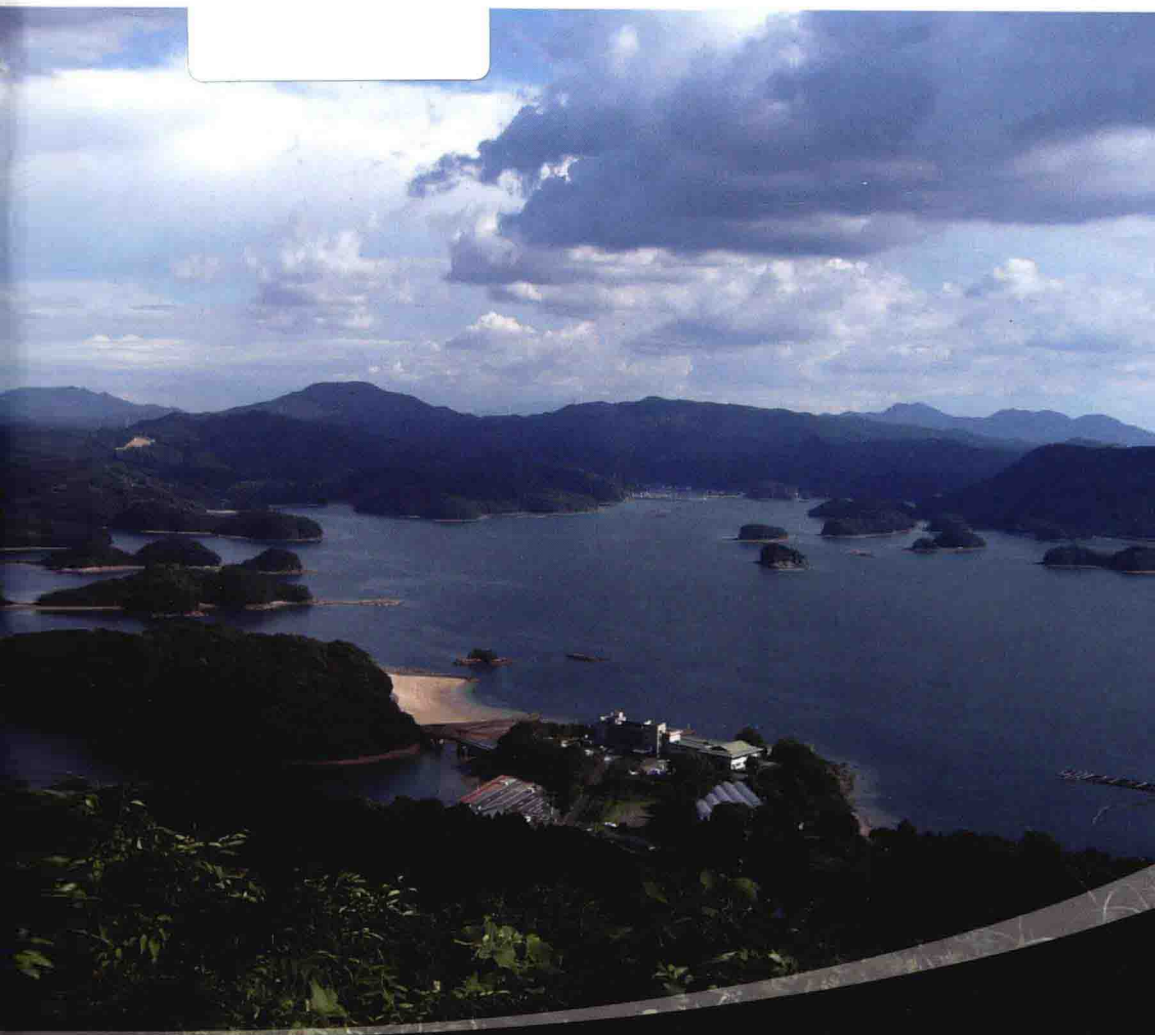


Differential Geometry of Submanifolds and its Related Topics

Editors

Sadahiro Maeda
Yoshihiro Ohnita
Qing-Ming Cheng



Differential Geometry of Submanifolds and its Related Topics

Proceedings of the International Workshop
in Honor of S. Maeda's 60th Birthday

Saga University, Saga, Japan, 4 – 6 August 2012



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Sadahiro Maeda (Saga University, Japan)

Yoshihiro Ohnita (Osaka City University, Japan)

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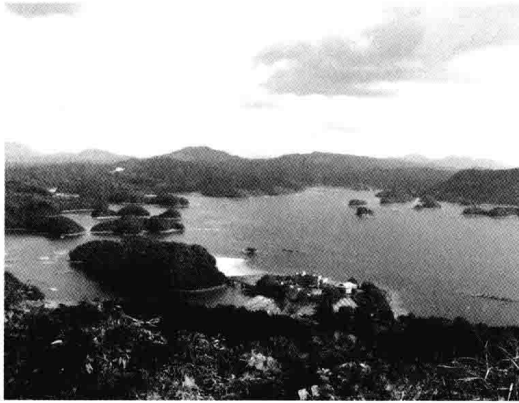
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FOREWORD

Professor Sadahiro MAEDA
and his friends in Mathematics

It is a great honor for me to have this opportunity to write this biographical profile on Professor Dr. Sadahiro MAEDA. On behalf of his friends, students and colleagues, I will give you a little history of this man and his achievements and describe his particular brand of mathematics. He completed his undergraduate studies in mathematics at Saga University in March, 1975 and went on to obtain his MA from Kumamoto University in March, 1978. During his master's studies, he met Professor Koichi Ogiue of Tokyo Metropolitan University, who came to Kumamoto University to give a series of lectures on differential geometry of submanifolds in complex space form. This was the key turning point of Sadahiro MAEDA's career. He immediately decided to apply for admission to enter the doctoral course at Tokyo Metropolitan University under Professor Ogiue. He was able to pass the entrance examination successfully, and began to study the geometry of submanifolds there. Among the many topics he was interested in at the time, he was particularly fascinated with real hypersurfaces in complex projective spaces and extrinsic appearances of geodesics of the submanifolds. In 1989, he published two joint papers. The first, entitled "On real hypersurfaces of a complex projective space", *Math. Z.* 202 (1989), 299-311, written with Professor Makoto Kimura. The second, entitled "Helical geodesic immersions into complex space forms", *Geom. Dedicata* 30 (1989), 93-114, written with Professor Yoshihiro Ohnita. MAEDA successfully defended his thesis to obtain his doctoral degree of mathematics from Tokyo Metropolitan University in May, 1989. He was offered his first position as a lecturer at Kumamoto Institute of Technology in April, 1985 and collaborated with several colleagues in mathematics, Professors Kazumi Tsukada, Yoshihiro Ohnita, Makoto Kimura, Toshikai Adachi and myself on a variety of projects. This marked the starting point of MAEDA's greatest achievements. It was then that he began studying extrinsic shapes of submanifolds

from a geometrical perspective, and he subsequently conducted several intriguing and innovative geometrical studies. His papers are a plethora of geometrical ideas coming from his geometrically refined sense of taste. With these achievements in hand, he was offered a position as an associate professor in 1991 at Nagoya Institute of Technology where Professor T. Adachi was working. The paper “Circles in a complex projective space”, *Osaka J. Math.* 32 (1995), 709-719, coauthored with Toshiaki Adachi and I, marked the beginning of a fruitful collaboration with Toshiaki Adachi. In 1995, MAEDA was awarded a professorship of mathematics at Shimane University. After some time, Makoto Kimura joined him there as a colleague in the mathematics department. By then, Professor Sadahiro MAEDA had become a mature geometer. In 2002, he wrote a paper with Professors T. Adachi and M. Yamagishi entitled “Length spectrum of geodesic spheres in a non-flat complex space form”, *J. Math. Soc. Japan* 54 (2002). In that study, he discovered that the estimation on the number of the congruency classes of geodesics in geodesic spheres was determined by the length of geodesics. He has also collaborated with Professor K. Ogiue, Professor Ryoichi Takagi, Professor Bang-Yen Chen and others on many more studies. MAEDA has been a professor of Mathematics at Saga University since April, 2007. No less than 120 academic papers have been published since his graduation from Saga University, and Professor Sadahiro MAEDA is now recognized as a world-famous geometer comparable to his respected teacher Professor Koichi Ogiue. Not only known as a researcher but also as an accomplished educator MAEDA has sent several fine geometers into mathematics world. Nobutaka Boumuki, Hiromasa Tanabe and Kazuhiro Okumura were supervised by Professor Sadahiro MAEDA when they were predoctoral students.

This volume was planned to be published on the occasion of his 60th birthday.

In closing, Professor Sadahiro MAEDA expresses his heartfelt appreciation to all of his teachers, friends, students, the contributors of the present volume and especially to his wife, his son and two daughters for their kind support over the years.

Seiichi UDAGAWA

PREFACE

The workshop on Differential Geometry of Submanifolds and its Related Topics was held at Saga University, Saga City, Japan, during the period of 4-6 August, 2012 in honor of Professor Sadahiro Maeda (Saga University) for his 60th birthday, which was supported by the warm hospitality of Professor Q. M. Cheng (Fukuoka University).

This international workshop provided opportunities for geometers in Japan, China and Korea to exchange ideas and discuss geometry quite freely and in a homely atmosphere. This volume is concerned with minimal surfaces, real hypersurfaces of a nonflat complex space form, submanifolds of symmetric spaces and curve theory. Each participant shows his new results or gives a brief survey in these areas.

The publication of this volume is financially supported by the official fund of the Japanese government, which is entitled “Research on Submanifold Geometry and Harmonic Map Theory in Symmetric Spaces” with JSPS Grant-in-Aid for Scientific Research (C) No. 24540090, (2012-2014) whose principal investigator is Professor Y. Ohnita (Osaka City University and OCAMI). The purpose of this academic program is to study the submanifold geometry and harmonic map theory in symmetric spaces from viewpoints of the geometric variational problems, integrable systems and Lie theory of finite and infinite dimensions. This volume is published to realize such a purpose. We gratefully acknowledge Professor T. Hamada (Fukuoka University) for his devotion as the TeX Editor of our volume. The organizers would like to dedicate the present volume to Professor Sadahiro Maeda.

The Organizing Committee
May 5, 2013

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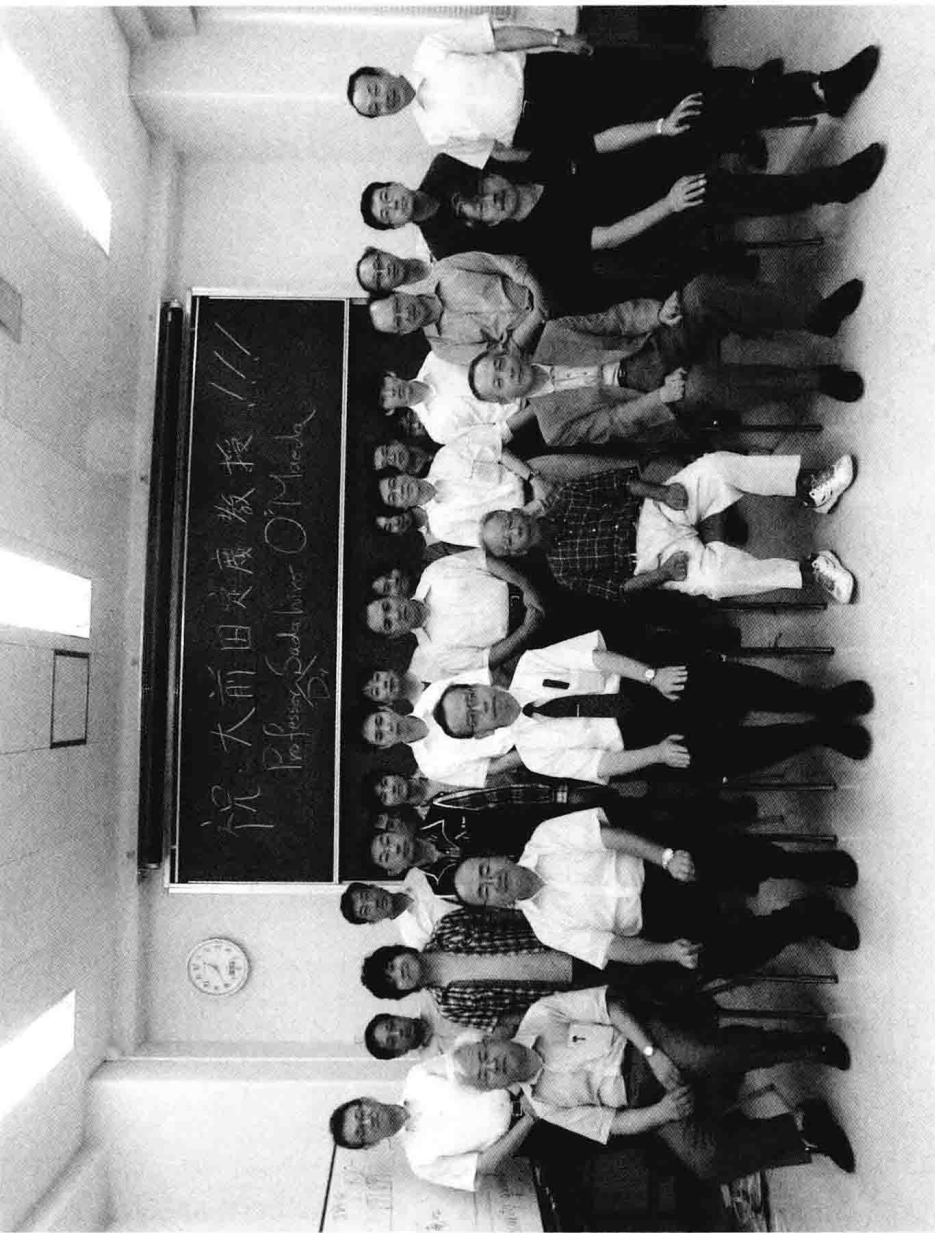
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3. **Lingzhong Zeng** (Saga University),
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12. **Kazuhiro Okumura** (Asahikawa National College of Technology),
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