中国科学院研究生教学丛书

极端顺序统计量的渐近理论

（第二版）

THE ASYMPTOTIC THEORY OF EXTREME ORDER STATISTICS (SECOND EDITION)

Janos Galambos

科学出版社 KRIEGER

影印版

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内 容 简 介

本书是中国科学院推荐的研究生原版教材之一．书中对极端顺序统计量的渐近理论作了详细介绍，尤其注重数学方法和结论在各实际领域中的应用，第二版还特别补充了该领域的一些最新发展．本书只要求读者具有微积分和概率论的知识，因此适合理工类各专业的研究生使用。

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《中国科学院研究生教学丛书》序

在21世纪曙光初露，中国科技、教育面临重大改革和蓬勃发展之际，《中国科学院研究生教学丛书》——这套凝聚了中国科学院新老科学家、研究生导师们多年心血的研究生教材面世了。相信这套丛书的出版，会在一定程度上缓解研究生教材不足的困难，对提高研究生教育质量起着积极的推动作用。

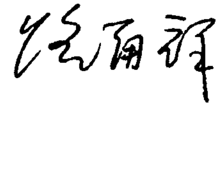
21世纪将是科学技术日新月异，迅猛发展的新世纪，科学技术将成为经济发展的最重要的资源和不竭的动力，成为经济和社会发展的首要推动力量。世界各国之间综合国力的竞争，实质上是科技实力的竞争。而一个国家科技实力的决定因素是它所拥有的科技人才的数量和质量。我国要想在21世纪顺利地实施“科教兴国”和“可持续发展”战略，实现邓小平同志规划的第三步战略目标——把我国建设成中等发达国家，关键在于培养造就一支数量宏大、素质优良、结构合理、有能力参与国际竞争与合作的科技大军。这是摆在我国高等教育面前的一项十分繁重而光荣的战略任务。

中国科学院作为我国自然科学与高新技术的综合研究与发展中心，在建院之初就明确了出成果出人才并举的办院宗旨，长期坚持走科研与教育相结合的道路，发挥了高级科技专家多、科研条件好、科研水平高的优势，结合科研工作，积极培养研究生；在出成果的同时，为国家培养了数以万计的研究生。当前，中国科学院正在按照江泽民同志关于中国科学院要努力建设好“三个基地”的指示，在建设具有国际先进水平的科学研究基地和促进高新技术产

业发展基地的同时，加强研究生教育，努力建设好高级人才培养基地，在肩负起发展我国科学技术及促进高新技术产业发展重任的同时，为国家源源不断地培养输送大批高级科技人才。

质量是研究生教育的生命，全面提高研究生培养质量是当前我国研究生教育的首要任务。研究生教材建设是提高研究生培养质量的一项重要的基础性工作。由于各种原因，目前我国研究生教材的建设滞后于研究生教育的发展。为了改变这种情况，中国科学院组织了一批在科学前沿工作，同时又具有相当教学经验的科学家撰写研究生教材，并以专项资金资助优秀的研究生教材的出版。希望通过数年努力，出版一套面向21世纪科技发展、体现中国科学院特色的高水平的研究生教学丛书。本丛书内容力求具有科学性、系统性和基础性，同时也兼顾前沿性，使阅读者不仅能获得相关学科的比较系统的科学基础知识，也能被引导进入当代科学研究的前沿。这套研究生教学丛书，不仅适合于在校研究生学习使用，也可以作为高校教师和专业研究人员工作和学习的参考书。

“桃李不言，下自成蹊。”我相信，通过中国科学院一批科学家的辛勤耕耘，《中国科学院研究生教学丛书》将成为我国研究生教育园地的一丛鲜花，也将似润物春雨，滋养莘莘学子的心田，把他们引向科学的殿堂，不仅为科学院，也为全国研究生教育的发展作出重要贡献。



Preface to the Second Edition

Since the first edition of this book there has been an active development of extreme value theory. This is reflected both in the added material, and in the increased number of references.

Substantial changes have been made on the material related to the von Mises conditions, on the estimates of the speed of convergence of distribution, and in Chapter 5 on multivariate extremes. In particular, James Pickands III's proof of the representation theorem is included with the kind permission of Pickands, for which I am grateful.

I am also indebted to R.H. Berk, R. Mathar, R. Mucci, J. Tiago de Oliveira and I. Weissman, who pointed out errors in the book, and to M. Falk, L. de Haan, R.D. Reiss, E. Seneta and W. Vervaat for their comments either on the original book or on the new material.

JANOS GALAMBOS

Willow Grove, Pa.

Preface

The asymptotic theory of extreme order statistics provides in some cases exact but in most cases approximate probabilistic models for random quantities when the extremes govern the laws of interest (strength of materials, floods, droughts, air pollution, failure of equipment, effects of food additives, etc.). Therefore, a complicated situation can be replaced by a comparatively simple asymptotic model if the basic conditions of the actual situation are compatible with the assumptions of the model. In the present book I describe all known asymptotic models. In addition to finding the asymptotic distributions, both univariate and multivariate, I also include results on the almost sure behavior of the extremes. Finally, random sample sizes are treated and a special random size, the so-called record times, is discussed in more detail. A short section of the last chapter dealing with extremal processes is more mathematical than the rest of the book and intended for the specialist only.

Let me stress a few points about the asymptotic theory of extremes. I have mentioned that an asymptotic model may sometimes lead to the exact stochastic description of a random phenomenon. Such cases occur when a random quantity can be expressed as the minimum or maximum of the quantities associated with an arbitrarily large subdivision (for example, the strength of a sheet of a metal is the minimum of the strengths of the pieces of the sheet). But whether a model is used as an exact solution or as an approximation, its basic assumptions decide whether it is applicable in a given problem. Therefore, if the conclusions for several models are the same, each model contributes to the theory by showing that those conclusions are applicable under different circumstances. One of the central problems of the theory is whether the use of a classical extreme value distribution is justified—that is, a distribution which can be obtained as the limit distribution of a properly normalized extreme of independent and identically distributed random variables. Several of the models of the book give an affirmative answer to this question. In several other cases, however, limiting distributions are obtained that do not belong to the three classical types. This is what Bayesian statisticians and reliability scientists expected all along (and they actually used these distributions without appealing to extreme value theory). I sincerely hope that these distributions will be widely used in other fields as well.

One more point that is not encountered in most cases of applied statistics comes up in the theory of extremes. Even if one can accept that the basic random variables are independent and identically distributed, one cannot make a decision on the population distribution by standard statistical methods (goodness of fit tests). I give an example (Example 2.6.3) where, by usual statistical methods, both normality and lognormality are acceptable but the decision in terms of extremes is significantly different depending on the actual choice of one of the two mentioned distributions. It follows that this choice has to be subjective (this is the reason for two groups coming to opposite conclusions, even though they had the same information).

The book is mathematically rigorous, but I have kept the applied scientist in mind both in the selection of the material and in the explanations of the mathematical conclusions through examples and remarks. These remarks and examples should also make the book more attractive as a graduate text. I hope, further, that the book will promote the existing theory among applied scientists by giving them access to results that were scattered in the mathematical literature. An additional aim was to bring the theory together for the specialists in the field. The survey of the literature at the end of each chapter and the extensive bibliography are for these purposes.

The prerequisites for reading the book are minimal; they do not go beyond basic calculus and basic probability theory. Some theorems of probability theory (including expectations or integrals), which I did not expect to have been covered in a basic course, are collected in Appendix I. The only exception is the last section in Chapter 6, which is intended mainly for the specialists. By the nature of the subject matter, some familiarity with statistics and distribution functions is an advantage, although I introduce all distributions used in the text. The book can be used as a graduate text in any department where probability theory or mathematical statistics are taught. It may also serve as a basis for a nonmathematical course on the subject, in which case most proofs could be dropped but their ideas presented through special cases (e.g., starting with a simple class of population distributions). In a course, or at a first reading, Chapter 4 can be skipped; Chapters 5 and 6 are not dependent on it.

No books now in print cover the materials of any of Chapters 1 or 3–6. The only overlap with existing monographs is Chapter 2, which is partially contained in the book by Gumbel (1958) and in the monograph of de Haan (1970) (see the references). It should be added, however, that Gumbel's book has an applied rather than theoretical orientation. His methods are not applicable when the restrictive assumptions of Chapter 2 are not valid.

Although many proofs are new here, the credit for the theory, as it is known at present, is due to those scientists whose contributions raised it to its current level. It is easy to unify and simplify proofs when the whole material is collected at one place.

I did not have time to thank the many scientists individually who responded to my requests and questions. My heartiest thanks go to them all over the world. My particular thanks are due to those scientists who apply extreme value theory and who so patiently discussed the problems with me either in person or in letters.

I am indebted to Professor David G. Kendall, whom I proudly count among my friends, for presenting my plan to John Wiley & Sons, Inc. I should also like to thank Mrs. Mittie Davis for her skill and care in typing the manuscript.

JANOS GALAMBOS

Willow Grove, Pennsylvania
March 1978

Notations and Conventions

$X_1, X_2, \ldots, X_n$	basic random variables.
Z_n	maximum of $X_1, X_2, \ldots, X_n$.
W_n	minimum of $X_1, X_2, \ldots, X_n$.
$F(x) = P(X < x)$	distribution function of X.
$H_n(x)$	distribution function of Z_n.
$L_n(x)$	distribution function of W_n.
$\alpha(F)$	$\inf\{x : F(x) > 0\}$.
$\omega(F)$	$\sup\{x : F(x) < 1\}$.
$t \to \omega(F)$	means $t < \omega(F)$ and $t \to \omega(F)$
$t \to \alpha(F)$	means $t > \alpha(F)$ and $t \to \alpha(F)$
$H(x)$	limit of $H_n(a_n + b_n x)$ with some constants a_n and $b_n > 0$.
$L(x)$	limit of $L_n(c_n + d_n x)$ with some constants c_n and $d_n > 0$.
$X_{r:n}$	the rth order statistic of $X_1, X_2, \ldots, X_n$. Thus $X_{1:n} \leqslant X_{2:n} \leqslant \ldots \leqslant X_{n:n}$ and $X_{1:n} = W_n$ and $X_{n:n} = Z_n$.
$\sum_{k=1}^{t}, \prod_{k=1}^{t}$	summation and product, respectively, from one to the integer part of t.
$H_{1,\gamma}(x)$	defined at (11) on p. 53.
$H_{2,\gamma}(x)$	defined at (13) on p. 53.
$H_{3,0}(x)$	defined at (18) on p. 54.
$L_{1,\gamma}(x)$	defined at (28) on p. 58.
$L_{2,\gamma}(x)$	defined at (30) on p. 58.
$L_{3,0}(x)$	defined at (35) on p. 59.
A' or A^c	the complement of the set A.
i.i.d.	independent and identically distributed.
i.o.	infinitely often.

Contents

CHAPTER 1

Introduction: Estimates in the Univariate Case

We shall describe a number of situations where the extremes govern the laws that interest us. Both practical and theoretical problems will be listed, which will then be unified by a general mathematical model. Our aim is to investigate this mathematical model and to describe our present stage of knowledge about it under different sets of assumptions. The beauty of this subject matter is that it leads us to the understanding of regularities of extreme behavior—an expression that seems to contradict itself.

After the introduction of the mathematical model, the present chapter is devoted to inequalities which involve the distribution of extremes in a set of observations. Those inequalities serve two purposes. On one hand, they may provide good bounds on the distribution of extremes for a given number of observations without resorting to an asymptotic theory. On the other hand, they will constitute some of the basic tools of the asymptotic theory to be developed in later chapters. It should be emphasized that in several situations, contrary to general belief, an asymptotic theory may provide the exact model, while a fixed number of observations can be used only as an approximation. The reader is referred to Section 1.2 for a specific example.

1.1. PROBLEMS LEADING TO EXTREME VALUES OF RANDOM VARIABLES

We now list a number of cases when a mathematical solution to the problems involved is in terms of the largest or the smallest "measurements."

Natural disasters. Floods, heavy rains, extreme temperatures, extreme atmospheric pressures, winds and other phenomena can cause extensive human and material loss, if society is unprepared for them. While such disasters cannot be completely avoided, communities can take preventive action to minimize their effects. In dams, dikes, canals, and other structures the choice of building materials and methods of architecture can take some of these disasters into account. Engineering decisions that confront such problems should be based on a very accurate theory, because inaccuracies can be very expensive. For example, dams built at a huge expense may not last long before collapsing.

Failure of a piece of equipment. Assume that a piece of equipment fails if one of its components fails. In other words, we consider only those of its components, the failure of any one of which leads to a halt in its operation. This is an extreme situation in the sense that the weakest component alone makes the equipment fail. While this assumption may seem a simplification, the most general failure model of a complicated piece of equipment can be reduced to this model. As a matter of fact, if one first considers groups of components where the failure of a group results in the failure of the equipment, then the weakest group of components with the assumed property effects the first failure of the equipment.

Service time. Consider a piece of equipment with large number of components and assume that components can be serviced concurrently. Then the time required for servicing the equipment is determined by that component which requires the longest service.

Corrosion. We say that a surface with a large number of small pits fails due to corrosion if any one of the pits penetrates through the thickness of the surface. Initially the pits are of random depths, which increase in time due to chemical corrosion. Again one extreme measurement, the deepest pit, causes failure.

Breaking strength. An absolutely homogeneous material would break under stress by a deterministic law. However, no material is absolutely homogeneous; indeed, engineering experience shows that the breaking strength of materials under identical production procedures varies widely. The explanation is that each point, or at least each small area, has a random strength, and thus varying amounts of force will be needed to break the material at different points. Evidently the weakest point will determine the strength of the whole material.

Air pollution. Air pollutant concentration is expressed in terms of proportion of a specific pollutant in the air. Concentrations are recorded at equal time intervals (present investigations are based on data obtained at five-minute intervals), and the aim of society is to keep the largest measurement below given limits.

Statistical samples. Observations are made on a given quantity; often one would like to know how large or how small a measurement can be expected.

Statistical estimators. After the collection of observations, the data are used to calculate estimators of certain characteristics of the quantity under observation. One would like to estimate these characteristics as accurately as possible, but over- or underestimation is unavoidable. Of considerable interest, therefore, is the investigation of the largest or the smallest estimator.

These problems, though they do not exhaust the possibilities, indicate that any successful theory of extremes unifies a great number of interesting topics. The theory to be developed can also show the beauty of mathematical abstraction: a single language will speak to the engineer, the physicist, the service person, the statistician, and others.

Further examples of fields for application of the theory will be spread in the text and among the problems for solution. Problems leading to multivariate extremes are postponed until Chapter 5.

1.2 THE MATHEMATICAL MODEL

In all the examples of the preceding section we were faced with a number n of random measurements $X_1, X_2, \dots, X_n$, and the behavior of either

$$Z_n = \max(X_1, X_2, \dots, X_n)$$

or

$$W_n = \min(X_1, X_2, \dots, X_n)$$

was of interest.

As a matter of fact, in terms of floods, X_j may denote the water level of a given river on day j, "day 1" being, for example, the day of publication of this book. Since we do not know the water levels in advance, they are random to us. A question such as, "How likely is it that in this century the water level of our river remains below 230 cm?" is evidently asking the value $P(Z_n \leq 230)$, the probability of the event $\{Z_n \leq 230\}$. Here n is, of course, the number of days remaining in this century after the publication of this book. On the other hand, if we want to use the river as a source of