Problems; Acoustics; Sensitivity Analysis and Optimization; Electromagnetics; Inverse Problems; Engineering Applications and Pre- and Post-Processing. The sections in the companion volume deal with Mathematical and Computational Aspects; Potential and Diffusion Problems; Stress Analysis; Plates and Shells and Dynamics.

The Organizing Committee wishes to express its gratitude to the organizations sponsoring the conference, i.e. the Japan Society for Computational Methods in Engineering (JASCOME); the Computational Mechanics Institute (CMI) of Technology; the International Society of Boundary Elements (ISBE). They are also grateful to the members of the Scientific Advisory Committee who made this Conference successful. Thanks are also due to Kozo Keikaku Engineering Inc. for its helpful support service at the JASCOME Office, in sharing the management of all the matter inherent to the conference secretariat together with the Wessex Institute of Technology secretariat.

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M. Tanaka
T. Honma

September 1990

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SECTION 1: FLUID MECHANICS

A Hybrid Panel Method for Aerofoil Aerodynamics

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ABSTRACT

This paper presents a hybrid panel method for aerofoils in irrotational flow, which uses a panel method in conjunction with an analytical solution based on the method of velocity singularities developed in References [1-4]. The basic panel method uses uniform source panels on the aerofoil contour and linearly-variable doublet panels on the camberline. The method is validated by comparison with the exact solution derived by conformal transformation for Karman-Trefftz aerofoils. The results obtained with this hybrid approach were found to be in better agreement with the exact solution than the basic panel method, which requires a larger number of panels to achieve comparable accuracy. As a result, the present hybrid panel method has a substantially better computing efficiency in comparison with the basic panel method.

1. INTRODUCTION

The boundary element methods in aerofoil aerodynamics lead to potential flow solutions which can be used in conjunction with real fluid flow calculations resulting from boundary layer corrections, compressibility corrections and estimation of the flow separation effects. In this sense, several panel methods based on source, vortex, doublet or higher order singularities have been widely used in applied aerodynamics (especially in the preliminary design stages).

In general, the panel methods provide very accurate potential flow solutions, but they are computationally less efficient than the analytical methods of solution, which however are usually less accurate especially at high angles of attack and when thick and cambered aerofoils are involved.

This paper presents a hybrid panel method for aerofoil aerodynamics, combining the advantages of the panel methods and of the analytical solutions, that is a high accuracy combined with a good computing efficiency.

In the first part, the paper presents briefly the method of velocity singularities which was recently used to develop a nonlinear analytical solution for aerofoils in irrotational flow [1]. The method of velocity singularities was initially developed as a linear theory by Mateescu and Newman [2,3] and applied to the analysis of flexible-membrane and jet-flapped aerofoils [3,4]. This method, which is used in the present hybrid panel method, determines the perturbation velocity field by considering the singular contributions of the geometrically important elements of the aerofoil, namely the leading edge and the ridges (points where the aerofoil surface slope changes), which are mathematically derived to satisfy the basic boundary conditions and the Kutta condition at the trailing edge. This type of approach is somewhat similar to that developed by Carafoli and Mateescu [5,6] for wings in supersonic flow, although for that case the singular contributions were derived in a crossflow plane from initially different governing equations.

The paper presents next a panel method using uniform source panels on the aerofoil contour and linearly-variable doublet panels on the camberline, with a special leading edge treatment. The results obtained with this panel method, as well as the nonlinear analytical solution developed in Reference [1], were found in general to be in good agreement with the exact solution obtained by conformal transformation for Karman - Trefftz aerofoils. However, the panel method is less accurate near the leading edge and requires usually a large number of panels on the aerofoil contour and on the camberline for accurate results.

In order to obtain a better accuracy with a reduced number of panels, a hybrid panel method is developed in this paper. In this hybrid method, the panel method is used in conjunction with an analytical solution based on the method of velocity singularities [1-4], which leads to improved accuracy and computing efficiency.

2. ANALYTICAL SOLUTIONS BASED ON THE METHOD OF VELOCITY SINGULARITIES

The fluid velocity around an aerofoil placed at an angle of attack α in an incompressible uniform flow of velocity U_∞, can be expressed as

$$\mathbf{V} = \hat{\mathbf{i}}\ (U_\infty \cos\alpha + u) + \hat{\mathbf{j}}\ (U_\infty \sin\alpha + v)\ , \tag{1}$$

where u and v are the perturbation velocity components parallel and normal to the chord, respectively, and $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ are the unit vectors of the axes x and y directed along the aerofoil chord and normal to it. In a potential flow, u and v are harmonic functions (satisfying the Laplace equation) and can be expressed as the real and imaginary parts of a complex conjugate perturbation velocity defined as

$$W(z) = u(x,y) - i\ [v(x,y) - v_0]\,, \qquad z = x + iy\ , \tag{2}$$

where v_0 is a conveniently chosen constant and where the complex variable z is defined by the coordinates x and y of the physical plane.

A typical problem for the linear theory is represented by a thin flapped aerofoil shown in Figure 1, characterized by a sudden change β of the aerofoil slope at the ridge $R(x = s)$ and defined by the boundary conditions on the chord line

$$v = \begin{cases} v_0 & \text{for} \quad 0 < x < s \\ v_0 + \Delta v & \text{for} \quad s < x < c \end{cases} \tag{3}$$

$$u = 0 \quad \text{for} \quad x < 0 \quad \text{and} \quad x > c\ , \tag{4}$$

where

$$v_0 = [-\sin\alpha + \cos\alpha \tan\alpha]\ U_\infty\ , \tag{5}$$

$$\Delta v = \cos\alpha\ [\tan(\delta - \beta) - \tan\delta]\ ; \tag{6}$$

the condition (4) implies that the chordwise perturbation velocity is, in the linear theory, antisymmetric with respect to the chordline.

The solution for this prototype problem was derived in the complex plane $z = x + iy$, as shown in References [1,2], in the form

$$W(z) = A\sqrt{\frac{c-z}{z}} - \frac{2}{\pi}\cosh^{-1}\sqrt{\frac{(c-z)s}{c(s-z)}} \tag{7}$$

where

$$A = -v_0 - \frac{2}{\pi}\Delta v\cosh^{-1}\sqrt{\frac{s}{c}}\,. \tag{8}$$

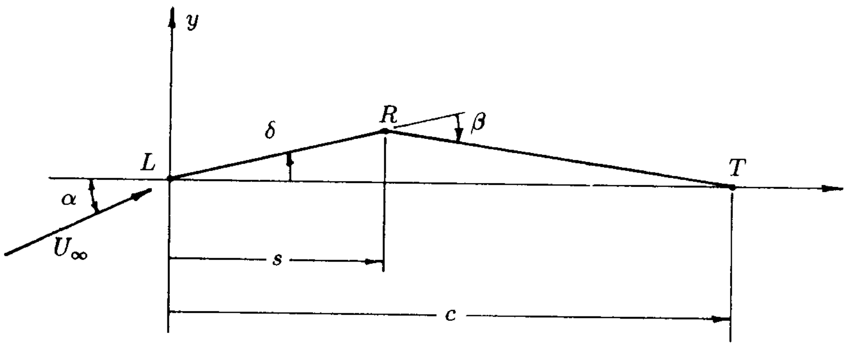

Figure 1. Geometry of a typical thin flapped aerofoil at incidence.

On the upper part of the aerofoil, the chordwise perturbation velocity, u, may be expressed as

$$u(x) = A\sqrt{(c-x)/x} - \Delta v\, G(c,s,x)\,, \tag{9}$$

where

$$G(c,s,x) = \begin{cases} \dfrac{2}{\pi}\cosh^{-1}\sqrt{\dfrac{(c-x)s}{c(s-x)}} & \text{for} \quad 0 < x < s \\ \dfrac{2}{\pi}\sinh^{-1}\sqrt{\dfrac{(c-x)s}{c(x-s)}} & \text{for} \quad s < x < c \\ 0 & \text{for} \quad x < 0,\ \ x > c\,. \end{cases} \tag{10}$$

The first and second terms of equations (7) and (9) represent the singular contributions of the leading edge ($x = 0$) and ridge ($x = s$), respectively, which implicitly satisfy the Kutta condition at the trailing edge ($x = c$).

The solution for a continuously-cambered thin aerofoil, which is defined by the camberline slope $h'(x)$ and by the boundary condition

$$v_A(x) = U_\infty\,[-\sin\alpha + h'(x)\cos\alpha]\,, \tag{11}$$

can be obtained by superimposing infinitesimally flapped aerofoils in the form

$$\begin{aligned} u_A(x) &= -\left[v_A(0) + \frac{2}{\pi}\int_0^c v_A'(s)\cos^{-1}\sqrt{\frac{s}{c}}\,ds\right]\sqrt{\frac{c-x}{x}} \\ &\quad - \int_0^c v_A'(s)\,G(c,s,x)\,ds\,. \end{aligned} \tag{12}$$

When a polynomial expansion (as adopted for NACA aerofoils [7]) is assumed for the camberline slope,

$$h'(x) = \sum_{k=0}^{n} h_k \left(\frac{x}{c}\right)^k , \tag{13}$$

the solution becomes

$$u_A(x) = U_\infty \left[\sin\alpha - \cos\alpha \sum_{k=0}^{n} h_k \sum_{j=0}^{k} q_{k-j} \left(\frac{x}{c}\right)^j\right] \sqrt{\frac{c-x}{x}} , \tag{14}$$

where

$$q_k = \frac{(2k)!}{2^{2k}\,(k!)^2} . \tag{15}$$

Similarly, for an aerofoil of symmetrical thickness at zero angle of attack defined by its slope $g'(x)$ and by the boundary condition $v_S(x) = U_\infty\, g'(x)$, the linear solution for the chordwise perturbation velocity is obtained in the form

$$u_S(x) = \frac{1}{\pi}\left[v_S(0)\,\ln x + \int_0^c v_S'(s)\,\ln(x-s)\,ds - v_S(1)\,\ln(x-c)\right] . \tag{16}$$

However, depending on the expansion considered for $g'(x)$, the leading and trailing edges may be singular points in this pure linear solution, instead of being stagnation points, although for slender aerofoils this solution may be sufficiently accurate for the most part of the aerofoil except the extremities. The correct flow at the leading and trailing edges is better predicted by the nonlinear approach or by the local linearization method.

Local linearization method. A correct behaviour of the flow field at the leading and trailing edges can be obtained with a simple but very accurate solution [1] based on the local linearization of the boundary conditions with respect to the tangential component of the free stream velocity relative to the aerofoil surface, $U_{\infty_t} = U_\infty\, \cos(\tau - \alpha)$, where τ is its slope angle with respect to the chord and α is the incidence. In this local linearization method, the boundary conditions are expressed using the tangential and normal components of the perturbation velocity, u_t and v_n, defined as

$$u_t = V - U_{\infty_t}, \qquad v_n = U_{\infty_t}\, \tan(\tau - \alpha) . \tag{17}$$

Following Mateescu and Nadeau [1], the boundary condition for a symmetrical aerofoil at zero incidence can be expressed as

$$v_n \simeq U_{\infty_t}\, \tan\tau = U_{\infty_t}\, g'(x), \qquad g'(x) = \sqrt{\frac{c-x}{x}} \sum_{k=0}^{N} g_k \left(\frac{x}{c}\right)^k , \tag{18}$$

leading to the solution for the tangential perturbation velocity component

$$u_t = U_{\infty_t} \sum_{k=0}^{N} g_k \left[q_k - \frac{c-x}{c} \sum_{j=0}^{k-1} q_{k-j-1} \left(\frac{x}{c}\right)^j\right] . \tag{19}$$

The local linearization solution for the pressure coefficient on the aerofoil,

$$C_p = 1 - \frac{V^2}{U_\infty^2} = 1 - \frac{(1 + u_t/U_{\infty_t})^2}{1 + [g'(x)]^2} , \tag{20}$$

was found to be in very good agreement with the exact solution derived by conformal transformation for symmetrical Jukovski aerofoils, as shown in Reference [1].

Nonlinear solution. The boundary conditions on the upper and lower surfaces of a general aerofoil at incidence α may be expressed as

$$\frac{U_\infty \sin\alpha + v_u}{U_\infty \cos\alpha + u_u} = h'(x) + g'(x), \quad \frac{U_\infty \sin\alpha + v_l}{U_\infty \cos\alpha + u_l} = h'(x) - g'(x)\,, \tag{21}$$

where $h'(x)$ is the camberline slope and $\pm g'(x)$ is the slope associated with the aerofoil thickness, considered symmetrically distributed with respect to the camberline.

As shown in Reference [1], the flow field around the aerofoil may be decomposed into an antisymmetric flow field ($\pm u_A$ and v_A) and a symmetric one (u_S and $\pm v_S$), i.e.

$$\begin{aligned} u_u &= u_A + u_S\,, & v_u &= v_A + v_S\,, \\ u_l &= -u_A + u_S\,, & v_l &= v_A - v_S\,. \end{aligned} \tag{22}$$

In this manner, conditions (21) may be recast to define the boundary conditions for the antisymmetric and symmetric flows in the form

$$v_A(x) = -U_\infty \sin\alpha + [U_\infty \cos\alpha + u_S(x)]\; h'(x) + u_A(x)\; g'(x)\,, \tag{23}$$

$$v_S(x) = [U_\infty \cos\alpha + u_s(x)]\; g'(x) + u_A(x)\; h'(x)\,. \tag{24}$$

Since in the nonlinear formulation the antisymmetric and symmetric flows are strongly coupled (by contrast with the linear theory where they are completely separated), one cannot expect to find analytical functions for the unknowns $u_A(x)$, $v_A(x)$, $u_S(x)$ and $v_S(x)$ to identically satisfy equations (23), (24), as well as (12) and (16). The strategy adopted to solve this nonlinear problem was to consider convenient expressions for $v_A(x)$ and $v_S(x)$, containing appropriate singularities and with *a priori* unknown coefficients, which are used to derive $u_A(x)$ and $u_S(x)$ by integration from equations (12) and (16). These solutions are then inserted in equations (23) and (24) which are imposed to be rigorously satisfied at a certain number N of discrete points x_j $(j = 1, 2, \ldots, N)$ on each side of the aerofoil, leading to a system of algebraic equations which is solved to determine the *a priori* unknown coefficients included in the assumed expansions of $v_A(x)$ and $v_S(x)$.

This approach led to very accurate nonlinear solutions, as shown in Reference [1], where a typical number $N = 10$ discrete points was chosen on each side of the aerofoil.

3. BASIC PANEL METHOD

The basic panel method in this analysis uses source panels on the aerofoil contour and linearly-variable doublet panels on the camberline, as proposed by Hunt [8]; the wake is replaced in this approach by a panel with a uniform doublet distribution of intensity m_W equal to the circulation around the aerofoil. The fluid velocity can be expressed as

$$\mathbf{V} = U_\infty \left(\hat{\mathbf{i}} \cos\alpha + \hat{\mathbf{j}} \sin\alpha\right) + \nabla\varphi\,, \tag{25}$$

where $\varphi(x,y)$ represents the perturbation velocity potential determined with the panel method. The singularities associated with the source and doublet panels are derived from the application of Green's theorem, which leads to the representation of the perturbation potential in the form

$$\varphi(x,y) = \int_S q\, f_S(r)\, dS + \int_{S_c \cup S_W} m\, f_D(r)\, dS\,, \tag{26}$$

where q is the intensity of the source distribution on the aerofoil contour, S, m is the intensity of the doublet distribution on the camberline and wake, S_c and S_W, and where

$f_S(r)$ and $f_D(r)$ are the Green's singular functions associated with the source and doublet distributions defined as

$$f_S(r) = \frac{1}{2\pi} \ln r \,, \tag{27}$$

$$f_D(r) = -\mathbf{n} \cdot \nabla f_S = -\frac{1}{2\pi} \frac{\mathbf{n} \cdot \mathbf{r}}{r^2} \,, \tag{28}$$

in which $\mathbf{r}$ is the position vector of the point (x, y) where the potential is evaluated with respect to the source or doublet location, and $\mathbf{n}$ is the unit vector normal to the surface (camberline or wake) containing the doublet distribution.

The actual aerofoil contour and the camberline are then divided in $2N$ and, respectively, N panels (or boundary elements) along which uniform source distributions, q_j, or linearly-variable doublet distributions, $m_j + \gamma_j\, s_j$, are considered. Thus, the potential φ evaluated at a control point $(x_i,\ y_i)$ can be expressed as

$$\begin{aligned} \varphi(x_i,\ y_i) &= \sum_{j=1}^{2N} q_j \frac{1}{2\pi} \int_{-S_j}^{S_j} \ln\ r_{ij}\ ds_j \\ &+ \sum_{j=1}^{N+1} \frac{-1}{2\pi} \int_{-S_j}^{S_j} (m_j + \gamma_j s_j)\ \mathbf{n}_j \cdot \nabla\ (\ln\ r_{ij})\ ds_j \,, \end{aligned} \tag{29}$$

where

$$r_{ij} = \sqrt{(x_i - x_j - s_j\ \cos\beta_j)^2 + (y_i - y_j - s_j\ \sin\tau_j)^2} \,, \tag{30}$$

$$\mathbf{r}_{ij} = \hat{\mathbf{i}}\ (x_i - x_j - s_j\ \cos\tau_j) + \hat{\mathbf{j}}\ (y_i - y_j - s_j\ \sin\tau_j) \,, \tag{31}$$

in which x_j, y_j are the coordinates of the mid-panel points, s_j is a local coordinate along the panel with respect to its midpoint, S_j represents the semi-length of the panel and τ_j is the panel slope angle with respect to the chordwise axis x; the wake doublet panel, denoted by $j = N + 1$, has a uniform doublet distribution and hence $\gamma_{N+1} = 0$.

The linearly-variable panels can be conveniently replaced by an equivalent system of uniform vortex panels and concentrated vortices at the panel extremities, according to the equation

$$\begin{aligned} &-\frac{1}{2\pi} \int_{-S_j}^{S_j} (m_j + \gamma_j\ s_j)\ \mathbf{n}_j \cdot \nabla\ (\ln\ r_{ij})\ ds_j = \\ &= (m_j - \gamma_j\ S_j)\ \Theta_{i,j} - (m_j + \gamma_j\ S_j)\ \Theta_{i,j+1} + \gamma_j \int_{-S_j}^{S_j} \theta_{ij}\ ds_j \,, \end{aligned} \tag{32}$$

where

$$\Theta_{i,j} = \frac{1}{2\pi} \tan^{-1} \frac{y_i - y_j + S_j\ \sin\tau_j}{x_i - x_j + S_j\ \cos\tau_j} \,, \tag{33}$$

$$\theta_{ij} = \frac{1}{2\pi} \tan^{-1} \frac{y_i - y_j - s_j\ \sin\tau_j}{x_i - x_j - s_j\ \cos\tau_j} \,. \tag{34}$$

One can notice that the contribution of the wake panel ($m_W = m_{N+1}$ and $\gamma_{N+1} = 0$), reduces to the contribution of a concentrated vortex situated at the trailing edge of the aerofoil (the other extremity of the wake panel being situated at infinity).

The boundary conditions are imposed in this basic approach at the midpoints of the panels situated on the aerofoil contour, and the Kutta condition is implemented at a very small distance δ behind the trailing edge on the camberline extension. These provide the following $(2N+1)$ boundary conditions

$$\sum_{j=1}^{2N} q_j\, C_{ij} + \sum_{j=1}^{N} [(m_j - \gamma_j\, S_j)\, \mathbf{n}_i \cdot \nabla\, \Theta_{ij} - (m_j + \gamma_j\, S_j)\, \mathbf{n}_i \cdot \nabla\, \Theta_{i,j+1} + \gamma_j\, K_{ij}]$$

$$+ m_{N+1}\, \mathbf{n}_i \cdot \nabla\, \Theta_{i,N+1} = U_\infty\, \sin(\tau_i - \alpha), \quad \text{for } i = 1, 2, \ldots, 2N+1\,, \tag{35}$$

where C_{ij} and K_{ij} are the influence coefficients of the source and vortex panels defined as

$$C_{ij} = \frac{1}{2\pi} \int_{-S_j}^{S_j} \frac{\mathbf{n}_i \cdot \mathbf{r}_{ij}}{r_{ij}^2}\, ds_j\,, \tag{36}$$

$$K_{ij} = \int_{-S_j}^{S_j} \mathbf{n}_i \cdot \nabla\, \theta_{ij}\, ds_j = \frac{1}{2\pi} \int_{-S_j}^{S_j} \frac{\mathbf{t}_i \cdot \mathbf{r}_{ij}}{r_{ij}^2}\, ds_j\,. \tag{37}$$

However, the singularities strengths q_j, m_j and γ_j, representing $(4N+1)$ unknowns, are not uniquely determined by the above $(2N+1)$ boundary conditions, and hence $2N$ additional conditions have to be imposed to render the problem determined. The following strategy regarding the additional conditions, as suggested by Hunt [8], is adopted in this basic approach:

(i) Equal strengths, $q_j = q_{2N+1-j}$, are considered for the *opposed* source panels situated on the upper and lower surfaces of the aerofoil, assuming that the main rôle of the source panels is to simulate the effect of the aerofoil thickness.

(ii) The slopes of the linearly-variable doublet distributions on the camberline panels (with the main aim to simulate the effect of the incidence and camber) are defined as

$$\gamma_j = \frac{m_{j+1} - m_j}{S_j + S_{j+1}}\,, \tag{38}$$

with $S_{N+1} = 0$ and $\gamma_{N+1} = 0$ for the wake panel.

With these additional conditions, equations (35) can now be solved for the singularities strengths q_j and m_j. The velocity at the midpoint $(x_i,\ y_i)$ of the panel i can be expressed, since $V_i = \mathbf{t}_i \cdot \mathbf{V}_i$, as

$$V_i = U_\infty\, \cos(\beta_i - \alpha) + \sum_{j=1}^{2N} q_j\, T_{ij} + m_{N+1}\, \mathbf{t}_i \cdot \nabla\, \Theta_{i,N+1}$$

$$+ \sum_{j=1}^{N} [(m_j - \gamma_j\, S_j)\, \mathbf{t}_i \cdot \nabla\, \Theta_{ij} - (m_j + \gamma_j S_j)\, \mathbf{t}_i \cdot \nabla\, \Theta_{i,j+1} + \gamma_j\, Q_{ij}]\,, \tag{39}$$

where

$$T_{ij} = \frac{1}{2\pi} \int_{-S_j}^{S_j} \frac{\mathbf{t}_i \cdot \mathbf{r}_{ij}}{r_{ij}^2}\, ds_j\,, \tag{40}$$

$$Q_{ij} = \int_{-S_j}^{S_j} \mathbf{t}_i \cdot \nabla\, \theta_{ij}\, ds_j\,. \tag{41}$$