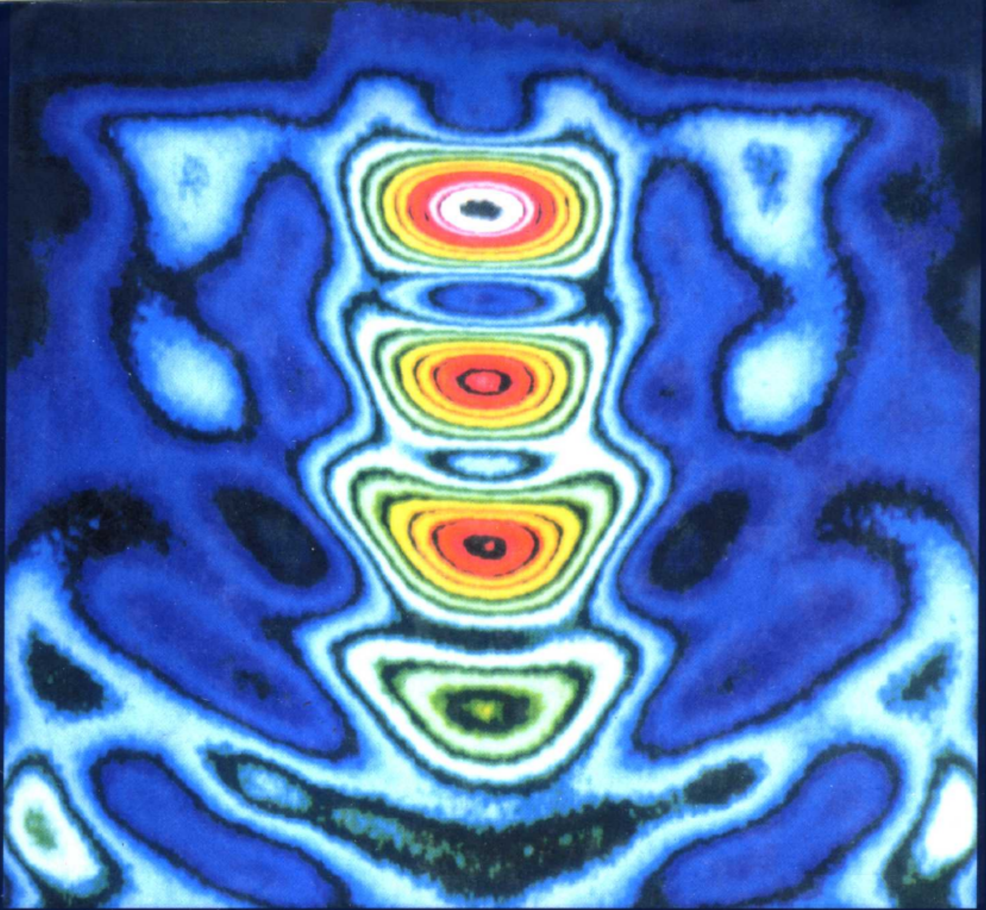

Quantum Chaos

an introduction

量子混沌导论

H-J Stöckmann

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QUANTUM CHAOS

An Introduction

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Preface

This monograph is based on the script of a lecture series on the quantum mechanics of classically chaotic systems given by the author at the University of Marburg during the summer term 1995. The lectures were attended by students with basic knowledge in quantum mechanics, including members of the author's own group working on microwave analogous experiments on quantum chaotic questions.

When preparing the lectures the author became aware that a comprehensive textbook, covering both the theoretical and the experimental aspects, was not available. The present monograph is intended to fill this gap.

The basic concepts of the quantum mechanics of classically chaotic systems, termed 'quantum chaos' for short, are easy to grasp by any student of physics. The mathematical apparatus needed, however, often tends to obscure the physical background. That is why the theoretical results will be illustrated by real experimental or numerical data whenever possible.

Chapter 1 gives a short introduction on the essential ideas of semiclassical quantum mechanics, which is illustrated by two examples taken from the microwave billiards and the kicked rotator.

Chapter 2 treats the different types of billiard experiments. Methods to study vibrating solids and liquids are presented. The main part of this chapter deals with microwave techniques, as by far the most experiments have been performed in microwave billiards. The chapter ends with a discussion of mesoscopic billiards including quantum corrals.

Chapter 3 introduces random matrix theory. It was developed in the sixties, and is treated in several reviews and monographs. Therefore only the fundamental concepts are discussed, such as level spacing distributions and spectral correlation functions. The last part of the chapter introduces supersymmetry techniques, which have become more and more important in recent years.

In Chapter 4 systems with periodic time dependences are discussed, with special emphasis on dynamical localization, i.e. the suppression of classical chaos by quantization. Since dynamical localization is a special case of Anderson localization, which is observed for electrons in disordered lattices, the chapter includes a short discussion of the latter systems.

Chapter 5 deals with the analogy between the dynamics of the eigenvalues of a chaotic system as a function of an external parameter and the dynamics of a one-dimensional gas, a model introduced by Pechukas and Yukawa. When varying the parameter, the phases of the wave functions change as well. These so-called geometrical phases, also called 'Berry's phases', are treated in the last section of this chapter.

In Chapter 6 scattering theory is introduced to describe the influence of coupled antennas on the spectrum, with special emphasis on microwave billiards. For depths and widths of the resonances Porter–Thomas distributions are found, these have been well-known in nuclear physics for many years. The chapter ends with a discussion of the fluctuations of scattering matrix elements, known as Ericson fluctuations in nuclear physics, and as universal conductance fluctuations in mesoscopic systems.

In Chapter 7 semiclassical quantum mechanics is developed. Starting with the Feynman path integral for the quantum mechanical propagator, stationary phase approximations are applied to obtain semiclassical expressions for the propagator and the Green function. The main result of the chapter is the Gutzwiller trace formula expressing the quantum mechanical spectrum in terms of the classical periodic orbits.

In Chapter 8 several applications of periodic orbit theory are presented. It starts with a discussion of Fourier transform techniques to extract the contributions of periodic orbits from the spectra. The semiclassical theory of spectral rigidity establishes a link between periodic orbit and random matrix theory. Subsequently resummation schemes are developed allowing the calculation of quantum mechanical spectra from the periodic orbits under favourable conditions. The chapter ends with a discussion of billiards on a metric with constant negative curvature, and of the Selberg trace formula, the non-Euclidean equivalent of the Gutzwiller trace formula.

From the very beginning, when starting the microwave experiments in the late eighties, I have been supported and encouraged by many colleagues from theory. Prof. S. Großmann, Marburg, roused my interest in nonlinear dynamics. In the following years Prof. B. Eckhardt, Marburg, and Prof. F. Haake, Essen, have been my main interlocutors in theoretical problems. Moreover, I want to thank all my coworkers, especially J. Stein, with whom I started the experiments, and U. Kuhl, my senior coworker. The experiments have been supported by the Sonderforschungsbereich 'Nichtlineare Dynamik' of the Deutsche Forschungsgemeinschaft, both financially and scientifically. Here I want to mention in particular the groups of Prof. T. Geisel, Göttingen, and Prof. A. Richter, Darmstadt. The Fachbereich Physik at the University of Marburg has provided me with the local support necessary to perform the experiments.

I further want to thank the authors and the publishers who gave permission to reproduce figures from their publications. In addition most of the authors have provided me with good copies of the figures. Some of the figures have been prepared by U. Kuhl. My coworker M. Barth has cared for the regular updating of our internal quantum chaos bibliography used for this monograph. Finally I am indebted to my wife E.-B. Stöckmann for her critical reading of the whole manuscript and stylistic corrections.

Hans-Jürgen Stöckmann

Contents

1
Introduction

From the very beginning classical nonlinear dynamics has enjoyed much popularity even among the noneducated public as is documented by numerous articles in well-renowned magazines, including nonscientific papers. For its nonclassical counterpart, the quantum mechanics of chaotic systems, termed in short 'quantum chaos', the situation is completely different. It has always been considered as a more or less mysterious topic, reserved to a small exclusive circle of theoreticians. Whereas the applicability of classical nonlinear dynamics to daily life is comprehensible for a complete outsider, quantum chaos, on the other hand, seems to be of no practical relevance at all. Moreover, in classical nonlinear dynamics the theory is supported by numerous experiments, mainly in hydrodynamics and laser physics, whereas quantum chaos at first sight seems to be the exclusive domain of theoreticians. In the beginning the only experimental contributions came from nuclear physics [Por65]. This preponderance of theory seems to have suppressed any experimental effort for nearly two decades. The situation gradually changed in the middle of the eighties, since when numerous experiments have been performed. An introductory presentation also suited to the experimentalist with no or only little basic knowledge is still missing.

It is the intention of this monograph to demonstrate that there is no reason to be afraid of quantum chaos. The underlying ideas are very simple. It is essentially the mathematical apparatus that makes things difficult and often tends to obscure the physical background. Therefore the philosophy adopted in this presentation is to illustrate theory by experimental results whenever possible, which leads to a strong accentuation of billiard systems for which a large number of experiments now exist. Consequently, results on microwave billiards obtained by the author's own group will be frequently represented. This should not be misunderstood as an unappropriate preference given to their own work. The billiard, though being conceptually simple, nevertheless exhibits the full complexity of nonlinear dynamics, including its quantum mechanical aspects. Probably there is no essential aspect of quantum chaos which cannot be found in chaotic billiards.

The nonexpert for whom this book is mainly written may ask whether quantum chaos is really an interesting topic in its own right. After all, quantum mechanics has now existed for more than 60 years and has probably become the best tested physical theory ever conceived. Quantum mechanics can handle not only the hydrogen atom which is classically integrable but also the classically nonintegrable helium atom. We may even ask whether there is anything like quantum chaos at all. The Schrödinger equation is a linear equation leaving no room for chaos. The correspondence principle, on the other hand, demands that in the semiclassical region, i.e. at length scales large compared to the de Broglie wavelength, quantum mechanics continuously develops into classical mechanics.

That is why there has even been a debate whether the term 'quantum chaos' should be used at all. In 1989 the leading scientists in the field came together to discuss these questions at a summer school in Les Houches [Gia89]. The proceedings are titled 'chaos and quantum physics' thus avoiding the dubious term. Berry [Ber89] once again proposed the term 'quantum chaology', introduced by him previously [Ber87]. This would obviously have been a much better choice than 'quantum chaos', but was not generally accepted. In the following years the debate ceased. Today the term 'quantum chaos' is generally understood to comprise all problems concerning the quantum mechanical behaviour of classically chaotic systems. This view will also be adopted in this book. For billiard experiments another aspect has to be considered. Most of them are analogue experiments using the equivalence of the Helmholtz equation with the stationary Schrödinger equation. That is why the term 'wave chaos' is sometimes preferred in this context. Most of the phenomena discussed in this book indeed apply to all waves and are not primarily of quantum mechanical origin.

The problems with the proper definition of the term 'quantum chaos' have their origin in the concept of the trajectory, which completely loses its significance in quantum mechanics. Only in the semiclassical region do the trajectories eventually reappear, an aspect of immense significance in the context of semiclassical theories. For purposes of illustration, let us consider the evolution of a classical system with N dynamical variables $x_1, \ldots, x_N$ under the influence of an interaction. Typically the x_n comprise all components of the positions and the momenta of the particles. Consequently the number of dynamical variables is $N = 6M$ for a three-dimensional M particle system.

Let $\mathbf{x}(0) = [x_1(0), \ldots, x_N(0)]$ be the vector of the dynamical variables at the time $t = 0$. At any later time t we may write $\boldsymbol{x}(t)$ as a function of the initial conditions and the time as

$$\boldsymbol{x}(t) = \boldsymbol{F}[\boldsymbol{x}(0), t]. \tag{1.1}$$

If the initial conditions are infinitesimally changed to

$$\boldsymbol{x}_1(0) = \boldsymbol{x}(0) + \boldsymbol{\xi}(0), \tag{1.2}$$

then at a later time t the dynamical variables develop according to

$$\boldsymbol{x}_1(t) = \boldsymbol{F}[\boldsymbol{x}(0) + \boldsymbol{\xi}(0), t]. \tag{1.3}$$

The distance $\boldsymbol{\xi}(t) = \boldsymbol{x}_n(t) - \boldsymbol{x}(t)$ between the two trajectories is obtained from Eqs. (1.1) and (1.3) in linear approximation as

$$\boldsymbol{\xi}(t) = (\boldsymbol{\xi}(0)\boldsymbol{\nabla})\boldsymbol{F}[\boldsymbol{x}(0), t], \tag{1.4}$$

where $\boldsymbol{\nabla}$ is the gradient of $\boldsymbol{F}$ with respect to the initial values. Written in components Eq. (1.4) reads

$$\xi_n(t) = \sum_m \frac{\partial F_n}{\partial x_m} \xi_m(0). \tag{1.5}$$

The eigenvalues of the matrix $M = (\partial F_n/\partial x_m)$ determine the stability properties of the trajectory. If the moduli of all eigenvalues are smaller than one, the trajectory is stable, and all deviations from the initial trajectory will rapidly approach zero. If the modulus of at least one eigenvalue is larger than one, both trajectories will exponentially depart from each other even for infinitesimally small initial deviations $\boldsymbol{\xi}(0)$. Details can be found in every textbook on nonlinear dynamics (see Refs. [Sch84, Ott93]).

In quantum mechanics this definition of chaos becomes obsolete, since the uncertainty relation

$$\Delta x \Delta p \geqslant \tfrac{1}{2}\hbar \tag{1.6}$$

prevents a precise determination of the initial conditions. This can best be illustrated for the propagation of a point-like particle in a box with infinitely high walls. For obvious reasons these systems are called billiards. They will accompany us throughout this book. For the quantum mechanical treatment two steps are necessary. First the Schrödinger equation

$$-\frac{\hbar^2}{2m}\Delta\psi = \imath\hbar\frac{\partial\psi}{\partial t} \tag{1.7}$$

has to be solved with the Dirichlet boundary condition

$$\psi|_S = 0, \tag{1.8}$$

where S denotes the walls of the box. Stationary solutions of the Schrödinger equation are obtained by separating the time dependence,

$$\psi_n(x, t) = \psi_n(x)e^{\imath\omega_n t}. \tag{1.9}$$

Insertion into Eq. (1.7) yields

$$(\Delta + k_n^2)\psi_n(x) = 0 \tag{1.10}$$

where ω_n and k_n are connected via the dispersion relation

$$\omega_n = \frac{\hbar}{2m} k_n^2. \tag{1.11}$$

Equation (1.10) is also obtained if we start with the wave equation

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\psi = 0, \tag{1.12}$$

where c is the wave velocity, and if we separate again the time dependence by means of the ansatz (1.9). In contrast to the quadratic dispersion relation (1.11) for the quantum mechanical case we now have the linear relation

$$\omega_n = ck_n \tag{1.13}$$

between ω_n and k_n. It is exactly this correspondence between the stationary Schrödinger equation and the stationary wave equation, also called the Helmholtz equation, which has been used in many billiard experiments to study quantum chaotic problems using wave analogue systems (see Chapter 2).

As soon as the stationary solutions of the Schrödinger equation are known, a wave packet can be constructed by a superposition of eigenfunctions,

$$\psi(x, t) = \sum_n a_n \psi_n(x) e^{-i\omega_n t}. \tag{1.14}$$

For a Gaussian shaped packet centred at a wave number $\bar{k}$ and of width Δk the coefficients a_n are given by

$$a_n = a \exp\left[-\frac{1}{2}\left(\frac{k_n - \bar{k}}{\Delta k}\right)^2\right], \tag{1.15}$$

where a is chosen in such a way that the total probability of finding the particle in the packet is normalized to one. If the a_n are known at time $t = 0$, e.g. by a measurement of the momentum with an uncertainty of $\Delta p = \hbar\Delta k$, the quantum mechanical evolution of the packet can be calculated for any later time with arbitrary precision. Moreover, to construct wave packets with a given width, the sum in Eq. (1.14) can be restricted to a finite number of terms. Apart from untypical exceptions, the resulting function is not periodic, since in general the ω_n are not commensurable, but *quasi-periodic*. Thus the wave packet will always reconstruct itself, possibly after a long period of time. The exponential departure of neighbouring trajectories known from classical nonlinear dynamics has completely disappeared.

The wave properties of matter do not provoke an additional spreading of the probability density as we might intuitively think. On the contrary, in systems where the classical probability density continuously diffuses with time, e.g. by a random walk process, quantum mechanics tends to freeze the diffusion and to localize the wave packet [Cas79]. This has been established in numerous calculations and has even been demonstrated experimentally [Gal88, Bay89, Moo94]. The phenomenon of quantum mechanical localization will be discussed in detail in Chapter 4.

To demonstrate how the wave packet just constructed evolves with time, we now take the simplest of all possible billiards, a particle in a one-dimensional box with infinitely high walls. Taking the walls at the positions $x = 0$ and $x = l$, the eigenfunctions of the system are given by

$$\psi_n(x) = \sqrt{\frac{2}{l}} \sin k_n x, \qquad n = 1, 2, 3, \ldots \tag{1.16}$$

with the wave numbers

$$k_n = \frac{\pi n}{l}. \tag{1.17}$$

Insertion into Eq. (1.14) yields

$$\psi(x, t) = a\sqrt{\frac{2}{l}} \sum_{n=1}^{\infty} \exp\left[-\frac{1}{2}\left(\frac{k_n - \overline{k}}{\Delta k}\right)^2\right] \sin k_n x \, e^{-i\omega_n t}. \tag{1.18}$$

This equation holds for the propagation of both particle packets and ordinary waves, provided that the respective dispersion relations (1.11) or (1.13) are obeyed. The calculation is somewhat easier for ordinary waves. Therefore this situation will now be considered by putting $\omega_n = ck_n$. For particle waves the calculation follows exactly the same scheme. To simplify the calculation it will be further assumed that the average momentum is large compared to the width of the distribution, i.e. $\overline{k} \gg \Delta k$. Then the sum can be extended from $-\infty$ to $+\infty$, and we can apply the *Poisson sum relation*

$$\sum_{n=-\infty}^{\infty} f(n) = \sum_{n=-\infty}^{\infty} g(n), \tag{1.19}$$

where

$$g(n) = \int_{-\infty}^{\infty} f(n) e^{2\pi i nm} \, dn \tag{1.20}$$

is the Fourier transform of $f(n)$. Application to Eq. (1.18) yields

$$\psi(x, t) = a\sqrt{\frac{2}{l}} \sum_{m=-\infty}^{\infty} \frac{l}{\pi} \int_{-\infty}^{\infty} \exp\left[-\frac{1}{2}\left(\frac{k - \overline{k}}{\Delta k}\right)^2\right] \sin kx \, e^{i(2lm - ct)k} \, dk, \tag{1.21}$$

where the integration variable n has been substituted by $k = n\pi/l$. The integration is easily carried out using the well-known relation

$$\int_{-\infty}^{\infty} \exp[-(ak^2 + 2bk + c)]\, dk = \sqrt{\frac{\pi}{a}} \exp\left(\frac{b^2}{a} - c\right) \tag{1.22}$$

for Gaussian integrals which also holds for complex values of a, b, c provided that $\mathrm{Re}(a) > 0$. The result is

$$\psi(x, t) = \sum_{m=-\infty}^{\infty} [\phi(x - ct_m) - \phi(l - x - ct_{m+1})], \tag{1.23}$$

where

$$t_m = t - m\frac{2l}{c}, \tag{1.24}$$

and

$$\phi(x) = 2a\sqrt{\frac{l}{\pi}}\Delta k \exp\left[\imath\bar{k}x - \frac{1}{2}(x\Delta k)^2\right]. \tag{1.25}$$

Equation (1.23) allows a straightforward interpretation. It describes the propagation of a Gaussian pulse with width $\Delta x = 1/\Delta k$ and velocity c, passing to and fro between the two walls and changing sign upon every reflection. For the propagation of particle waves the situation is qualitatively similar, but now the quadratic dispersion relation (1.11) leads to a spreading of the pulse with time and a pulse width $\Delta x(t)$ given by

$$\Delta x(t) = \frac{1}{\Delta k}\left[1 + \left(\frac{\hbar(\Delta k)^2 t}{m}\right)^2\right]^{1/4}. \tag{1.26}$$

For time $t = 0$ we obtain $\Delta x \Delta k = 1$ as for ordinary waves. This is just the quantum mechanical uncertainty relation.

By means of the Poisson sum relation two different expressions for $\psi(x, t)$ have been obtained. First, in Eq. (1.18), it is expressed in terms of a sum over the eigenfunctions of the systems, second, in Eq. (1.23), it is written as a pulse propagating with the velocity c being periodically reflected at the walls. This reciprocity, with the quantum mechanical spectrum on the one side and the classical trajectories on the other, will become one of the main ingredients of the semiclassical theory, in particular of the Gutzwiller trace formula. In the special example presented here the applied procedure worked especially well, since the set $\{k_n\}$ of eigenvalues was equidistant, leading to a perfect pulse reconstruction after every reflection. In the general case the pulse will be destroyed after a small number of reflections, but pulse reconstructions are still possible. The correspondence between classical and quantum mechanics will be demonstrated by two examples.

Figure 1.1 shows the propagation of a microwave pulse in a cavity in the shape of a quarter stadium [Ste95]. The measuring technique will be described in detail in Section 2.2.1. A circular wave is emitted from an antenna, propagates through the billiard, and is eventually reflected by the walls, thereby undergoing a change of sign (this can be seen especially well in Fig. 1.1(d) for the reflection of the pulse at the top and the bottom walls). After a number of additional reflections the pulse amplitude is distributed more or less equally

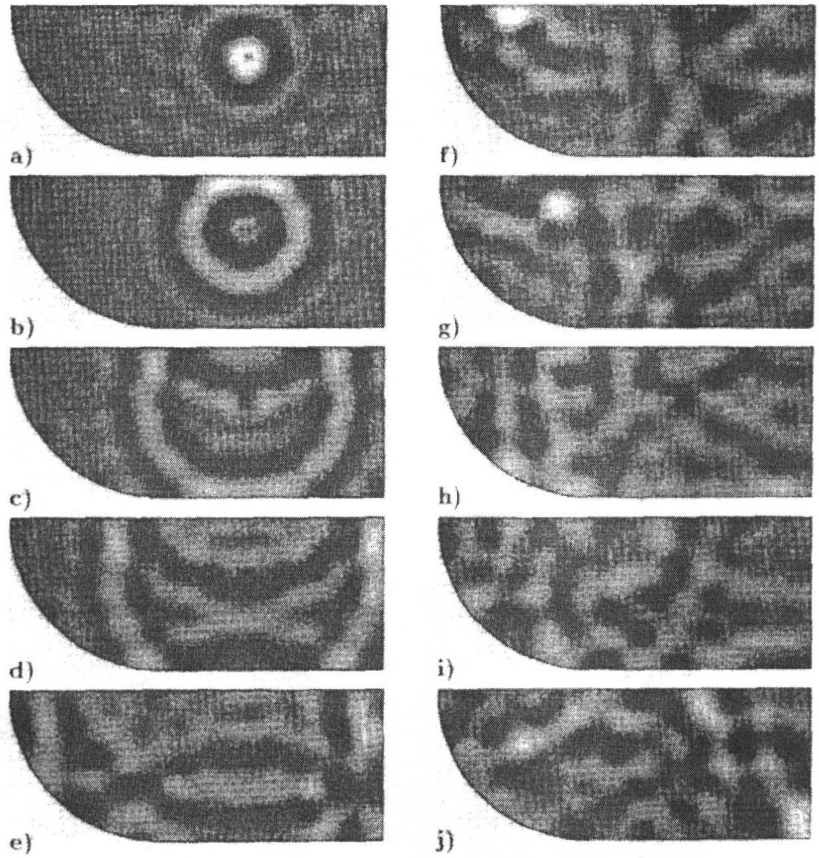

Figure 1.1. Propagation of a microwave pulse in a microwave cavity in the shape of a quarter stadium (length of the straight part $l = 18$ cm, radius $r = 13.5$ cm, height $h = 0.8$ cm) for different times $t/10^{-10}$ s: 0.36 (a), 1.60 (b), 2.90 (c), 3.80 (d), 5.63 (e), 9.01 (f), 10.21 (g), 12.0 (h), 14.18 (i), 19.09 (j) [Ste95] (Copyright 1995 by the American Physical Society).

over the billiard. But after some time the pulse suddenly reappears (see Fig. 1.1(f)). This is even more evident in Fig. 1.2 where the pulses are shown in a three-dimensional representation for two snapshots corresponding to Figs. 1.1(a) and (f). This reconstruction has nothing to do with the quantum

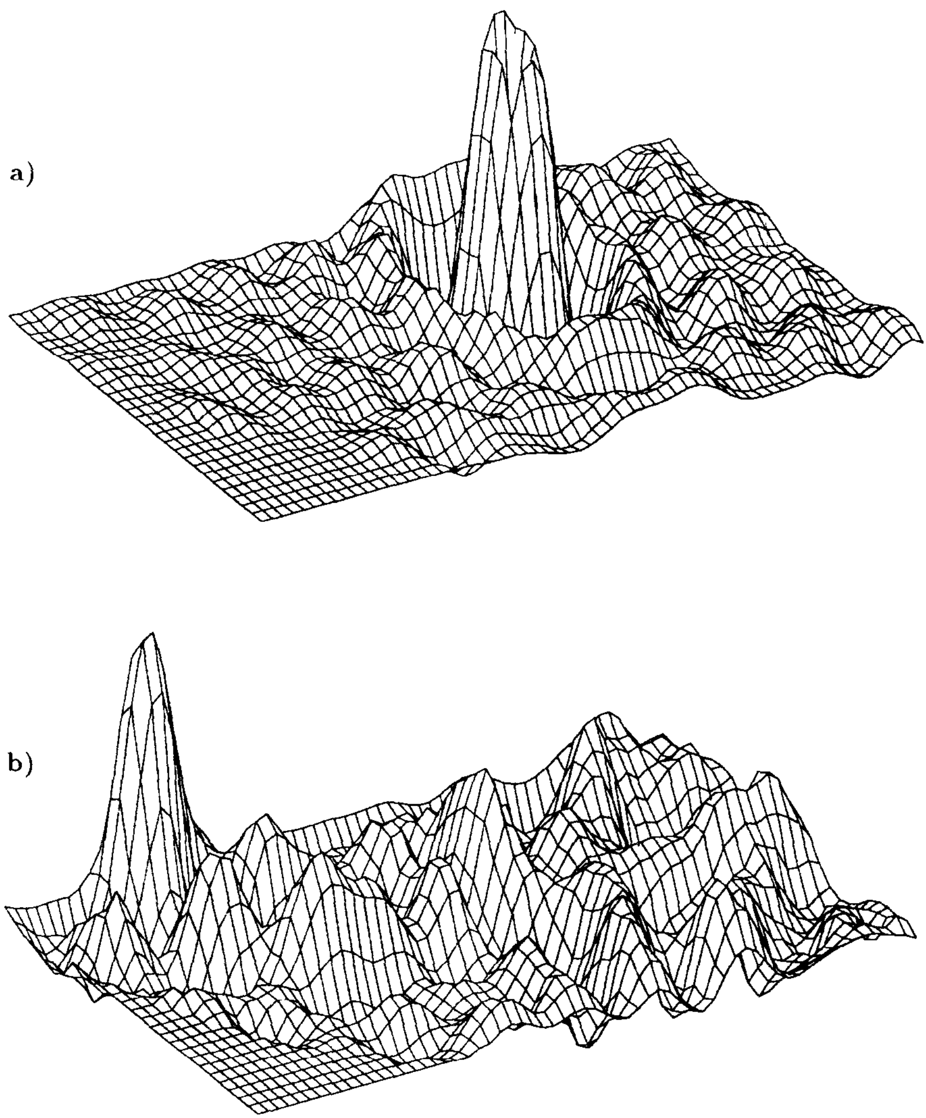

Figure 1.2. Three-dimensional view of the pulse propagation shown in Fig. 1.1 for two times, corresponding to Figs. 1.1(a) and (f), respectively [Ste95] (Copyright 1995 by the American Physical Society).