Model Predictive Control

Theory, Practices and Future Challenges

Corrine Wade
Editor

Mechanical Engineering Theory and Applications

NOVA

MECHANICAL ENGINEERING THEORY AND APPLICATIONS

MODEL PREDICTIVE CONTROL

THEORY, PRACTICES AND FUTURE CHALLENGES

CORRINE WADE
EDITOR

nova publishers
New York

NOTICE TO THE READER

The Publisher has taken reasonable care in the preparation of this book, but makes no expressed or implied warranty of any kind and assumes no responsibility for any errors or omissions. No liability is assumed for incidental or consequential damages in connection with or arising out of information contained in this book. The Publisher shall not be liable for any special, consequential, or exemplary damages resulting, in whole or in part, from the readers' use of, or reliance upon, this material. Any parts of this book based on government reports are so indicated and copyright is claimed for those parts to the extent applicable to compilations of such works.

Independent verification should be sought for any data, advice or recommendations contained in this book. In addition, no responsibility is assumed by the publisher for any injury and/or damage to persons or property arising from any methods, products, instructions, ideas or otherwise contained in this publication.

This publication is designed to provide accurate and authoritative information with regard to the subject matter covered herein. It is sold with the clear understanding that the Publisher is not engaged in rendering legal or any other professional services. If legal or any other expert assistance is required, the services of a competent person should be sought. FROM A DECLARATION OF PARTICIPANTS JOINTLY ADOPTED BY A COMMITTEE OF THE AMERICAN BAR ASSOCIATION AND A COMMITTEE OF PUBLISHERS.

Additional color graphics may be available in the e-book version of this book.

Library of Congress Cataloging-in-Publication Data

Model predictive control : theory, practices and future challenges / [edited by] Corrine Wade.
pages cm. -- (Mechanical engineering theory and applications)
Includes bibliographical references and index.
ISBN 978-1-63463-859-3 (hardcover)
1. Predictive control. I. Wade, Corrine, editor.
TJ217.6.M63 2015
629.8--dc23
2014048429

Published by Nova Science Publishers, Inc. † New York

MECHANICAL ENGINEERING THEORY AND APPLICATIONS

MODEL PREDICTIVE CONTROL

THEORY, PRACTICES AND FUTURE CHALLENGES

MECHANICAL ENGINEERING THEORY AND APPLICATIONS

Additional books in this series can be found on Nova's website under the Series tab.

Additional e-books in this series can be found on Nova's website under the e-books tab.

CONTENTS

PREFACE

Although industrial processes are inherently nonlinear, many contributions for controller design for those plants are based on the assumption of a linear model of the system. However, in some cases it is difficult to represent a given process using a linear model. Model Predictive Control (MPC) is an optimal control approach which can effectively deal with constraints and multivariable processes in industries. Because of its advantages, MPC has been widely applied in automotive and process control communities. This book discusses the theory, practices and future challenges of model predictive control.

Among the several predictive control approaches proposed in the literature (hysteresis-based, trajectory-based, deadbeat, etc.), Model Predictive Control (MPC) surely represents the most promising one due to its inherent flexibility and versatility. In fact, MPC cost function generally consists of a sum of several terms, whose weights can be tuned in accordance with different criteria. As a result, an accurate choice of MPC cost function enables an appropriate optimization of several non-linear systems (electrical, mechanical, chemical, etc.), especially those characterized by several inputs and/or outputs. One of the most important feature of MPC consists of more easily taking into account both input and output constraints compared to other predictive control approaches, because they can be accounted by the MPC cost function. However, this solution does not guarantee that they are always satisfied, may impairing system performance optimization at the same time.

A viable solution consists of appropriately managing input and output constraints in order to guarantee system optimization. In particular, system operating boundaries should be determined at first, within which a number of MPC objective functions should be minimized in a descending order of priority. Although such an approach is generally quite complex compared to conventional MPC, it leads to a better system exploitation. In addition, higher computational efforts can be easily handled by means of fast processing units, like Field Programmable Gate Arrays (FPGA); in fact, they allow very fast execution times, even for advanced MPC-based control systems, making them particularly suitable in replacing traditional control techniques in industrial applications.

Chapter 1 addresses the problem of accurate management of input and output constraints for MPC. Thus, firstly referring to a generic system, problem statement and formulation are firstly introduced and briefly discussed. Subsequently, reference is made to a case study, namely a Surface-Mounted Permanent Magnet Synchronous Machine (SPM). In particular, its mathematical modelling is briefly introduced at first, as well as its operating constraints (voltage saturation, current limitations, etc.). Subsequently, the design of an MPC algorithm is reported, which is based on accurate management of SPM operating constraints in order to guarantee optimal SPM performances, over both steady-state and dynamic operation. Both simulation and experimental results are also enclosed in order to highlight the effectiveness of this MPC approach; in particular, simulations are carried out by means of Matlab-Simulink, whereas experiments refer to the employment of an appropriate FPGA-based control board.

Chapter 2 deals with a hybrid actuator composed by a piezo and a hydraulic part and with a Robust Model Predictive Control (RMPC) structure combined with a feedforward control in camless engine motor applications. A combination between a feedforward control based on an inversion of the system and an MPC structure is considered. To perform a feedforward regulator an identification of the start condition of the piezo actuator is needed. This start condition of the piezo actuator is due to some structural constructive aspects which generate an offset into the piezo position. The feedforward regulator ends up being an affine function to compensate for this offset. A procedure for its identification is proposed. The idea behind the conception of the proposed new actuator is to use the advantages of both the high precision of the piezo and the force of the hydraulic part. In fact, piezoelectric actuators (PEAs) are commonly used for precision positionings, despite the fact that PEAs present nonlinearities, such as hysteresis, saturations, and creep. In the control problem such nonlinearities must be taken into account. In this paper the Preisach dynamic model with the abovementioned nonlinearities is considered together with a feedforward control combined with a RMPC. Simulations of the implementation of the MPC structure together with the feedforward regulator and the abovementioned start condition of the piezo actuator with real data are shown.

Mobile robotics is a notable case of such evolution. The robotics community has developed sophisticated analysis and control techniques to meet increasing requirements for the control of motions of mechanical systems. These increasing requirements are motivated by higher performance specifications, notably an increasing number of degrees-of-freedom. Chapter 3 proposes a controller for the motion of the Robotino. The proposed controller takes under consideration a non-interacting control strategy realized using a geometric approach. Horizontal, vertical and angular motions are considered and once the decoupling between these motions is obtained, a Model Predictive Control (MPC) strategy is used in combination with a Feedforward controller. The approach used to obtain a decoupling consists of a geometric approach. In the past three decades, research on the geometric approach to dynamic systems theory and control has allowed this approach to become a powerful and a thorough tool for the analysis and synthesis of dynamic systems. Simulation results using real data of the Robotino are shown.

There are very few controller design techniques that can be proven to stabilize processes in the presence of nonlinearities and constraints. Model predictive control (MPC) is one of these techniques. For this reason, there has been much interest in nonlinear model-based control within the process engineering community. A critical step in the application of these methods is the development of a suitable model for the process dynamics. In this sense, block-oriented models have proved to be useful as simple nonlinear models for a vast number of applications. They are described as a cascade of linear dynamic and nonlinear static blocks. They have emerged as an appealing proposal due to their simplicity and the property of being valid over a larger operating region than a linear time invariant (LTI) model. A typical block-oriented model found in the literature is the Hammerstein model. In Chapter 4 a nonlinear memoryless block is followed by a linear dynamics. A broad type of dynamic processes can be described by such representations consisting of these two simple elements usually referred to as subsystems. This chapter deals with robust control for uncertain Hammerstein models. The starting point for the controller design is a Hammerstein model which describes the systems dynamics in the presence of uncertainty. This model is employed to design a model based predictive controller. The mathematical problem involved in the development of the algorithm is stated in the context of Linear Matrix Inequalities (LMI) theory. The straightforward use of Hammerstein models for designing the Model Predictive controller would lead to a nonlinear optimization problem due to the static nonlinearity. From the point of view of the implementation, this could result in high computational complexity and be a very time-consuming process. This can be avoided by exploiting the structure of the Hammerstein model, which is a novel approach. This strategy developed in this chapter takes advantage of the static nature of the nonlinearity which allows being transformed into polytopic representation and, therefore, to solve the control problem by focusing only in the linear dynamics. This formulation results in a simplified design procedure, because the original nonlinear Model Predictive Control problem turns into a linear one. At the end of the chapter, different simulation examples are presented to illustrate the controller design procedure.

In model predictive control (MPC) the basic notions are a trajectory and a set of feasible trajectories. By using these notions the author's describe the MPC procedure in Hilbert space. It is shown that a feasible set of trajectories can be associated with a bounded positive self-adjoint linear operator in Hilbert space. This operator gives a complete description of CS (at a given time or a state). In the same way a target of control is described. Chapter 5 shows that the principal stage of MPC - reconstruction of a new trajectory - can be formalized as the Newton transformation of a given trajectory. The author's show how these constructions in Hilbert space can be transformed into a mathematical (identified) model. For this an operator defined by relations between two bases (in Hilbert and finite coordinate spaces) is introduced and called a realization operator. This approach is generalized on a collection of trajectories (in Hilbert and coordinate spaces respectively) and naturally leads to frame theory. A predictive frame that gives a complete description of CS (at a given time or a state) is introduced in Hilbert space. By using the realization operator the author's transform the predictive frame into a table of numbers and we call it data predictive matrix. The data predictive matrix is a basis of a new approach to predictive control which the author's call data predictive control (DPC). The author's analyze the stages of DPC and show how by using different control strategies the construction of data predictive matrix can be adopted to goals of control. Finally, the author's consider a construction of the feasible trajectory which is the closest to a target (but unfeasible) trajectory (this procedure is similar to the consistent reconstruction in frame theory).

Consider a complex large-scale control system which is composed of many spatially distributed subsystems. Each subsystem interacts with some other subsystems by their states and/or inputs, e.g. large-scale chemical process, smart grid, distributed generation systems. The control objective is to achieve a specific global performance of the entire system or a common goal of all subsystems.

To control this class of system, the Distributed Model Predictive Control (DMPC), which controls each subsystem by a separate local Model Predictive Control (MPC), has become more and more popular since it not only inherits MPC's ability to explicitly accommodate constraints but also possesses the advantages of the distributed framework of good flexibility and good error tolerance. On the other hand, with the development of communication network technologies in process industries, which allow a distributed controller to access and send information throughout the system, also helps to promote distributed control solutions. However, as point in many articles, the performance of a DMPC is, in most cases, not as good as that of a centralized MPC.

The flexibility (or error tolerance) and global performance is two important characteristics of a DMPC. To improve the optimization performance, the existing methods usually increase the coordination degree (the range of cost that each subsystem-based MPC minimized). With the increasing of the coordination degree, the performance of entire system becomes better and better. However, with the increasing of the coordination degree, the network connectivity become more and more complicity, and consequently the error tolerance and high flexibility become weaker and weaker. It is not expected. Can the author's find a method which could improve the global performance or coordination degree without any increasing of network connectivity?

In Chapter 6, a novel coordination strategy, where each subsystem-based model predictive control (MPC) added a quadratic function of the affection of the current subsystem's input to its down-stream neighbors into its optimization index, is proposed for improving the optimization performance of entire system. This method is able to increase the coordination degree without any increasing of network connections comparing to the methods which do not use this coordination strategy. The consistency constraints, which limit the error between the state predicted at the previous time instant, referred to as the presumed state, and the state predicted at the current time instant within a prescribed bound, are designed and included in the optimization problem of each subsystem-based MPC. These constraints guarantee the recursive feasibility of each subsystem-based MPC. In the meantime, a stabilization constraint and the dual mode predictive control strategy are adopted to result in a stabilizing DMPC.

A set of feasible trajectories for a control system is analyzed. Chapter 7 considers real trajectories (realized by a given control system or calculated by a computer) and possible trajectories (subjected to the equations and constraints of a given mathematical model). Therefore the author's have in model predictive control (MPC) the space of real trajectories and the space of possible trajectories. It is shown that the basic notion for the space of possible trajectories is an equivalence relation and associated with this relation a set of feasible trajectories (a ball in a Banach space). This ball with some additional requirements of symmetry determines a positive inner product; thus the space of possible trajectories becomes a Hilbert space (an Euclidean space for finite dimensional space). It is shown that the basic notion for the space of real trajectories is an order relation and associated with this relation a positive cone. Under some additional requirements of symmetry the cone becomes an elliptical cone that determines a pseudo-Euclidean metric in the space of real trajectories. In the last part of the paper the space of trajectories is considered as a direct sum of two spaces introduced above. A control possibilities set is introduced in this space. It is shown that the projections of this set onto the above spaces define the structure of control in MPC. This structure is described by the composition of two multivalued mappings: a given trajectory to a set of possible trajectories and a set of possible trajectories to a new (optimal) trajectory called pilot transformation. The author's describe pilot transformation by using the basic constructions of frame theory.

In: Model Predictive Control
Editor: Corrine Wade, pp. 1-39
ISBN: 978-1-63463-859-3

Chapter 1

MODEL PREDICTIVE CONTROL WITH INPUT AND OUTPUT CONSTRAINTS

Alessandro Serpi, Gianluca Gatto, Alfonso Damiano and Ignazio Marongiu
Department of Electrical and Electronic Engineering, University of Cagliari, Piazza d'Armi, Cagliari, Italy

Abstract

Among the several predictive control approaches proposed in the literature (hysteresis-based, trajectory-based, deadbeat, etc.), Model Predictive Control (MPC) surely represents the most promising one due to its inherent flexibility and versatility. In fact, MPC cost function generally consists of a sum of several terms, whose weights can be tuned in accordance with different criteria. As a result, an accurate choice of MPC cost function enables an appropriate optimization of several non-linear systems (electrical, mechanical, chemical, etc.), especially those characterized by several inputs and/or outputs. One of the most important feature of MPC consists of more easily taking into account both input and output constraints compared to other predictive control approaches, because they can be accounted by the MPC cost function. However, this solution does not guarantee that they are always satisfied, may impairing system performance optimization at the same time.

A viable solution consists of appropriately managing input and output constraints in order to guarantee system optimization. In particular, system operating boundaries should be determined at first, within which a number of MPC objective functions should be minimized in a descending order of priority. Although such an approach is generally quite complex compared to conventional MPC, it leads to a better system exploitation. In addition, higher computational efforts can be easily handled by means of fast processing units, like Field Programmable Gate Arrays (FPGA); in fact, they allow very fast execution times, even for advanced MPC-based control systems, making them particularly suitable in replacing traditional control techniques in industrial applications.

This chapter addresses the problem of accurate management of input and output constraints for MPC. Thus, firstly referring to a generic system, problem statement and formulation are firstly introduced and briefly discussed. Subsequently, reference is made to a case study, namely a Surface-Mounted Permanent Magnet Synchronous Machine (SPM). In particular, its mathematical modelling is briefly introduced at first, as well as its operating constraints (voltage saturation, current limitations, etc.). Subsequently, the design of an MPC algorithm is reported, which is based on accurate management of SPM operating constraints in order to guarantee optimal SPM performances, over both steady-state and dynamic operation. Both simulation and experimental results are also enclosed in order to highlight the effectiveness of this MPC approach; in particular, simulations are carried out by means of Matlab-Simulink, whereas experiments refer to the employment of an appropriate FPGA-based control board.

Introduction

Considering a generic system, its continuous-time mathematical model can be expressed in terms of state variables as

$$\begin{aligned} \dot{x} &= f(x,u,t) \quad , \quad x(t_0) = x_0 \\ y &= g(x,u,t) \end{aligned} \tag{1}$$

where x and u denote state and input vectors respectively, y being the output vector. In particular, for electrical systems, x generally consists of inductor currents, magnetic flux linkages and/or capacitor voltages, whereas u accounts for voltage and current supplies mostly. Regarding the output vector y, it generally depends on both x and u and consists of those variables whose time-evolution should be imposed appropriately in order to optimize system performances. In this context, apart from (1), a number of input and output constraints have to also be taken into account, which can generally be expressed as:

$$\mathcal{C}_i(x,u,t) \le 0 \quad , \quad i = 1..n \,. \tag{2}$$

In particular, input constraints usually account for bounded values of input variables, such as maximum supply voltage and/or current. As a result, input constraints generally affect both dynamic and steady-state system performances, e.g. by increasing response times and preventing the achievement of some operating conditions at the same time. Whereas state and output constraints affect steady-state operations mostly because they account for hazardous conditions that cannot be held continuously. For example, considering an electrical machine, such constraints consist of high currents flowing into electrical machine windings or excessive speed values. The firsts may lead to overheating, which, in turn, may deteriorate winding isolation. Whereas the latter may introduce mechanical problems on shaft and bearings, thus leading to unsuitable vibrations and noise. However, since such operating conditions can be sustained for short periods of time, state and output constraints do not generally affect dynamic system operation significantly.

Thus, based on(1) and (2), or on their corresponding sampled-data versions, several control systems can be designed and implemented in order to achieve system performance optimization, depending on inherent features of the system to be controlled, as well as on specific application needs and requirements. In general, a reference output evolution is imposed (y^*), whose tracking has to be accomplished in accordance with different optimization criteria (fastest response, minimum losses, etc.). Consequently, different optimal control laws (u^*) can be achieved, also in accordance with the kind of control system employed (hysteresis, PI-based, predictive control, etc.). In this context, it is worth noting that hysteresis control systems provide good dynamic responses, being robust to parameter variations and uncertainties too.

Unfortunately, steady-state evolutions are generally characterized by strong ripple, which prevents an optimal tracking of y^*. Differently, PI-based control systems provide good steady-state performances and a fair insensitivity to system parameter variations and uncertainties, but they do not generally guarantee high dynamic performances. Unlike the above mentioned control techniques, predictive control can assure good performances over both dynamic and steady-state operation, but an accurate knowledge of system model and parameters is required. In fact, since predictive control algorithms are generally designed based on sampled-data models of the controlled system, they are badly affected by parameter variations and uncertainties, which may lead to un-optimized system performances. However, this drawback can be overcome by employing appropriate on-line parameter identification and/or adaptive control approaches.

Among the several kinds of predictive control proposed in the literature (hysteresis-based, trajectory-based, deadbeat, etc.), Model Predictive Control (MPC) surely represents the most promising one due to its inherent flexibility and versatility. In fact, referring to Fig. 1, conventional MPC consists in synthesizing the most suitable control law u^* in order to minimize an appropriate cost function, which can be expressed as

$$\Phi(x,u,t,y^*) = \sum_j \lambda_j \cdot \varphi_j(x,u,t,y^*) \quad , \qquad j = 1..m. \tag{3}$$

Particularly, MPC cost function generally consists of a sum of several objective functions, each of which (φ_j) should be minimized in accordance with its corresponding weight (λ_j). As a result, an accurate choice of λ_j enables an appropriate optimization of several non-linear systems, especially those characterized by several inputs and/or outputs. In this context, it is worth noting that different choices of λ_j can lead to different system optimization, making MPC very flexible. In addition, MPC more easily takes into account both input and output constraints compared to other predictive control approaches. In fact, they can be generally included into MPC cost function, leading to

$$\tilde{\Phi}(x,u,t,y^*) = \sum_j \lambda_j \cdot \varphi_j(x,u,t,y^*) + \sum_i \psi_i \cdot \mathcal{C}_i(x,u,t) \quad , \qquad \begin{cases} j = 1..m \\ i = 1..n \end{cases} \tag{4}$$

where each ψ_i denotes the weight of $\mathcal{C}_i$. As a result, the minimization of $\tilde{\Phi}$ instead of Φ should guarantee the achievement of an appropriate control law u^* that is able to track y^*optimally, by satisfying all input and output constraints at the same time.

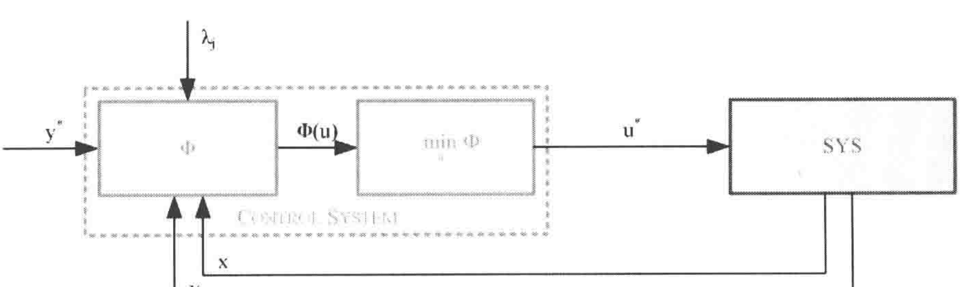

Figure 1. Schematic representation of a conventional Model Predictive Control system.

Although such an MPC approach is quite simple, effective and easy to be implemented, it also reveals some critical issues, which may prevent system performance optimization. In particular, the minimization of Φ defined by (3) always requires a trade-off among φ_j minimizations, each of which can be more or less prioritized by employing high or low λ_j value respectively. Consequently, the employment of $\tilde{\Phi}$ in place of Φ may lead to sub-optimal solutions; in fact, high ψ_i values makes $\tilde{\Phi}$ very sensitive to input and output constraints mostly, thus system performance optimization, which represents the main goal of MPC, may be unsuitably impaired. On the other hand, if ψ_i are chosen relatively low compared to λ_j, $\tilde{\Phi}$ almost equals Φ, thus input and output constraints become quite irrelevant. However, regardless of the choice of ψ_i, the minimization of $\tilde{\Phi}$ as a whole may not comply with all input and output constraints, some of which may be thus not satisfied over both dynamic and steady-state operations.

On the basis of the previous considerations, another MPC approach should be followed in order to assure an appropriate system performance optimization and full compliance with all input and output constraints, over both dynamic and steady-state operation. It consists of carrying out an accurate input and output constraint management at steady-state operation at first, leading to define the following steady-state subset X_0:

$$X_0 : \ \forall x \in X_0 \ \rightarrow \ \mathcal{C}_i(x,u,t) \le 0 \ , \qquad i = 1..n. \tag{5}$$

In particular, since only x within X_0satisfy all input and output constraints, the optimal steady-state solution (x^*) must lie within X_0 for any given y^*. As a result, system performance optimization could be achieved by minimizing Φ within X_0, since there is no need of introducing $\tilde{\Phi}$ further. In addition, assuming that φ_j can be ordered by decreasing priority, several steady-state subset can be introduced as

$$X_j \subseteq X_{j-1} : \ \forall x \in X_j \ \rightarrow \ \varphi_j(x,u,t) = \min_{x \in X_{j-1}} \{\varphi_j(x,u,t)\} \ , \ j = 1..m. \tag{6}$$

Thus, referring to Fig. 2, it can be stated that the optimal solution x^* can be found without resorting to conventional MPC cost functions, like that expressed by (3). Consequently, the introduction of weights, whose tuning is generally hard to be accomplished and which may lead to unsuitable operating conditions, is not required further. A similar approach can be followed over dynamic operation; in fact, once x^* is determined, its tracking can be accomplished referring to appropriate dynamic subsets ($\tilde{X}$). These last can be defined in accordance with (5) and (6), but based on dynamic operating constraints and objective functions, which may differ from the corresponding steady-state ones.

The design and implementation of this novel MPC approach is shown in the following by referring to a case study, i.e. a Surface-Mounted Permanent Magnet Synchronous Machine (SPM) fed by a three-phase inverter. In particular, SPM mathematical models are briefly introduced at first, as well as its dynamic and steady-state operating constraints. Subsequently, an MPC algorithm is designed based on the above-mentioned approach, whose effectiveness is appropriately highlighted by both simulation and experimental results.

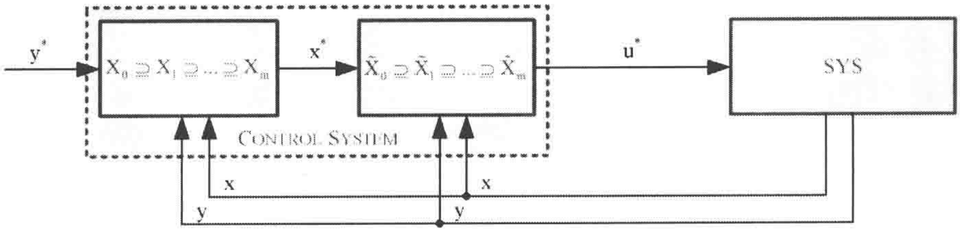

Figure 2. Schematic representation of a novel Model Predictive Control approach.

Surface-Mounted Permanent Magnet Synchronous Machine

Permanent Magnet Synchronous Machines (PMSMs) are nowadays widely employed in a variety of applications, such as electric vehicles, electric ship propulsion, robotics, aerospace and power generation. This is due to their high torque and power density, high efficiency and reliability, which are obtained by employing appropriate permanent magnet materials and machine topologies [1]. Among PMSMs, Surface-Mounted Permanent Magnet Synchronous Machines (SPMs) are very widespread because they are relatively simple to be manufactured and easy to be controlled in comparison with other kinds of PMSMs. However, full exploitation of SPM dynamic and steady-state performances also needs advanced control systems, which should manage SPM operating constraints appropriately, especially at high-speed.

In this context, several predictive control algorithms have been proposed in the literature for power electronic converters and electrical drives [2]-[23], some of which specifically designed for PMSM [12]-[23]. All these aims to enhance dynamic and steady-state performances compared to other control systems. It is worth noting that although the application of predictive control technique for such electrical systems was already suggested a long time ago [2]-[4], its high computational effort prevented it from being widely employed in the past decades. However, it has recently gained more and more interest due to the increasing development of fast processing units, like Field Programmable Gate Arrays (FPGA) [24]-[28]. In particular, FPGAs match the predictive control computational demands well, especially in terms of execution time, even allowing real-time implementation of advanced predictive control algorithms. In addition, FPGAs also enable the implementation of appropriate on-line parameter identification procedures in order to overcome predictive control issues due to parameter variations and uncertainties [29]. However, in spite of this, predictive control algorithms proposed in the literature are generally designed on simplified sampled-data models, as well as on approximated input and output constraints in order to ease algorithm implementation. This prevents the full exploitation of SPM performances, especially above its rated speed. In fact, in this last case, appropriate flux-weakening control strategies have to be employed, which should entail an appropriate management of input and output constraints, over both dynamic and steady-state operation [30]-[46].

On the basis of all the previous considerations, an MPC algorithm for an SPM is presented in the following. In particular, an accurate SPM sampled-data model is introduced at first, which allows the minimization of modelling errors due to the discretization procedure, resulting in enhanced MPC performances [47]. In addition, the MPC algorithm is designed in order to appropriately account for both input and output constraints (current limitation, voltage saturation, etc.), leading to fully exploit SPM performances, over both dynamic and steady-state operation[48]-[50]. The effectiveness of this novel MPC algorithm is validated by means of both simulations and experiments; in particular, simulations are carried out in the Matlab Simulink environment, whereas experiments are performed by means of an FPGA-based control board. These last also regard the comparison between the novel MPC algorithm and a conventional PI-based control system, i.e. the well-known voltage follower based on a PI voltage compensator (VF-PI)[43]-[46], in order to highlight the enhanced SPM performances achievable by the former.

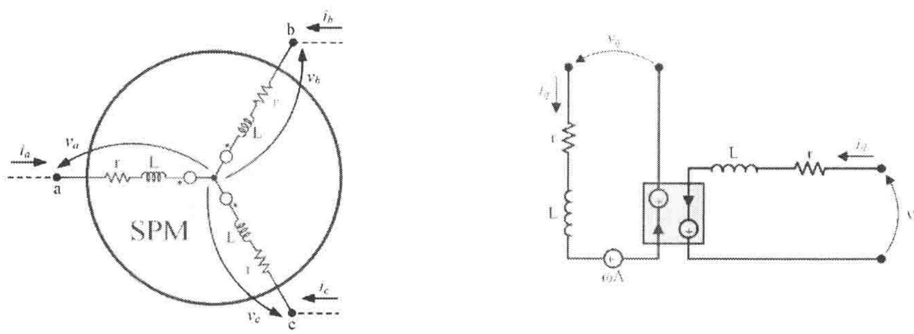

Figure 3. Schematic representation of a three-phase SPM: abc and dq equivalent circuits (on the left and on the right respectively).

Mathematical Modelling

Referring to the schematic representation of three-phase SPM depicted in Fig. 3, where *{a,b,c}* denote the phases of the stator winding, SPM electrical equations can be expressed as

$$v_n = r\,i_n + L\frac{di_n}{dt} + \frac{d\lambda_n}{dt} \quad , \qquad n \in \{a,b,c\} \tag{7}$$

in which r and L denote the phase resistance and the synchronous inductance, whereas v_n and i_n are phase voltages and currents respectively, λ_n being the magnetic flux linkages due to permanent magnets. It is worth noting that (7) is assumed in conditions of negligible magnetic anisotropy and saturation effects, as generally occurs for SPM. Thus, it is possible to define voltage and current space vectors based on their corresponding phase quantities as

$$x_{\alpha\beta} = \frac{2}{3}\left(x_a + x_b\, e^{j\frac{2}{3}\pi} + x_c\, e^{j\frac{4}{3}\pi} \right) \quad , \quad x \in \{v, i, \lambda\}\,. \tag{8}$$

Therefore, by substituting (7) in (8) and assuming each λ_n sine-shaped, the continuous-time electrical equation of SPM in the stationary αβ reference frame can be achieved as

$$v_{\alpha\beta} = r\,i_{\alpha\beta} + L\frac{di_{\alpha\beta}}{dt} + \frac{d\lambda_{\alpha\beta}}{dt} \quad , \qquad \lambda_{\alpha\beta} = \Lambda e^{j\theta} \tag{9}$$