Numerical and Asymptotic Techniques in Electromagnetics

Edited by R. Mittra

With Contributions by
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With 112 Figures

Springer-Verlag Berlin Heidelberg New York 1975

Professor Raj Mittra

University of Illinois at Urbana-Champaign, College of Engineering
Department of Electrical Engineering, Electromagnetics Laboratory
Urbana, IL 61801, USA

ISBN 3-540-07072-9 Springer-Verlag Berlin Heidelberg New York
ISBN 0-387-07072-9 Springer-Verlag New York Heidelberg Berlin

Library of Congress Cataloging in Publication Data. Mittra, Raj. Numerical and asymptotic techniques in electromagnetics. (Topics in applied physics; v. 3.) Includes bibliographical references and index. 1. Antennas (Electronics). 2. Electromagnetic waves. 3. Electric engineering – Mathematics. I. Deadrick, F. J. II. Title. TK7871.6.M59. 621.38'0283. 74-30146.

Monophoto typesetting, offset printing and bookbinding: Brühlsche Universitätsdruckerei, Gießen

Preface

During the past two decades a large number of books have been written on the general subject of electromagnetics. Most of these publications have dealt with the classical approaches to solving electromagnetic boundary value problems. There are only a few notable exceptions to these, for instance, a monograph by HARRINGTON on the application of the moment method for the solution of field problems and a text by MITTRA on computer techniques for solving electromagnetic scattering and radiation problems. Since the appearance of these two books much progress has been made in the areas of computerated electromagnetics and the application of ray-optical techniques in the low-frequency and high-frequency regions, respectively. This book attempts to present a comprehensive description of some of these important recent developments and to illustrate the application of these techniques to a variety of problems of practical interest, e.g., design of dipole arrays and reflector antennas. Almost all of the material appearing in the book is relatively new and has not appeared elsewhere in any other publication except in the form of journal articles. It is hoped that the book will serve to fill the gap that exists in the current literature on numerical and asymptotic techniques in electromagnetics and will be found useful both as a convenient reference and as a practical tool for investigating electromagnetic radiation scattering problems. All of the contributing authors are well known throughout the world for their many contributions on moment methods, numerical aspects of computerated solution, ray-optical techniques and other topics covered in the book and they have made every attempt to present the material in the text in a coherent and easily readable form. It is their fervent hope that the reader will not only find the book useful as a tool for modern electromagnetic analysis and design, but will also enjoy the clarity of presentation of the advanced material discussed in the book.

Finally, the editor (R. M.) would like to thank all of the authors for their excellent cooperation and the publishers for their extremely efficient production schedule.

Urbana, Illinois, January 1975

R. MITTRA

Contributors

DEADRICK, FREDERICK J.

Lawrence Livermore Laboratory, P.O. Box 808, Livermore, CA 94550, USA

HARRINGTON, ROGER F.

Syracuse University, Electrical Engineering Department, Syracuse, NY 13210, USA

IMBRIALE, WILLIAM A.

TRW Systems Group, One Space Park, Redondo Beach, CA 90278, USA

KLEIN, CHARLES A.

University of Illinois at Urbana-Champaign, College of Engineering, Department of Electrical Engineering, Electromagnetics Laboratory, Urbana, IL 61801, USA

KOUYOUMJIAN, ROBERT G.

The Ohio State University, Department of Electrical Engineering, 2015 Neil Avenue, Columbus, OH 43210, USA

MILLER, EDMUND K.

Lawrence Livermore Laboratory, P.O. Box 808, Livermore, CA 94550, USA

MITTRA, RAJ

University of Illinois at Urbana-Champaign, College of Engineering, Department of Electrical Engineering, Electromagnetics Laboratory, Urbana, IL 61801, USA

RUSCH, WILLARD V. T.

University of Southern California, Department of Electrical Engineering, Los Angeles, CA 90007, USA

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1. Introduction

R. Mittra

In the recent past, much of the effort in electromagnetics research has been directed toward two broad areas, viz., numerical and asymptotic techniques for solving boundary value problems arising in antennas, scattering, propagation, an so on. Not surprisingly, both of these areas are relatively new in the sense that the significant advances in these areas have been made only within the last fifteen years or less. The advent of the computer triggered a prolific surge of interest in the numerical solution of many practical electromagnetic analysis and synthesis problems that were previously regarded as too complex to be analytically tractable using the available theoretical methods. Ever since the publication of the now classic paper "Matrix Methods for Field Problems" by Harrington [1.1] there has been an almost exponential growth of the number of publications in the area of computerated solution of electromagnetic problems. The genesis of modern asymptotic electromagnetics is the pioneering work by Keller [1.2] on the Geometrical Theory of Diffraction, or GTD in abbreviated form. Though the growth of research effort on GTD has not been nearly as dramatic as in numerical electromagnetics, the advances have been highly significant nonetheless.

As with any field which has seen a rapid expansion in a relatively short period of time, the results of recent research efforts in electromagnetics can only be located in highly specialized journal publications which may not be easily accessible to the interested user in the form of a convenient collection. In addition, presentation in these papers may be too abstruse for an average reader to receive any benefit. The only exceptions are two relatively recent publications on numerical electromagnetics, viz. a monograph entitled "Field Computation by Moment Methods" by Harrington [1.3] published in 1968 and a text edited by Mittra [1.4] which bears the title "Computer Techniques for Electromagnetics". No such comparable text or monograph currently exists on the asymptotic techniques for electromagnetics, or, more specifically on the GTD methods.

The present book represents an effort to at least partially fill in this gap, by attempting to present the basic tools of numerical and asymptotic techniques available to the user in a coherent and convenient form. Since the basic principles and applications of the moment method may be found in [1.3], and a host of applications of this technique to the problems of wire antennas, and general three-dimensional scattering from solid surfaces has been adequately covered in [1.4], the topics of discussion in the present text have been carefully selected to avoid duplication of the material already published. Instead, this book deals with the important topic of the application of the moment method to antenna arrays in Chapter 2, and the problem of numerically deriving the characteristic modes of antennas and scatterers in Chapter 3. The knowledge of the modes for a given body shape is not only directly useful for solving radiation and scattering problems from these bodies, but it also provides great insight into the mechanism of such radiation. Hence, the characteristic mode concept is useful for investigating the antenna synthesis problems as well.

Chapters 4 and 5 of this book examine various computational aspects of the moment method. These chapter discuss the problems of numerical modeling, dependence of the numerical solution on the manner in which the integral equation is transformed into a matrix equation, questions of convergence, stability of matrix solution, difficulties at interior resonant frequencies, and so on. Various computational aspects which may affect the validity of a numerical solution for a thin wire structure are considered in Chapter 4 and some additional questions pertaining to numerical analysis are discussed in Chapter 5. Data based on extensive numerical experiments with the method of moments are presented in these chapters and some important conclusions are drawn on the basis of these results.

Although the two currently available texts [1.3, 4] serve as excellent background material for the discussion on numerical aspects of electromagnetics presented in Chapters 2 through 5 of this book, as mentioned earlier no such counterpart is available in the literature on the subject of asymptotic solutions in electromagnetics. Thus, an entire chapter, viz. Chapter 6, has been devoted to developing the background on ray techniques and bringing the reader up-to-date on the latest methods available for constructing ray-optical solutions of radiation and scattering problems. It is believed that this chapter fills an important need in providing the background and reference material, which is essential for following the proliferation of recent journal publications. This chapter also serves as an excellent introduction to Chapter 7 which deals with the topic of reflector antennas and which makes extensive use of ray optical techniques for computing the characteristics of reflector antennas that are typically very large in terms of the illuminating wavelengths.

It is a well recognized fact that the numerical techniques are limited in application to radiation and scattering from bodies that are not electrically large. This is because the precipitous growth of the matrix size and the associated processing time on the computer become prohibitive at smaller wavelengths, that is in the resonance region or above. In contrast, the ray-optical techniques which are high-frequency asymptotic solutions, obviously work best when the body size is large compared to the wavelength. Thus, in providing the analytical and numerical tools for solving the electromagnetic problems by approaching the spectrum from both ends, the book will hopefully serve the needs of a broad range of users who would obviously benefit from the wide array of techniques presented in the text. Also, it is hoped that the availability of a text such as this will spark future research interest in electromagnetics with the result that numerical and asymptotic electromagnetics will be brought even closer together, and perhaps a new arsenal of methodologies that bridge the gap between the low and high frequency techniques will emerge from this happy merger.

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2. Applications of the Method of Moments to Thin-Wire Elements and Arrays

W. A. IMBRIALE

With 32 Figures

The use of high-speed digital computers with recently developed numerical methods makes possible the solution of many electromagnetics problems of practical interest which, by classical techniques, would be virtually impossible. A major advantage of numerical techniques is that they may be applied to a body of arbitrary shape and are generally only limited by the wavelength size of the body. This limitation is practical rather than theoretical, in that a set of linear equations describing the system can be generated but this set may be too large to be solved in a convenient manner. A principle objective in solving a large electromagnetics problem is to obtain a minimum set of equations for a given degree of accuracy.

The purpose of this chapter is to present a practical method for solving antennas constructed with arrays of wire elements.

Wire antennas are solved using a moments solution where the method of subsectional basis is applied with both the expansion and testing functions being sinusoidal distributions. Using sinusoidal basis functions the terms of the generalized impedance matrix become equivalent to the mutual impedances between the subsectional elements. These basis functions are extremely useful for the analysis of large arrays of dipoles since the use of one subsegment per dipole is equivalent to the induced EMF method of calculating mutual impedances and gives a physically meaningful results. For an array of N dipoles this allows the minimum matrix size of $N \times N$ to achieve a good first-order approximation to the solution. Of the many applications discussed, this first-order approximation is shown to be an adequate representation of the solution.

Section 2.1 examines the numerical convergence of the moments solution using sinusoidal basis functions. Generally an equivalent radius is used in the evaluation of the self-impedance term to reduce computation time. It is shown that only for very thin subsegments is the correct equivalent radius independent of length and that the use of an incorrect self-term is responsible for the divergence of numerical solutions as the number of subsections in increased.

Section 2.2 discusses the solution of single and multiple Log-Periodic Dipole (LPD) antennas. The analysis is formulated in terms of impedance and admittance matrices for the dipole and transmission line networks. The first order approximation is shown to be equivalent to the original solution proposed by CARRELL [2.1] and is adequate for the practical range of operation of LPD antennas. In addition, the analysis for the detailed characteristics of the antenna over a wider range of operating conditions is provided by the complete moments solution. Also, the effects of feeder booms on the performance of multiple LPD antennas is examined.

Section 2.3 combines the method of moments for wire antennas with the physical optics scattering from reflectors to provide a description of LPD antennas as feeds for parabolic reflectors. Utilizing the numerical solutions, design curves for optimizing the gain of LPD fed reflectors are generated. In addition, a technique for reducing mutual coupling between multiple LPD feeds is presented.

2.1. Method of Moments Applied to Wire Antennas

The method of moments is applied to wire antennas, as discussed in other papers [2.2, 3], but carried to a higher order of approximation which allows treating the case where the length to radius ratio is small. The theory will address the straight wire antenna but the extension to wires of arbitrary shape is straightforward.

In the moments solution the method of subsectional basis is applied with both the expansion and testing functions being sinusoidal distributions [2.4]. This allows not only a simplification of near-field terms but also the far-field expression of the radiated field from each segment, regardless of its length. Using sinusoidal basis functions, the terms of the generalized impedance matrix are the mutual impedances between the subsectional elements and can be computed using the induced EMF method.

In the induced EMF method an equivalent radius is usually used in the evaluation of the self-impedance term to reduce computation time. It is shown, however, that only for very thin subsegments is the correct equivalent radius independent of length. When the radius to length ratio (a/L) is not small, an expansion for the equivalent radius in terms of a a/L is given for the self-impedance term. The use of incorrect self term, obtained by using a constant equivalent radius term, is shown to be responsible for divergence of numerical solutions as the number of subsections is increased. This occurrence is related to the ratio of a/L of the subsections and hence becomes a problem for moderately thick wire antennas even for a reasonably small number of subsegments per wavelength. Examples are given showing the convergence with the correct self terms and the divergence when only a length independent equivalent radius is used.

Sinusoidal basis functions are extremely useful for the analysis of large arrays of dipoles since the use of one subsegment per dipole is equivalent to the induced EMF method of calculating mutual impedances and gives a physically meaningful result. For an array of N dipoles this allows the use of the minimum matrix size of $N \times N$ to achieve a good "first-order" approximation to the solution. If n subsegments are required to characterize the behavior of each dipole in the array then the minimum matrix size required for a solution would be $n \cdot N \times n \cdot N$. For large N and moderate n this matrix size quickly exceeds the capability of most computer systems. Of the many applications discussed in later sections, the use of the "first-order" approximation is shown to be an adequate representation of the solution.

2.1.1. Basic Theory

Figure 2.1 shows a straight section of wire of circular cross section, and defines the coordinate system. The wire extends from $z=0$ to $z=L$ along the z-axis and is of radius a. It is assumed that the radius is small compared to a wavelength but the ratio of a to L need not be small. The only significant component of current on the wire is then the axial

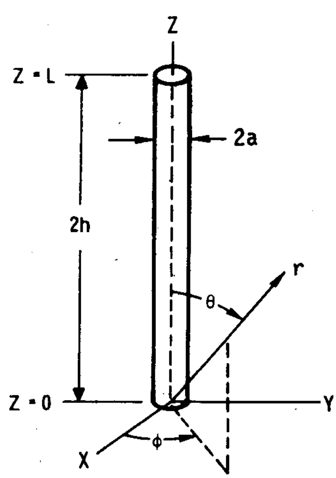

Fig. 2.1. Straight wire and coordinate system

component, which can be expressed in terms of the net current $I(z)$ at any point z along the wire. The current distribution will then be modeled as an infinitely thin sheet of current forming a tube of radius a, with the density of current independent of circumferential position on the tube.

An operator equation for the problem is given by

$$L\{I(z)\} = j(4\pi\omega\varepsilon)^{-1}(d^2/dz^2 + k^2)\oint_c \int_0^L [\exp(-jkR)/R] \cdot I(z')\,dz'\,dc = E_z^i(z), \tag{2.1}$$

where $E_z^i(z)$ is the z component of the impressed electrical field at the wire surface, $I(z')$ is surface current density, $\oint_c dc$ represents the integration around the circumference, and R is the distance from the source point to the field point.

The procedure is basically one for which the wire is divided into subsections, and a generalized impedance matrix $[Z]$ obtained to describe the electromagnetic interactions between subsections. The problem is thus reduced to a matrix one of the form

$$[Z][I] = [V], \tag{2.2}$$

where $[I]$ is related to the current on the subsections, and $[V]$ to the electromagnetic excitation of the subsection.

Matrix inversion is a simple procedure for high-speed digital computers, and hence the problem is considered solved once a well-conditioned matrix $[Z]$ is obtained. Of considerable importance is the ease and speed of evaluating the matrix elements and the realization of a well conditioned matrix $[Z]$.

The solution to be described uses sinusoidal subsectional currents and Galerkin's method [2.5], which is equivalent to the reaction concept [2.6], and the variational method [2.7]. Let the wire be broken up into N segments each of length $2H$ and let $I(z)$ be expanded in a series of sinusoidal functions

$$I(z) \approx \sum_{n=1}^{N-1} I_n S(z - nH), \tag{2.3}$$

where I_n are constants and

$$S(z) = \begin{cases} \sin k(H - |z|), & |z| < H \\ 0, & |z| > H. \end{cases} \tag{2.4}$$